Making Sense of How Students Come to an Understanding of Physics: An Example from Mechanical Waves

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ABSTRACT

While physics education research (PER) has traditionally focused on introductory physics, little work has been done to organize and develop a model of how student come to make sense of the material they learn. By understanding how students build their knowledge of a specific topic, we can develop effective instructional materials. In this dissertation, I describe an investigation of student understanding of mechanical and sound waves, how we organize our findings, and how our results lead to the development of curriculum materials used in the classroom.

The physics of mechanical and sound waves at the introductory level (using the small-amplitude approximation in a dispersionless system) involves fundamental concepts that are difficult for many students. These include: distinguishing between medium properties and boundary conditions, recognizing local phenomena (e.g. superposition) in extended systems, using mathematical functions of two variables, and interpreting and applying the mathematics of waves in a variety of settings. Student understanding of these topics is described in the context of wave propagation, superposition, use of mathematics, and other topics. Investigations were carried out using the common tools of PER, including free response, multiple-choice, multiple-response, and semi-guided individual interview questions.

Student reasoning is described in terms of primitives generally used to simplify reasoning about complicated topics. I introduce a previously undocumented primitive, the object as point primitive. We organize student descriptions of wave physics around the idea of patterns of associations that use common primitive elements of reasoning. We can describe students as if they make an analogy toward Newtonian particle physics. The analogy guides students toward describing a wave as if it were a point particle described by certain unique parts of the wave. A diagnostic test has been developed to probe the dynamics of student reasoning during the course of instruction.

We have replaced traditional recitation instruction with curriculum materials designed to help students come to a more complete and appropriate understanding of wave physics. We find that the research-based instructional materials are more effective than the traditional lecture setting in helping students apply appropriate reasoning elements to the physics of waves.

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Introduction

Investigations of student difficulties with physics are growing in number and sophistication. As researchers gain deeper insight into student understanding of the material taught in the classroom, they are able to create curriculum materials that are more effective in improving a student's actual understanding. At the same time, growing understanding of student reasoning provides the opportunity to find more systematic descriptions of how students come to make sense of the physics. By evaluating student performance in a modified curriculum setting, researchers can then develop an understanding of not only the curriculum's effects on student learning, but also the manner in which a curriculum *can* affect student learning.

There are many goals when investigating student understanding of physics. As researchers, we aim to recognize and "diagnose" specific difficulties while also finding the most common difficulties related to a specific topic. We try to help individual students more effectively while also creating curriculum that helps the highest number of students overcome the most common difficulties. The general goal is to help students understand what it means to understand physics.

In this dissertation, I will discuss the above ideas in the context of student difficulties with the physics of mechanical waves. The physics of mechanical waves is common to most introductory physics curricula at the university level and provides many interesting topics in which to discuss how students come to an understanding of physics. My investigations have taken place at the University of Maryland (UMd) with engineering students taking a required three-semester sequence of physics classes. The students discussed in this dissertation were in the second semester of the sequence.

In this chapter, I summarize the dissertation by giving an overview of the issues that affect the discussion of physics education research on student understanding of mechanical waves. Rather than discussing the contents of each chapter in order of its appearance, I will describe some of the issues that play a role in the dissertation while pointing out where a discussion of these issues can be found.

Physics Education Research

The field of Physics Education Research (PER) has come about in reaction to the growing need for innovative methods in education that address student difficulties with the difficult material they are required to learn in our physics classrooms. PER involves

- investigations of specific aspects of student understanding of the physics,
- the development of investigative probes to help fulfill this goal,
- the development of statistical methods that help researchers organize, analyze, and present their findings,

- the design and implementation of curriculum materials that provide a more effective learning environment for students, and
- evaluations of the effectiveness of these materials.

In chapter 2, I give an overview of PER by summarizing work done by some of the leaders in the field. In the course of the analysis, I discuss different methods of analysis that have been used, including focusing both on specific aspects of understanding only and on broader descriptions of common student difficulties. In addition, I discuss how PER can lead to curriculum materials that can be demonstrated to be effective in addressing student difficulties with the physics. The remainder of the dissertation presents my own work, done as part of the Physics Education Research Group (PERG) at UMd, which involved all the aspects of PER as described above.

