— Theory of Numbers —

MAT 465, Taught by Jakob Streipel
Monday, Wednesday, Friday, 10:00–10:50 AM

Some of the simplest objects we learn about as children — the whole numbers 1, 2, 3, 4, 5, and so on — remarkably contain within them a cornucopia of interesting, surprising, and sometimes remarkably difficult questions.

For example, we can try writing the numbers as sums of two squares:

$$1 = 1^2 + 0^2, \quad 2 = 1^2 + 1^2, \quad 3 = \ldots?, \quad 4 = 2^2 + 0^2,$$
$$5 = 2^2 + 1^2, \quad 6 = \ldots?, \quad 7 = \ldots?, \quad 8 = 2^2 + 2^2,$$

and so on. Is there a pattern to when we can and when we cannot do this? How does the answer change if we use three squares? Four?

Or: There are infinitely many prime numbers 2, 3, 5, 7, 11, 13, ..., but how many prime numbers are there up to 1,000? Up to 1,000,000? Up to X? Are there more or fewer than there are perfect squares? How many of the prime numbers differ by 2? What if you count only those that end with a 3?

There are wilder questions still which fall under this umbrella. Say, for instance, that you stand at the origin $(0, 0)$ of the xy-plane, and you plant a tree at every whole number point in the plane, $(1, 1), (1, 2), (2, 1), (2, 2), (3, 1)$, and so on and so forth. How many of those trees can you see from where you stand at the origin? In other words, how many trees aren't blocked from view by another tree from where you're standing?

Some of these questions have wonderfully beautiful answers, and some of them humanity has yet to answer in spite of centuries of combined effort.

In this course we will explore many of the tools required to study problems like these — and many others — and together discover some of the answers.

We will learn the language of modular arithmetic and explore precisely how whole numbers break down into products of prime numbers, we will use those ideas to answer the question about the two squares, we will learn the mathematics of how encryption keeps our data safe when we send it over the Internet, and how to find right angle triangles with whole number side lengths... and many more things besides!

The prerequisite for this course is a grade of C or better in MAT 261 – Introduction to Abstract Mathematics or permission from the department chair.