MAT 463: Abstract Algebra I

Spring 2025

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Time:	MWF 1PM–1:50PM	**Room:**	J102

Course description:

Abstract algebra is the study of the underlying structure of certain mathematical objects, most notably *groups*, *rings*, and *fields*. You've already seen examples of some of these in your previous classes. For instance, you've at some point encountered number systems (the integers $\mathbb{Z}$ and the real numbers $\mathbb{R}$) and polynomials over those number systems (the set of polynomials $\mathbb{Z}[x]$ in one variable x with integer coefficients). These are all examples of *rings*. Although $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{Z}[x]$ are different, they share some underlying structural properties.

In this course, we will focus on *groups*, which are relatively simple to describe but have a rich theory. Groups were developed in the 19th Century to solve classical problems in mathematics, such as proving that there is no closed form expression solving a polynomial of degree 5 or higher. The significance and utility of groups lies in their ability to encode the symmetry of other mathematical objects. For instance, they provide a conceptual framework for solving the Rubik's cube and Sudoku puzzles. In this course, we will build up the theory of groups, asking important questions along the way (what does it mean for two different groups to be essentially the same? How does a group *act* on another object?) and culminate in the beautiful Sylow theorems and their applications.

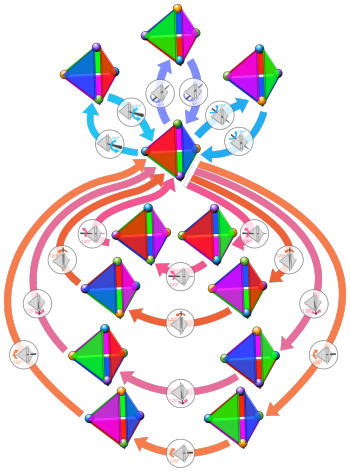

Figure 1: The symmetries of a tetrahedron via group theory