Wave Physics

In order to discuss student understanding of wave physics, it is necessary to describe in detail what it is that we want them to actually learn in our classes. A mechanical wave is a propagating disturbance to a system, such as a wave traveling on a long, taut string. At the introductory level, we teach students a very simplified model of waves. In this model, there are no large amplitude waves and there is no dispersion in the system. Any disturbance will propagate indefinitely. Mathematically, these traveling waves can be described by functions of the form $f(x \pm vt)$, where f is any function that describes the displacement of the mechanical wave from equilibrium.

In chapter 2, I discuss the generally accepted model of waves that is taught at the introductory level. Many of the ideas of wave physics are subtle and rarely addressed explicitly in textbooks. For example, an understanding of waves requires an understanding of the role of initial conditions to help create a wave, though the initial conditions do not affect the manner of the wave's propagation through the system. Also, propagation occurs due to local interactions between "nearest neighbors" in the system. Student understanding of the distinctions between local phenomena and global phenomena plays a central role in this dissertation.

After the discussion of the physics, I discuss previous research by summarizing the literature of investigations into student difficulties with wave physics. Very few studies have been published, and the common themes among those that are available suggests that wave physics is a rich topic for investigating more general patterns of reasoning that we find in students.

Student Difficulties with Wave Physics

The wave physics topics discussed in this dissertation include

- wave propagation speed (and how to change it),
- wave superposition (point-by-point addition of displacement),
- the mathematical description of waves (the $f(x \pm vt)$ dependence), and
- the physics of propagating sound waves.

In addition, other topics play a role, such as wave reflection, though these have not been investigated in as great detail.

The specific wave physics topics serve as a context in which to discuss concepts and ideas that are more general to physics as a whole. These general ideas (displacement from equilibrium, for example, or the role of initial conditions in describing the dynamics of a system) build on concepts that students have encountered in their previous mechanics classes and also play a role in students' future studies. As a result, the discussion of student difficulties in chapter 3 provides a context in which to discuss how students build on the knowledge that they bring into the classroom.

Organizing Student Difficulties

In addition to investigating student difficulties with specific physics concepts, we must also try to find ways to organize, explain, and discuss systematically how students are coming to their understanding of the material. An extensive literature has grown in the fields of education and cognitive studies in which these issues are addressed. In chapter 4, I give a summary of some of these ideas that help account for some of the difficulties we see students having with wave physics. Each of the different cognitive concepts that I discuss is presented with a typical example of how student reasoning has been interpreted in physics through the use of these different ideas. By presenting these cognitive ideas, I suggest a model of learning that is applicable to describing student difficulties with waves.

In chapter 5, I apply the proposed model of learning to the specific student difficulties first presented in chapter 3. We describe student reasoning in terms of fundamental and very simple ideas that are consistently and generally applied to many different situations. These *reasoning primitives* are generally applicable to many different situations in many different (possibly non-physics) settings. In physics, the same primitive may be applied in contradictory ways to a single physical situation. We refer to the application of a primitive in a context as a facet of reasoning.

In chapter 5, I discuss a primitive not previously presented in the literature, the *object as point* primitive. Students often apply this and other useful and reasonable primitives (helpful in mechanics, for example) inappropriately when thinking about the topic of wave physics. This provides us with a context in which to discuss different aspects of student reasoning. On the one hand, it is encouraging to see students trying to make sense of new topics in terms of the material that they have learned in previous semesters (though well-researched difficulties from mechanics arise again). On the other hand, we find that students do not have the ability to determine whether the primitives that they apply to a situation are appropriate in that setting.

Furthermore, many students seem to inappropriately use a set of primitives in conjunction with each other. We describe this as a pattern of association, meaning that students seem to use more than one primitive inappropriately to describe a single topic, but we do not claim that this is a robust model that students have in their head. The pattern of association may serve to help guide a student's choice of primitives in a given context. We refer to the pattern of association that many students use when discussing the physics of mechanical waves as the *Particle Pulses Pattern of*

Association, because student responses indicate a simplification of finite length waves to single points (rather than extended regions of displacement). In addition, students show great difficulties reasoning about force and motion with waves.

