MAT 562 : Advanced Linear Algebra **Spring 2025**

Much of modern mathematics plays out against a backdrop of linear algebra:

- ☐ analysts study differential operators on vector spaces of functions
- ☐ geometers approximate small patches of curved spaces by flat vector spaces
- ☐ topologists use dimensions of vector spaces to count holes in surfaces
- ☐ applied mathematicians use linear approximation to find efficient solutions to many different kinds of complex problems.

Methods of linear algebra are also vitally important to users of mathematics: statisticians, physicists, chemists, engineers, computer scientists, biologists, data scientists, economists, and many others.

One reason why linear algebra is so broadly useful is that it blends the abstract with the concrete. First courses in linear algebra, like UMaine's MAT 262, tend to focus on the concrete aspect of computations with finite matrices. MAT 562, on the other hand, investigates the very flexible theoretical framework that allows those powerful computational methods to be brought to bear on problems in the diverse fields listed above.

Content: The course will begin by introducing the abstract notions of vector spaces and linear maps, in both finite and infinite dimensions. We will then proceed to study dual spaces, quotient spaces, tensor products, chain complexes and homology, determinants, and canonical forms of linear maps. Then, specialising to matrices over the complex numbers, we will study the spectral theorem and the singular-value decomposition, ending with some applications to optimisation problems.

Prerequisites: MAT 262 (linear algebra), MAT 425 (real analysis 1) and MAT 463 (abstract algebra 1). The most important prerequisites are a first course in linear algebra, and some experience with abstract mathematics and proofs.

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