1 Contact Information

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1.1 Office Hours

I’m in my office most days, so for short questions, feel free to drop by if my door is open. Directly after class is probably the best time to catch me. You can also email or phone me for short questions or to set up an appointment. Email is usually the simplest way to get hold of me even when I’m not in the office. For all distance students, I’m happy to meet via skype, please send an email to set up an appointment.

2 Objectives and Topics

2.1 Course Objectives

• Introduce students to a variety of mathematical formalism (formal languages, mathematical structures and logical systems) to represent information;
• Equip students with the basic toolset to study more advanced formalism from mathematics and theoretical computer science on their own;
• Enable students to formally write up their ideas in a clear and well-structured manner;
• Associate mathematical formalisms to problems encountered in the student’s own work or research.
2.2 Learning Outcomes

The goal of the course is to improve the mathematical literacy of the student. Every student in the course is expected to learn to

- independently read, comprehend, and explain mathematical formalisms and simple proofs (formal or informal) presented in reference books or scholarly publications;
- concisely present thoughts in an organized way using standard mathematical notation and structures as well as algorithms, both in writing and in speaking;
- relate the basic concepts of set theory, functions, relations, sequences, and graphs to common problems in computer science;
- relate algorithmic thinking (“while there are bottles left, take a cap and screw in on the next bottle”) and declarative/functional thinking (“put a cap on all bottles”);
- discuss the theoretical foundations of computer systems and their limitations.

2.3 Covered Topics

We will cover the following topics with a focus on mathematical foundations but also on their applications to information systems:

1. Basic (discrete) mathematical structures:
   - Finite and infinite sets and their operations, ordered structures
   - Relations, functions/operations, properties of relations and functions
   - Cardinality and countability of sets
   - Equivalence relations, partially ordered and linearly ordered relations
   - Graphs and trees and important graph algorithms
   - Algebraic structures (groups, Boolean algebras) and morphisms
   - Regular languages and finite automata

2. Basics of logic:
   - Logical languages, formal proof systems, logical equivalence, logical consequence
   - Semantics (validity, satisfiability), interpretations, models
   - Propositional tableaux and its correct and completeness
   - Informal introduction to the axiomatic method using select examples of axiomatic theories chosen from number systems (Peano arithmetic), axiomatic set theory (Zermelo-Fraenkel set theory), geometry (incidence structures, Euclidean and non-Euclidean geometries)

3. Techniques of mathematical induction: proof by induction, recursive/inductively defined sets, recursive functions

4. Kinds of informal proof methods: direct proof, proof by cases, proof by contradiction, disproof
The topics will not be covered in order. Instead we will start with basic structures and basic ideas of logic and work our way up to more complex structures and to more advanced logical ideas in an interweaved manner. A small selection of advanced topics are chosen based on the students’ interests.

### 2.4 Schedule

<table>
<thead>
<tr>
<th>Week</th>
<th>Topics</th>
<th>Primary Reading Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>09/01</td>
<td>Expectations, Geometry as Example, Informal Introduction to Mathematica Logic</td>
<td>[Gre94, 1]</td>
</tr>
<tr>
<td>09/08</td>
<td>LaTeX &amp; Informal Introduction to Discrete Structures (Sets, Functions, Relations) &amp; Induction</td>
<td>[End77, 1], [Smu14, 1-4]</td>
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<tr>
<td>09/15</td>
<td>Propositional Logic (Informally &amp; Formally)</td>
<td>[Vel94, 1], [Smu14, 5]</td>
</tr>
<tr>
<td>09/22</td>
<td>Predicate Logic (Informally &amp; Formally)</td>
<td>[Vel94, 2], [Smu14, 8]</td>
</tr>
<tr>
<td>09/29</td>
<td>Mathematical Proofs: Proof Strategies</td>
<td>[Vel94, 3]</td>
</tr>
<tr>
<td>10/06</td>
<td>Axiomatic Method (Incidence Geometry); Automated Proofs (Propositional Tableaux); Metatheory</td>
<td>[Gre94, 2], [Smu14, 6]</td>
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<tr>
<td>10/13</td>
<td>Sets &amp; Relations (Formally)</td>
<td>[End77, 2-3]</td>
</tr>
<tr>
<td>10/20</td>
<td>Relations &amp; Functions (Formally)</td>
<td>[Vel94, 4-5]</td>
</tr>
<tr>
<td>10/27</td>
<td>Mathematical Induction, Recursion</td>
<td>[Vel94, 6]</td>
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<tr>
<td>11/03</td>
<td>Recursively Defined Sets, Recursive Functions</td>
<td>[Hei09, 3.1-2]</td>
</tr>
<tr>
<td>11/10</td>
<td>Applications: Closures &amp; Natural Numbers</td>
<td>[End77, 4.1]</td>
</tr>
<tr>
<td>11/17</td>
<td>Applications: Cardinality &amp; Countability</td>
<td>[End77, 6.1-2], [Vel94, 7.1]</td>
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<tr>
<td>11/24</td>
<td>Applications: Languages &amp; Automata</td>
<td>[Hei09, 3.3, 11.1]</td>
</tr>
<tr>
<td>12/01</td>
<td>Graphs &amp; Trees</td>
<td>[Hei09, 1.4]</td>
</tr>
<tr>
<td>12/08</td>
<td>Algebraic Structures</td>
<td>[Sto73, 1.7-2.3], [Hei09, 10.1-2]</td>
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</tbody>
</table>


3 Course Materials

3.1 Required Books

[Smu14] and [Vel94] are very accessible introductions to the formal (“computational”) and informal (as in mathematical proofs) aspects of logic and discrete mathematical structures whereas [End77] is more concise (the formality of presentation that we ultimately aim to achieve) and shows the true elegance of discrete mathematics. Because large parts of [Smu14] and [Vel94] are required readings and both books are very reasonably priced, I advise you to buy them. For the required readings from [End77], I will provide electronic copies. I still highly recommend the book if you are interested in doing research in this area.