Designing Curriculum to Address Student Needs

To help students in their learning of physics, we have begun to develop and implement a set of instructional materials that are designed to address specific student difficulties with the material. These materials, known as *tutorials*, are based on a design by the University of Washington (UW) Physics Education Group, under the guidance of Lillian McDermott. In tutorials, students participate in a process in which we

- *elicit* their difficulties, through questions that ask for their predictions or descriptions of a physical situation,
- *confront* students with a weak understanding of the material with evidence to show that their predictions were incorrect, and
- help students *resolve* their difficulties through guided questions and activities designed to let the student build their own robust understanding of the material.

The process of the development of research-based curriculum materials is described in chapter 2 in the context of a UW tutorial. Tutorials developed at UMd to address student difficulties with wave physics are described in chapter 6. This description includes a summary of how the computerized videos used in the instructional materials were created. In addition, in chapter 6 data are presented to indicate how effective the curriculum materials have been in addressing student difficulties. We have compared student performance from the beginning of an instructional sequence (pre-instruction on waves), the middle of the instructional sequence (after lecture instruction, but before tutorial instruction), and the end of instruction (after all instruction in the class has been completed). We find that student performance improves greatly as a result of research-based curriculum materials.

Investigating the Dynamics of Student Reasoning

To better describe how student reasoning about the physics changes over the course of instruction, we have developed a diagnostic test that allows us to gain deeper insight into student reasoning on many different aspects of the physics at once. By using many questions that ask about the same topic, we are able to see the extent to which students reason consistently about the physics. By using the same questions before and after instruction, we are able to compare the development of student reasoning as a result of different instructional materials.

We find that students are not consistent in their reasoning. We can describe two patterns that students use to guide their reasoning: the Particle Pulses Pattern of Association, mentioned above, and the community consensus model of waves. Through the use of a short diagnostic test, we have been able to describe the dynamics of student reasoning as moving from a primarily incorrect application of otherwise useful and reasonable primitives to a state where they use both types of reasoning.

The implications for instruction in physics are that students leave our classes with an incomplete understanding of when to use which reasoning while thinking about the physics. I discuss the dynamics of student reasoning in chapter 7 of this dissertation.

Chapter 1: The Need for Systematic Investigation of Student Understanding of Physics

Introduction

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Chapter 2: Review of Previous Research

Introduction

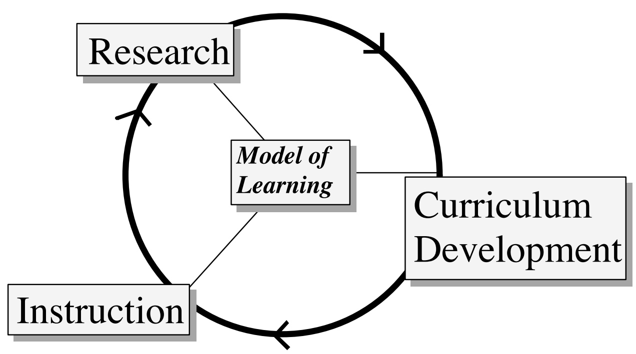
Work in physics education research (PER) seeks to understand how students come to understand physics and, as a result, how better to teach them. This work includes the development of relevant measurement techniques that investigate student reasoning in physics, the design of effective curriculum materials that address student difficulties, and the development of techniques and statistical methods to describe and organize our understanding of student difficulties. Research into student understanding forms a required basis for curriculum development. Instruction consists of the implementation of curriculum materials. The cycle of research used by the Physics Education Research Group (PERG) at the University of Maryland (UMd) is shown in Figure 2-1.[1] We believe that a model of learning can inform the research cycle at all of its stages.

Because a knowledge of the research methods of PER is important for an understanding of the later discussion, I begin this chapter with a brief overview of the different methods used to investigate student knowledge in physics. I then describe previous PER results which are relevant to my work, including previous investigations into how students make sense of introductory physics materials, student difficulties with wave physics, and the development of research-based curriculum materials.

