

MAT 523 HOMEWORK 1: DUE FRIDAY, SEPT. 14

Do the following, but **only hand in problems 1,2,3,5,7**.

- (1) Show that \mathbf{C} (the field of complex numbers) is not an ordered field.
- (2) Problem 2 on page 10.
- (3) Problem 6 on page 11.
- (4) In \mathbf{R} , find the sup and inf of the following sets: $\{e^r \mid r \in \mathbf{Q}\}$, $\{\sin x \mid x \in \mathbf{Q}\}$, $\{2 - \frac{1}{n} \mid n \in \mathbf{N}\}$
- (5) Let $E \subset \mathbf{R}$ and suppose $g \in \mathbf{R}$ is an upper bound for E . Prove that g is the least upper bound of E if and only if for all $\epsilon > 0$ $E \cap [g - \epsilon, g] \neq \emptyset$, i.e., there exists $x \in E$ such that

$$g - \epsilon \leq x \leq g.$$

- (6) Let $E \subset \mathbf{R}$ and suppose $a > 0$. Prove that

$$\sup\{ax \mid x \in E\} = a \sup E.$$

- (7) Let A be a bounded subset of \mathbf{R} , and let $B \subset \mathbf{R}$ be the set of upper bounds for A . Show that $\sup(A) = \inf(B)$.