Mathematics and the Arts

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TO TALK about art other than in the impersonal sense of history, is to talk about the moments when one has been confronted with beauty. Every essay on art that lights a hidden niche has its source in the life of the writer. You will then perhaps understand why I start with the mood of my childhood.

One hundred miles northeast of Derry, New Hampshire, lie the Belgrade Lakes, and out of the last and longest of these lakes flows the Messalonskee. I was born in its valley, "north of Boston" in the land of Robert Frost. The "Thawing Wind" was there, the "Snow," the "Birches," and the "Wall" that had to be mended: I was born on a sprawling farm cut by a pattern of brooks that went nowhere—and then somewhere. A hundred acres of triangles of timothy and clover, and twisted quadrilaterals of golden wire grass, good to look at, and good riddance. At ten I combed it all with horse and rake, while watching the traffic of mice beneath the horse's feet.

All that Frost has described was there—the meanness and generosity of men and women. A neighbor's house burned down in a wind, and everyone knew who held the grudge. The woman who must have killed her lover (so everyone thought) stood up in prayer meeting and testified, and there was no more judgment against her than was proper. The autumn winds were the prelude to the loon's strange song. There was time to think in the winter, to like some things better than others.

My mother's world was the world of music, and her world became mine. At thirteen I was playing the organ in church and wished the time in summer to study and practice. Somewhat reluctantly my father conceded me the mornings. He said that the grass was too wet to rake in the morning, and that I could walk the three miles from church to farm. And so I learned some of the Bach Fugues for the organ, and the moving Sonatas which Mendelssohn had written to honor the memory of Bach.

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Grecian art first became real to me in the shop of an old cabinet maker. I began to learn from him about cabinet making, and the history of his art. Sheraton chairs and tables were scattered about his shop, with their fluted columns and acanthus leaves. It came to me, all of a sudden, that *these* were fragments of Grecian temples.

Mathematics the Sister of the Arts

There was a copy of the *Cabinet Maker of Sheraton* in a remote library. It started with descriptive geometry and continued with a theory of ornaments. Cornices were constructed with ruler and compass; symmetry and perfection reigned throughout. Here was a meeting of mathematics and art, something final and universal, as it seemed to me then. It was very alive, because it was so new. But it was not mathematics as I know it today, and as it should be known; it was matter without the spirit. I made the same mistake that artists have made since the time of the Greeks, and placed mathematics alongside of the arts as their *handmaiden*. It is a humble and honorable position and very necessary; for one must begin with exactness in all the arts. But mathematics is the *sister*, as well as the *servant* of the arts and is touched with the same madness and genius. This must be known.

There was a German painter and engraver born in the fifteenth century with the name of Albrecht Dürer who wanted mathematics to be more than a handmaiden of art. His discontent on this account was unique among artists of all time. More completely than any other artist he formulated the rules of symmetry, perspective, and proportion, and used them in his art. But any one who thinks Dürer's spirit is bound by rules is mistaken. There is almost a shock in passing from his rugged, first engravings to the radiant classical beauty and slender proportions of his *Adam and Eve* of 1507.

Dürer was a creative mathematician as well as an artist. He wanted his geometric theories to measure up to his art. His great engraving *Melencolia I* is a psychological self-portrait. The perplexed and thoughtful heroine is the figure of geometry. Everything I have to say today is hidden in this engraving or may be derived from it by projection into the future. Let me quote from my colleague Erwin Panofsky.

The engraving *Melencolia I*, he says, ". . . typifies the artist of the Renaissance who respects practical skill, but longs all the more fervently for mathematical theory—who feels 'inspired' by celestial influences and eternal ideas, but suffers all the more deeply from his human frailty and intellectual finiteness . . . Dürer was an artist-geometer, and one who suffered from the very limitations of the discipline he loved. In his younger days, when he prepared the engraving *Adam and Eve*, he had hoped to capture absolute beauty by means of a ruler and a compass. Shortly before he composed the *Melencolia I* he was forced to admit: 'But what absolute beauty is, I know not. Nobody knows it except God.' "

In his dependence upon geometry Dürer was inspired by Leonardo but repudiated by Michelangelo. Later artists followed Dürer only half way or not at all; it is indeed hard to follow an inspiration. Leonardo himself had little of Dürer's divine discontent.

Back of Dürer and Leonardo in the distant past stands the Roman architect and geometer Vitruvius. The Mesopotamian artists also looked on geometry as an aid to art, and this was well known to the prophet Isaiah. Chapter 44 of Isaiah is written against idolatry; it is also an essay on aesthetics. The thirteenth verse reads: "The carpenter stretcheth out his rule; he marketh it out with a line; he fitteth it with planes, and he marketh it out with the compass, and maketh it after the figure of a man, according to the beauty of a man; that it may remain in the house."