Research Methods

Research results in PER depend on a rigorous and repeatable methodology that effectively probes student understanding of physics. A variety of methods has been developed to investigate student ideas, abilities, and concepts. These include: individual demonstration interviews, written questions on quizzes or exams, and specially designed diagnostic tests. When multiple research methods are used in conjunction, the researcher is able to gain deeper insight into students' reasoning patterns, providing detailed knowledge that can be used to develop more effective curriculum.

Figure 2-1



The iterative cycle of research, development, and instruction, centered around an understanding of student models of learning.

Observations of students often start through informal observations during lecture, office hours, help sessions, or discussion sections. Student comments may raise the interest of the researcher, or show where many students are having common problems with the material. Most often, informal observations are made in a setting where the goal is to help students arrive at the right answer or, in a sense, certify that they have stated the correct answer.

To describe how students approach physics, we must approach them with a different investigative method that does not attempt to teach them but rather gives us insight into their understanding. This requires that we go beyond trying to help students immediately and instead listen to what they are saying and doing. Observations of their understanding can come from listening to their descriptions of physical situations, asking them to explain their reasoning in solving problems, or continuing a series of questions that follow the thread of a concept that students seem to have difficulty with.

We call these investigations interviews. We ask for student volunteers who are then videotaped while answering questions in a one-on-one setting for approximately 45 minutes. The students are usually getting A's and B's in their physics classes. (We have found that weaker students are usually shyer about presenting their understanding of the physics.) In demonstration interviews, a researcher presents questions about a demonstration apparatus or situation to a single student. The researcher probes the student's understanding by following up on the student's predictions of the physical behavior of the system or by asking for clarifications of the student's descriptions of the physics. In problem interviews, a student solves (one or more) problems while the researcher asks questions that help elaborate the student's understanding of physics, reasoning methods, and the manner in which the student approaches the problem.

What sets interviews apart from informal observations is that the researcher can ask further, unscripted questions that probe deeper into student responses while flexibly adapting to the student's responses and not certifying or teaching the correct response. The power of the individual demonstration or problem solving interview lies in the fact that the researcher has chosen the context which the student must describe. By listening to many students describing the same physical situation, it is possible to compare their results and gain deeper insight into the common difficulties they are having. Interview data are used as the basis for coming to an understanding of students' reasoning processes and knowledge, forming a "state space" of possible student responses for other research investigations.

Interview videotapes must be transcribed, a time-consuming process. The transcript and the actual video of the interview are used to analyze student understanding, reasoning, and performance in the interview. Transcripts should be read and analyzed by multiple researchers so that personal bias of a single researcher does not skew the results.[2] Due to the amount of time required to carry out a large number of interviews (and often, the lack of students willing to volunteer to take part in this aspect of the research), interviews are rarely used as the sole source of data. The detailed student interview responses are used to help make sense of other data that is more easily collected.

Using written questions allows data to be gathered from many more students. Questions can be asked on examinations, in quizzes, in homework sets, or on pretests (an aspect of tutorials, which will be described below). Students are asked to answer a question or solve a problem and explain how they arrived at their answer. Student explanations on written questions are not as detailed as those which can be found through the use of interviews, but they help the researcher make a connection between student solutions to the written question and other students' explanations in interviews. The state space of student difficulties that was developed through interviews can help in interpreting student written responses.

I have used two types of written questions in my research. The first and most common is the free response (FR) question. Students are given an open-ended question and asked to give a response they believe is correct. We have found that students often do not give *all* answers they believe are correct, leading us to believe that the responses they give are filtered in some fashion. The second type of question is the multiple-choice, multiple-response (MCMR) question. Students are given a multiple-choice question with a long list of possible responses and asked to give *all* responses that they believe are correct. Individual students tend to use more explanations and give more responses on MCMR questions than on FR questions. It seems that the many offered responses trigger students into giving more of the explanations that they believe are correct than the free response leads them to. (I will address the issue of filtering and triggering in more detail in Chapters 3 and 5 of this dissertation.)

