— Introduction to Abstract Algebra II —

MAT 464, Taught by Jakob Streipel
Monday, Wednesday, Friday, 8:00–8:50 AM

Any sufficiently ~~advanced~~ abstract ~~technology~~ algebra is indistinguishable from magic

— Arthur C. Clarke (maybe)

In Abstract Algebra I, the focus was on the theory of groups, meaning (in some interpretation) the study of symmetries of objects. In another interpretation, it meant the study of sets with one operation (like $\times$) that behaved more or less nicely.

In Abstract Algebra II, the focus is on new objects called rings. These are sets with not one, not three, but two(!) operations (like $+$ and $\times$) that play well together. Think of the integers, or the real numbers, or the set of 2×2 matrices, for example.

In one direction, the goal is to understand what kind of structures can arise from this, and analyse what things we take for granted from e.g. the integers or the real numbers keep working, and what things fail. For example, if we encounter an equation like

$$ab = ac$$

we tend to want to "cancel" that a out… is this always a legal move? When is it a legal move, and how can we tell?

In another direction, algebra is in some way the study of how to express things. For example, we know from the Fundamental Theorem of Algebra (which is really an analysis result) that a degree $n \geq 1$ polynomial with complex coefficients has at least one complex root. Numerically, these roots are easy to approximate, but to find them "exactly" is more challenging. For quadratic polynomials it's not so hard, it just involves a sneaky square root, even for cubic or quartic ones it's doable. It turns out for a generic quintic, it cannot be done with sneaky square roots, and this is a result rooted deeply in algebra (and indeed one of the motivations for much of modern algebra's invention or discovery).

The prerequisite for this course is a grade of C or better in MAT 463 – Introduction to Abstract Algebra I.