Isaiah would minimize geometry in the arts, Dürer would maximize it. Neither Isaiah nor Dürer was content.

Let us turn to the relation between mathematics and music. The evolution of the scales, from the archaic sequences of tones of Euripedes to the whole tone scale of Debussy, shows that mathematics and music have much in common. And there is also the arithmetical basis for harmony. It is not too difficult to compose in the technical scheme of Debussy and thereby to get some of his naturalistic effects, but no one can explain the profound difference between the opera, *Péléas et Mélisande*, on the one hand, and *Tristan and Isolde*, on the other, by reference to whole tone scales or any other part of musical theory.

Geometric form imposed on music can have a null effect. As an example, I shall compare the First Prelude of Bach, as found in the *Well-Tempered Clavichord*, with the First Prelude of Chopin. The First Prelude of Bach is without melody, and consists of repeating ascending arpeggios with similar form and length. It is intended that the effect shall be harp-like. The musical text as a whole exhibits a design that appears in no one of the other forty-eight preludes. Looked at geometrically, the First Prelude of Chopin has a very similar geometric design, and if the Chopin prelude is played an octave higher than written, with perfect evenness of tone and tempo, the actual musical similarity of the two preludes is most striking. If, however, the Chopin prelude is played with the color and pulsating rhythm which it demands, all similarity to the Bach disappears.

Most convincing to me of the spiritual relations between mathematics and music, is my own very personal experience. Composing a little in an amateurish way, I get exactly the same elevation from a prelude that has come to me at the piano, as I do from a new idea that has come to me in mathematics.

Nature of Affinity

My thesis is prepared. It is that *the basic affinity between mathematics and the arts is psychological and spiritual and not metrical or geometrical.*

The first essential bond between mathematics and the arts is found in the fact that discovery in mathematics is not a matter of logic. It is rather the result of mysterious powers which no one understands, and in which the unconscious recognition of beauty must play an important part. Out of an infinity of designs a mathematician chooses one pattern for beauty's sake, and

Courtesy of The Art Institute of Chicago

Melencolia I (1514) ALBRECHT DÜRER

pulls it down to earth, no one knows how. Afterwards the logic of words and of forms sets the pattern right. Only then can one tell someone else. The first pattern remains in the shadows of the mind.

All this is like Robert Frost's "figure a poem makes." The poet writes: "I tell how there may be a better wildness of logic, than of inconsequence. But the logic is backward, in retrospect after the act. It must be more felt, than seen ahead like prophecy." Or again, "For me the initial delight is in the surprise of remembering something I didn't know I knew. I am in a place, in a situation, as if I had materialized from cloud, or risen out of the ground."

Compare this with the account of how the French mathematician Henri Poincaré came to make one of his greatest discoveries. While on a geologic excursion a mathematical idea came to him. As he says it came "without anything in my former thoughts seeming to have paved the way." He did not then have the time to follow up this idea. On returning from his geologic excursion he sought to verify the idea. He had no immediate success, and turned to certain other questions which interested him, and which seemed at the time to have no connection with the idea which he wished to verify. Here again he was unsuccessful. Disgusted with his failure he spent a few days at the seaside and thought of something else. One morning while walking on the bluff the final solution came to him with the same characteristics of brevity and suddenness as he had experienced on sensing the initial idea, and quite remarkably he had a sense of complete certainty. He made his great discovery.

An account of Gauss is similar. He tells how he came to establish a theorem which had baffled him for two years. Gauss writes: "Finally, two days ago, I succeeded, not on account of my painful efforts, but by the grace of God. Like a sudden flash of lightning, the riddle happened to be solved. I myself cannot say, what was the conducting thread, which connected what I previously knew, with what made my success possible."

These words of Frost, Poincaré, and Gauss show how much artists are in agreement as to the psychology of creation.

A second affinity between mathematicians and other artists lies in a psychological necessity under which both labor. Artists are distinguished from their fellows who are not artists by their overriding instinct of self-preservation as creators of art. This is not an economic urge as everyone knows who has a variety of artist friends. I shall illustrate this by the case of Johann Sebastian Bach and his son Philipp Emanuel.

Johann Sebastian's work culminates and closes a religious and musical epoch. It is inconceivable that Philipp Emanuel could have continued as a composer in the same sense as his father and have lived as an artist. He did in fact reject his father's musical canons. There is considerable evidence that his environment called for a new musical spirit. History justifies Philipp Emanuel; Mozart said of him, "He is the father, we are the children"; Haydn was inspired by him and Beethoven admired him. With all this to his credit, posterity can perhaps forgive him for calling his father an old wig.