Some student learning takes place when answering a physics question. Students who have participated in interviews may do better on written questions than those students who have not participated in interviews. As a result, we usually try to use different students in the same class or students from separate but identical classes (or sections) for interviews and for written questioning. At times, we have carried out interviews on students who have previously answered written questions on a physical topic. Most commonly, we do this to see if they are answering the question consistently in both settings. We use these interviews to better understand the links between the common written questions and common interview explanations.

In summary, using both written and interview questioning of students on a single topic gives a rich understanding of how students approach the physics of that topic. The purpose of interviews in PER is to gain deeper insight into student responses by providing the opportunity for deeper questioning through follow-up questions asked in response to student comments. The purpose of written questions such as the diagnostic test is to help researchers gain a better statistical overview of the distribution of student responses as understood from interviews.

Common Sense Physics

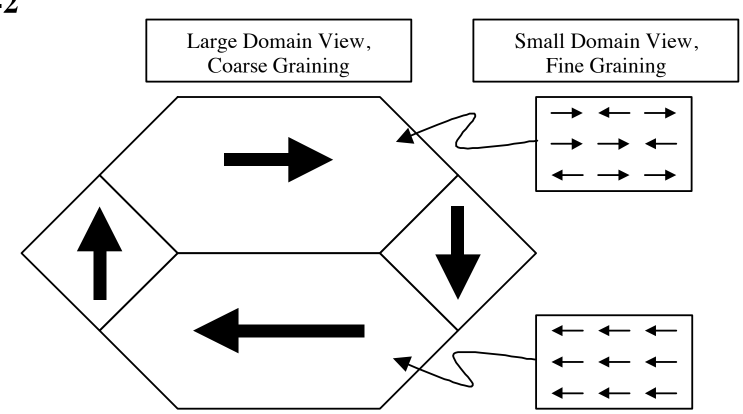
Previous PER has shown that students bring a common sense understanding of the world around them to their study of physics.[3] Many studies have investigated student understanding of specific topics to illustrate how common sense reasoning plays a role in how students come to understand physics. The studies summarized below take two different positions about the manner in which we should pay attention to student difficulties. The first position states that PER should pay attention to specific difficulties that students have and try to address these specific and profound difficulties such that students improve their understanding of fundamental physics ideas. The second position states that PER should focus on a broader understanding of student difficulties in terms of the general types of reasoning that they use to describe a large set of phenomena. In other words, the two positions differ on the issue of the domain size of the analysis.

By domain size, we mean the realm in which it is fruitful and meaningful to study student understanding of the physics. We use the term in an analogy with the physics of ferromagnetic materials. When considering a model of ferromagnets in terms of aligned atomic spins within the material, it is often found that regions of the material may have, on average, aligned spins (and therefore be magnetic) while the whole system is, on average, only weakly aligned (and barely magnetic). Two possible domain sizes with which one can study ferromagnets are at the level of the individual spins (fine graining) or at the level of the larger, aligned domains (coarse graining) (see Figure 2-2). Presently in PER, the investigation of student difficulties focuses primarily on individual difficulties and not on sets of difficulties that involve more general reasoning patterns.[4] The domain size of these investigations is at the "spin" (fine grain) level in the sense that only specific pieces of student knowledge are investigated, while the self-organized, aligned domains (course grain) are not investigated.

One goal of this dissertation is to show that an analysis at a larger domain size is productive and relevant to a study of student understanding of physics. Furthermore, there are connections between the different grain sizes used to investigate and describe student difficulties with physics.

As an example of a small domain investigation in PER, consider early work done by Clement.[5] As part of a larger investigation of student's understanding of force and motion, Clement investigated student descriptions of the forces acting on a coin tossed vertically in the air (see Figure 2-3). Students were given a brief description of

Figure 2-2



Blocking of spin states in a ferromagnet as an analogy to describe levels of analysis possible in a system. In the large domain view, a coarse graining creates an average over the system, while in the small domain view, each individual element of the system is considered and described.

each system and shown a diagram. They were asked to sketch vectors to show the direction of the force acting on the coin at different points in its trajectory.

