University of Maine – Department of Mathematics & Statistics
MAT 527 – Functions of a Complex Variable I
Fall 2023

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Lecture: MWF 2:00-2:50pm in Neville Hall 421

Credits: 3

Prerequisites: Undergraduate real or complex analysis

This course is an introduction to complex analysis at the graduate level. I will assume some familiarity with undergraduate analysis (either real or complex), but I will develop the theory from basic principles. Complex analysis has many elegant results that make a much cleaner theory than real analysis; in particular:

- Under some reasonable assumptions, a function which is (continuously complex-)differentiable is infinitely (complex-)differentiable.
- A function which is (continuously complex-)differentiable is given by a power series around each point.
- A function is (continuously complex-)differentiable if and only if the integral of the function around any closed loop is zero.
- A bounded function which is (continuously complex-)differentiable on all of $\mathbb{C}$ must be constant.
- Many integrals can be evaluated by finding the coefficient of z^{-1} in the Laurent series expansion at the singularities.

Some things in complex analysis become strangely less nice; in particular:

- The logarithm is multi-valued.

Time permitting, we will cover (in some order):

- Sequential and functional limits, continuity and series, differentiability.
- Multivalued functions, the complex logarithm, analytic continuation.
- Holomorphy, the Cauchy-Riemann equations, Cauchy's theorem, Cauchy's integral formula, Morera's theorem.
- Equivalence of holomorphy and analyticity, isolated singularities and Laurent series, the Casorati-Weierstrass Theorem.
- Liouville's theorem, the Argument Principle and Rouché's theorem.
- Conformal mapping and the Riemann Mapping Theorem.