3.2 Additional Resources

I have selected chapters from a few additional books that cover certain aspects in more detail or from a different perspective. I will provide electronic copies of these assigned readings. They are taken from:


You may find it useful to consult these books for other material as well, since they supplement the assigned readings.

If you come across other books that you personally find useful, please consult me first on whether they are appropriate.

Wikipedia provides very detailed and accessible presentations of many mathematical structures and concept. Feel free to use it to help with your reading, but don’t use Wikipedia as replacement for your readings.
4 Expectations and Assessments

I understand that everybody’s background will be quite diverse, many of you having no previous experience with logic or discrete mathematics. While no specific technical background is required, I expect a willingness to work your way through complex and formal material. To properly understand the material, you may have to reread it multiple times or to consult additional sources. You will need to extensively engage with the course material outside of class. This being a graduate course, we will go over basics fairly quickly, so that you may have to do extensive additional readings on your own to keep up with the pace of the course. Of course, I’m willing to help and guide you in this process.

4.1 Grading

Your grade for the course will be calculated from the following components:

- Participation and preparation (discussion questions and summaries): 15%
- Course notes in LaTeX: 15%
- Labs throughout the term: 30%
- Class presentation, discussion, and write-up of an assigned topic: 40%
  - Presentation and discussion in class: 15%
  - Initial write-up in LaTeX: 10%
  - Revised write-up in LaTeX: 15%

4.2 Participation and preparation

Class attendance and participation in class discussions are expected and count towards this portion of your grade. If you are absent due to illness or another important reason, please email me immediately prior to or after your absence.

Each week, I will list readings for next week’s classes. While you are not expected to understand every detail, you are expected to read it to sufficient details so that you can follow and contribute to the class discussion. Occasionally, I will ask for specific preparations for class such as a write up of discussion questions or a summary/reflection of the readings.

4.3 Course notes

For every class, one or two students will be asked to take detailed notes about the material presented in class and typeset it in Latex before the next class. I will give you feedback on how to improve your writing/presentation and ask you to produce a second, revised version of the notes. Grading will be based on how concise, complete, correct, and coherent your first and your revised versions of the notes are. Together, these notes will form a set of course notes for everyone that you can refer back to.

While this may seem arduous, typesetting mathematics is a great way to learn the material and to learn presenting mathematical formalism in a concise and clear way. While the initial learning curve may be a little steep, you will have a better learning experience and, at the
end, also have a highly useful skill. I will give an introduction to Latex at the beginning of the course.

4.4 Labs

Throughout the term, I will give exercises to you to be worked on at home on your own. These include practical skills directly related to the understanding and the application of the course content. Some exemplary lab assignments are:

- Writing a formal theory in a logical language (first-order logic or similar) or analyzing such a theory
- Using an automated theorem prover to formally deduce or disprove a logical statement
- Constructing, critiquing, or explaining a mathematical proof
- Constructing a model for a logical theory
- Evaluating an algorithm
- Implementing and analyzing an algorithm or a procedure

Your work must be handed in electronically (via dropbox). All writing must be typeset in LaTeX.

Labs submitted late will be deducted 10% per day. Submissions will not be accepted for credit more than 7 days late. In case of extraordinary circumstances, please make special arrangements for submission before a lab is due.

4.5 Advanced topic

Each student will be assigned a chapter to be read in detail, complemented by readings of related work (on the chapter’s topic) of the student’s choosing.

The student must prepare a 15 min class presentation on the paper’s topic explaining key ideas, formalisms, and results (including theorems or proofs). The presentation is judged on how concise, complete, correct, and coherent the topic is presented and to what extent suitable related material is included.

Additionally, each student must write a short paper (about 8-10 pages using the LaTeX article class) on the assigned topic. This paper should explore some questions or ideas presented in the assigned paper (such as a specific structure, proof, theorem, or method) in more detail, using other relevant sources. The student is free to choose which direction of the paper’s topic to pursue further, but must consult the instructor first to discuss the direction. The student’s paper must include a summary, discussion, and critique of the assigned paper. The final revised papers are due during the last week of class. Papers will be graded on how concise, complete, correct, and coherent the assigned paper is presented and on the discussion of the assigned material and related work (how relevant is it?, are the key ideas identified?, are the discussed questions/criticisms well laid out?).
5 Academic Honesty

Academic honesty is expected. Plagiarism—one form of academic dishonesty—is the handing in of work not substantially the student’s own. It is usually done without reference, but is unacceptable even in the guise of acknowledged copying. It is not cheating, however, to discuss ideas and approaches to a problem, nor is it cheating to seek or accept help with a program or with writing a paper. Indeed, a moderate form of collaboration is encouraged as a useful part of any educational process. Nevertheless, good judgement must be used, and students are expected to present the results of their own thinking and writing. Plagiarism is unacceptable in this course and will result in a failing grade.

6 Students with disabilities

If you have a disability for which you may be requesting an accommodation, please contact either me or Ann Smith, Director of Disabilities Services, 121 East Annex, 581-2319, as early as possible in the term.

7 Extended disruption

In the event of an extended disruption of normal classroom activities, the format for this course may be modified to enable its completion within its programmed time frame. In that event, you will be provided an addendum to the syllabus that will supersede this version.