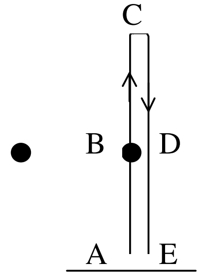
Clement studied the responses of a calculus-based engineering physics class on interviews and on a diagnostic test, both given before and after instruction. Before instruction, 34 students (group 1) participated in the research, while 43 different students (group 2) participated after they had received physics instruction. Clement points out that group 2 consisted of paid volunteers whose grades were all far above the course mean. A further 37 students (group 3) from another institution answered the question after having taken two semesters of physics. Eleven members of group 1 were interviewed. No members of the other groups were interviewed.

Before instruction, only 12% of the students described the forces on the coin correctly. After instruction, only 28% of group 2 and 30% of group 3 answered correctly. Clement states that, before instruction, "virtually all (90%) of the errors … involved showing an arrow labeled as a force pointing upwards" when the coin was on its upward path. Similar results were found in the post-instruction data.

Clement found that many students have what he called the "'motion implies a force' misconception." Evidence of this was found in student interview comments about the coin problem, done before instruction. Clement quotes students using phrases such as: "the 'force of the throw,' the 'upward original force,' the 'applied force,' the 'force that I'm giving it,' 'velocity is pulling upwards, so you have a net force in this direction [points upwards],' 'the force up from velocity,' and 'the force of throwing the coin up.'" Students had difficulty separating the motion of the coin in one direction from a force acting in another direction. Students had similar difficulties even after instruction. Clement states that "most errors are not due to random mistakes but rather are based on a stable misconception that is shared by many individuals." He adds that "the data support the hypothesis that for the majority of … students, the 'motion implies a force' preconception was highly resistant to change."

In focusing on the "motion implies force" misconception, Clement illustrates the small domain size of his research. He summarizes his findings in three comments. First, "continuing motion, even at a constant velocity, can trigger an assumption of the presence of a force in the direction of motion that acts on the object to cause the

Figure 2-3



A coin is tossed from point A straight up into the air and caught at point E. On the dot to the left of the drawing, draw one or more arrows showing the direction of each force acting on the coin when it is at point B. (Draw longer arrows for larger forces

The Clement coin toss problem. A correct answer would show only one force acting on the coin when at point B (the force of gravity, pointing downward). The most common student response was to include a force that pointed in the direction of the motion, often described as the "force of the throw."

motion." In other words, students will invent forces that point in the direction of motion. Second, "such invented forces are especially common in explanations of motion that continues in the face of an obvious opposing force. In this case the object is assumed to continue to move because the invented force is greater than the opposing force." Finally, students "may believe that such a force 'dies out' or 'builds up' to account for changes in an object's speed."

Whereas Clement and others focus on a small domain size of descriptions of student difficulties, Halloun and Hestenes[6] choose a larger domain size with which to analyze student difficulties. They speak of the need for "a more systematic and complete taxonomy of CS (common sense) beliefs" that goes beyond an identification of "specific CS beliefs" (i.e. specific student difficulties). Halloun and Hestenes investigate student understanding of Newtonian particle mechanics. Their study used data gathered from 478 university physics (calculus-based) students. These students answered a pre-instruction and post-instruction diagnostic test. From this population, 22 students were interviewed within a month of having taken the pre-instruction test. The analysis that follows is based on pre-instruction results. In a separate paper, the authors show that overall student scores on the diagnostic (as measured by correct responses) do not change very much over the course of a semester (from 51% to 64%).

Halloun and Hestenes state that "each student entering a first course in physics possesses a system of beliefs and intuitions about physical phenomena derived from extensive personal experience. This system functions as a *common sense theory* of the physical world which the student uses to interpret his experience." The authors use three descriptions for the most common student responses: Newtonian physics, Aristotelian physics, and impetus physics.

