

MAT 261 HOMEWORK 1: DUE FRIDAY, SEPT. 14

General guidelines for HW in this class: You are most welcome to collaborate with each other on HW, and ask me for help. However, your write-ups must be done on your own, without looking at your collaborators' write-ups. If, on a problem, you are asked a question, you will get no credit for simply answering the question. You need to explain why your answer is correct. Use complete sentences, but at the same time be concise- every line you write should advance the argument.

- (1) Prove that there are infinitely many prime numbers. (Feel free to closely follow what we did in class!)
- (2) How many positive numbers p have the property that p and $p + 1$ are both prime?
- (3) Find all positive integers n for which $6^n - 1$ is prime.
- (4) Show that the polynomial $x^n - y^n$ can be factored.
- (5) Prove that if $n > 1$ is not prime, then $3^n - 2^n$ is not prime.
- (6) What can you say about $3^p - 2^p$ if p is prime? (Investigate with some data!)