Quite analogous to the son's turning away from his father is the story of the relation of mathematician Henri Poincaré to his younger colleague, Lebesgue. Poincaré had used the materials of the nineteenth-century mathematics to revolutionize much of mathematics. He had gone so far in mathematics that it is doubtful whether his younger colleagues in France could go on in the same sense without introducing essentially new techniques. This was in fact what several of them did. One of the new fields was what is called "set theory," and one of the innovators Lebesgue.

Poincaré criticized the members of the new school rather severely. It is on record that at a Congress in Rome he made this prediction. "Later generations will regard set theory as a malady from which one has recovered." (One may remark parenthetically that the history of art records many maladies from which art has recovered.)

The response of Lebesgue to Poincaré was given on his elevation to a Professorship at the Collège de France. An older eminent colleague had praised the school of Lebesgue. Lebesgue made public reference to the "precious encouragement which had largely compensated for the reproaches" which his school had had to suffer.

I regard the reactions of both Poincaré and Lebesgue as dictated by instincts of self-preservation, typical of the artist. Such self-preservation was clearly to the advantage of mathematics as well. I am also one of the few mathematicians who think that Poincaré as well as Lebesgue was right, in that mathematics will return more completely to the great ideas of Poincaré with full appreciation of the innovations of Lebesgue, but with a truer understanding of the relation of mathematical technique to mathematical art.

Before coming to the third type of evidence of the affinity of mathematics with the rest of the arts it might be well to ask what is it that a mathematician wants as an artist. I believe that he wishes merely to *understand* and to *create*. He wishes to understand, simply, if possible—but in any case to understand; and to create, beautifully, if possible—but in any case to create. The urge to understand is the urge to embrace the world as a unit, to be a man of integrity in the Latin meaning of the word. A world which values great works of art, music, poetry, or mathematics, can only approve and honor the urge of any man capable of such activities, to create.

The third type of evidence of the affinity of mathematics with the arts is found in the comparative history of the arts. The history of the arts is the history of recurring cycles and sharp antitheses. These antitheses set pure art against mixed art, restraint against lack of restraint, the transient against the permanent, the abstract against the nonabstract. These antitheses are found in all of the arts, including mathematics.

In particular the antithesis of pure art and mixed art is very much in evidence in the relations between poetry and music. There have been those who wished to keep poetry and music separate at all times. Plato took sides when he said, "Poetry is the Lord of the Lyre," and music had to fight a long battle to obtain complete autonomy.

Quite analogously in mathematics there are those who would like to keep algebra and geometry apart, or would like to subordinate one to the other. The battle became acute when the discovery of analytic geometry by Descartes made it finally possible to represent all geometry by algebra. The battle between algebra and geometry has been waged from antiquity to the present.

Grecian art was of course restrained and a departure from restraint has always brought a reaction. Berlioz gave an example of extreme lack of restraint. To get the maximum effect of Doomsday trumpets in *The Last Day of the World*, Berlioz devised four full-fledged brass bands to play high in the four corners of St. Peters. One American composer even wanted to fire cannon on the beat.

Mathematicians too, are often unrestrained. In this direction are the grandiose cosmologies with more generality than reality. These fantasies are sometimes based neither on nature or logic. Mathematicians of *today* are perhaps too exuberant in their desire to build new logical foundations for everything. Forever the foundation and never the cathedral. Logic is now so well understood that the laying of foundations is not very difficult. The thing has gone so far that one of my Polish colleagues recently suggested that the right to lay foundations should be rationed, or put on the basis of the right to build one foundation for every genuine classical effort.

The antithesis between logic and intuition manifested itself in the days of the Greeks. Pythagoras had a mystical preference for whole numbers. The irrational numbers were not understood by the Greeks and hence avoided as much as possible. History has made a full turn and the nineteenth century saw the meteoric rise of a more sophisticated Pythagoras by the name of Kronecker. Kronecker laid down the rule "all results of mathematical analysis must ultimately be expressible in properties of integers."

This proclamation cut deeply into the life and work of Kronecker's colleague Weierstrass. Here are a few lines from Weierstrass's reproach.

> But the worst of it is that Kronecker uses his authority to proclaim, that all those who up to now have labored to establish the theory of functions, are sinners before the Lord—truly it is sad, and it fills me with a bitter grief, to see a man, whose glory is without flaw, let himself be driven by the well-justified feeling of his own worth, to utterances whose injurious effect upon others he seems not to perceive.