A single physical situation, such as Clement's coin toss (used in modified form by Halloun and Hestenes), can be described using all three theories. A Newtonian response would describe a constant force being exerted downward on the coin, causing an acceleration which causes a change in velocity such that the coin slows and reverses direction. As an example of an Aristotelian response, Halloun and Hestenes describe the idea that a force must act in the direction of motion to keep an object moving, and that the "force does not move an object unless it overcomes (exceeds) the object's *inertia*, an intrinsic resistance (mass) which is not distinguished from weight." An example of an impetus response would be "when an object is thrown, the active agent imparts to the object a certain immaterial motive power which sustains the body's motion until it has been dissipated due to resistance by the medium." Thus, the motive power eventually dies away, so that the object no longer moves. The impetus theory has been described in detail by McCloskey. He states, "the act of setting an object in motion imparts to the object an internal force or 'impetus' that serves to maintain the motion … [A] moving object's impetus gradually dissipates."[7]

Most students entering the course are not consistent in their use of theories. Halloun and Hestenes use their observations of student responses to describe students as predominantly Aristotelian (18%), predominantly impetus type (65%) or predominantly Newtonian (18%). Most of the students using theories inconsistently have predominantly non-Newtonian ideas. As the authors state, "no doubt much of the incoherence in the student CS systems is the result of vague and undifferentiated concepts." Common incorrect responses given during interviews were: "every motion has a cause," elaborating with statements such as "a force of inertia," "the force of velocity," or "it's still got some force inside" (this force is seemingly in the process of getting used up as time passes, showing evidence of the impetus theory). On the Halloun and Hestenes pretest, 65% of the students gave answers which Clement would describe as "motion implies force." Other students state "the speed is equal to the force of pull," or "the energy of blast has to be greater than the force" (indicative of an Aristotelian response). Many of these students do not distinguish between force and acceleration, think of force as a quantity that gets used up, and have difficulties distinguishing between related quantities whose distinctions help build a detailed understanding of physics.[8]

One weakness of the classification scheme used by Halloun and Hestenes is that they import "a ready-made classification scheme" taken from Newtonian mechanics. Student difficulties are interpreted in terms of the correct response, which we hope students will learn in our classrooms. But, Clement's research, described above, shows that students bring their own level of understanding to the classroom, and they are willing and able to invent forces to help analyze motion in their own framework. Thus, it may be that the description of student common sense beliefs according to a Newtonian taxonomy may not provide the most insight into student understanding or give the best guidance for our curriculum development.

Still, Halloun and Hestenes show that there is value in analyzing student performance on many questions in order to gain a more complete understanding of how students approach physics. The "coarse grain" analysis emphasizes both individual and specific difficulties while trying to analyze the whole system of reasoning that the student uses. Students answering pre-instruction questions have very little understanding of Newtonian mechanics. Individual students use many different theories to describe the same physics. Also, based on a comparison of pre- and post-instruction results, student reasoning does not seem to change very much during the course of instruction.

From the two summarized papers, one can conclude that the analysis of student difficulties with physics can lead to a meaningful and rich discussion of how students make sense of the physical world and the description of the physical world which they must learn in our classrooms. Both papers dealt with student understanding of mechanics. To improve student learning at all levels of instruction, issues of common sense physics and multiple theory use must be investigated in other areas of physics. In the next section, I will discuss the physics of waves and the previous research into student difficulties that forms the basis of the physics of this dissertation.

Wave Physics: Basic Concepts

A wave is a propagating disturbance to a medium. At the level of wave physics taught in introductory classes, mechanical wave phenomena occur via local interactions between neighbors and the propagation of disturbances can be described simply through spatial translations. The discussion below will use one-dimensional waves propagating on a taut string as an example (though also referring to sound waves when appropriate). One fundamental assumption of the introductory model of wave physics is that disturbances have only small effects on the system. In the case of transverse waves, this means that disturbances cause only small angle deviations from equilibrium. In the case of sound waves, it means that the deviations of pressure and density from equilibrium are very small compared to the unperturbed values.

Consequences of and problems with this assumption will be discussed below. The purpose of this section is to discuss the way that the professional physics community understands and describes wave physics. Assumptions, mathematical tools, and insights commonly used to think about wave physics are described below. Issues that might play a role in student understanding of wave physics will be raised.