The human documents which I have put before you are not concerned with processes which a machine can duplicate. One cannot decide between Kronecker and Weierstrass by a calculation. Were that the case, many of us would turn to another and truer art. As Dürer knew full well, there is a center and final substance in mathematics whose perfect beauty is rational, but rational "in retrospect." The discovery which comes before, those rare moments which elevate man, and the searchings of the heart which come after are not rational. They are gropings filled with wonder and sometimes sorrow.

Often, as I listen to students as they discuss art and science, I am startled to see that the "science" they speak of and the world of science in which I live are different things. The science that they speak of is the science of cold newsprint, the crater-marked logical core, the page that dares not be wrong, the monstrosity of machines, grotesque deifications of men who have dropped God, the small pieces of temples whose plans have been lost and are not desired, bids for power by the bribe of power secretly held and not understood. It is science without its penumbra or its radiance, science after birth, without intimation of immortality.

The creative scientist lives in "the wildness of logic" where reason is the handmaiden and not the master. I shun all monuments that are coldly legible. I prefer the world where the images turn their faces in every direction, like the masks of Picasso. It is the hour before the break of day when science turns in the womb, and, waiting, I am sorry that there is between us no sign and no language except by mirrors of necessity. I am grateful for the poets who suspect the twilight zone.

The more I study the interrelations of the arts the more I am convinced that every man is in part an artist. Certainly as an artist he shapes his own life, and moves and touches other lives. I believe that it is only as an artist that man knows reality. *Reality is what he loves, and if his love is lost it is his sorrow.*

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Author's Note

This essay was read at a conference in honor of Robert Frost in 1950 at Kenyon College. The subject of the Conference was "The Poet and Reality" and there were four speakers besides myself. Among these was L. A. Strong, a distinguished Irish poet and novelist. I single out this poet because it was certain sentences of his which caused me to rewrite the concluding paragraphs of my essay. Strong read the lines of one of his poems with great feeling and beauty of diction. When he had finished, he had occasion to refer to science, and I have no doubt he had in mind the opening of the atomic era. Placing his hand on his forehead he said, "Science is here," and then placing his hand upon his heart, he continued, "And poetry is here." The poet's words and gestures were pregnant with misunderstanding. Each scientist will have his own interpretation of what the speaker meant. I felt moved to rewrite much of what I had written, in the interest of science, and of poetry.

It is with some misgivings that I conceive of an essay originally intended for an audience of poets as now presented to an audience primarily of physicists. My doubts are somewhat lessened by a belief that many physicists, like mathematicians, are guided in the discovery and shaping of their theories by their sense of harmony and of beauty. I know, however, that there are some mathematicians and presumably some physicists, who do not feel any such guidance or at least do not regard the influence of the aesthetic in their groping for scientific law as important.

I do not understand such reluctance to admit the extra-empirical mental processes, particularly when one approaches such problems as those of particle physics, or of a reformulation of quantum mechanics. One cannot believe that the mathematical forms chosen to represent experimental fact are always uniquely determined by the empirical data. I shall illustrate this by referring to the bases of the general relativity theory. It is clear that this theory is much more flexible and more likely than the Newtonian theory, and that it is consistent with important experimental evidence. But admitting this, the form of the general relativity is not thereby uniquely determined. There is in the background of general relativity theory a tacit assumption that the paths of light which it is desired to represent correspond to a solution of the inverse problem of the calculus of variations, a problem which in general is known to have no solution, and which, with even less mathematical generality, has the geodesic solution which Einstein presupposes. The solution which Einstein has accepted may be a very good approximation to the observed physical universe for bounded space-time but at the same time may be infinitely in error when applied to a region of space-time which is unbounded; that is when applied to a cosmology which is not known to correspond to a closed and bounded universe.

There is here a tension between the aesthetically simple, and the mathematically general (sometimes less simple). One has to learn how properly to resolve this tension. But first of all one must admit the uncertainty and seek to understand it. To the extent to which there is a multiplicity of mathematical forms *a priori* available to express an empirically anchored physical law, to that extent one must call on further experimental evidence, or logic, or aesthetic judgment.

At stake is not only truth, but freedom. Such freedom of choice as exists must be acknowledged and comprehended, or else it is lost. In a mileu in which freedom of hypothesis is well understood the likelihood of intuitive discovery of high order will certainly be increased. The satisfactions of the physicist and the artist may be combined.—M. M.

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"The roads by which men arrive at their insights into celestial matters seem to me almost as worthy of wonder as those matters themselves."

—Johannes Kepler, tr. Arthur Koestler
(*Encounter,* December 1958)