Deriving the wave equation for mechanical waves

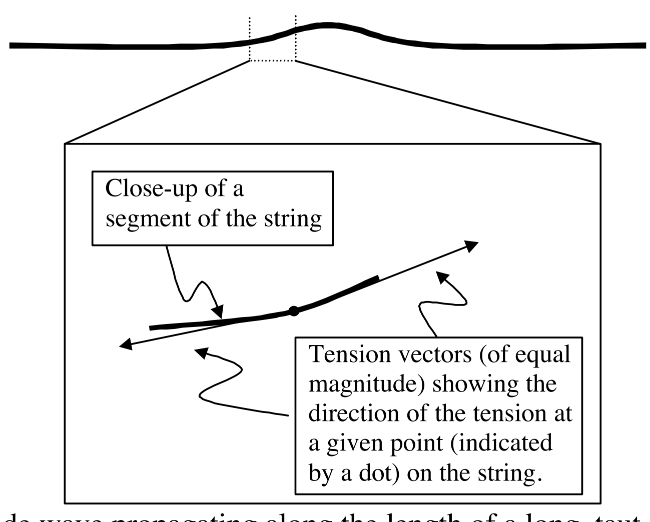
Consider a disturbance to a long, taut, ideal string consisting of a transverse displacement of the string from equilibrium (see Figure 2-4). In the case that the deviation from equilibrium is small, one can assume that the tension, T, in the string is constant. The sum of the forces exerted in the transverse direction, y, on a string element of mass density μ and length dx arises from the different angles at which the tension is being exerted on the element. Using Newton's Second Law, $\Sigma F = ma$, in the one transverse direction gives

$$(T\sin\theta_1 - T\sin\theta_2) = (\mu dx)\frac{\partial^2 y}{\partial t^2}. \qquad (2\text{-}1)$$

Because of the small angle approximation, the sine terms can be approximated by saying

$$\sin\theta \approx \theta \approx \tan\theta = \frac{dy}{dx}. \qquad (2\text{-}2)$$

Figure 2-4



A small amplitude wave propagating along the length of a long, taut string. The vertical displacement in this diagram has been exaggerated to emphasize the string's displacement.

Rewriting equation 2-1 in terms of this approximation gives

$$T\left[\left(\frac{\partial y}{\partial x}\right)_1 - \left(\frac{\partial y}{\partial x}\right)_2\right] = (\mu dx)\frac{\partial^2 y}{\partial t^2}\,. \tag{2-3}$$

Noting that the change in *dy/dx* on the left side of the equation is the change in the slope of the line on either side of the string element, we get the wave equation in its usual form,

$$T\frac{\partial^2 y}{\partial x^2} = \mu\frac{\partial^2 y}{\partial t^2} \tag{2-4}$$

which can be rewritten in a form analogous to Newton's Second Law,

$$Tdx\frac{\partial^2 y}{\partial x^2} = (\mu dx)\frac{\partial^2 y}{\partial t^2}\,. \tag{2-5}$$

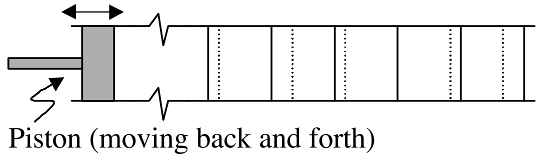
of the forces acting on the string element, while the right side of the equation contains mass and acceleration terms of the displacement from equilibrium.

Deriving the wave equation for sound waves

Equations like 2-4 can be constructed for other systems that show wave behavior. A detailed derivation of the wave equation for sound requires the use of fundamental concepts that may present some difficulty for students. Consider a tube filled with air extending infinitely in one direction with a movable piston at one end (see Figure 2-5). In the figure, the average equilibrium location of a plane of molecules and the average displacement of these molecules from equilibrium is shown. Note that not all planes are displaced an equal amount. Due to conservation of mass in the region between the planes, the density of the air inside the tube is no longer uniform when a sound wave is propagating through it. Students may have difficulty with the idea of mass conservation, since it is a fundamental concept that is rarely used explicitly in introductory physics.

Furthermore, the description of the motion of the air molecules may present difficulties for students. The air molecules are never motionless. The intrinsic motion of the medium is due to the temperature of the system (which is proportional to the average kinetic energy of the molecules in it). We can only describe the average equilibrium location of a plane of molecules and the average displacement from equilibrium of these molecules. Students may not recognize the distinction between

Figure 2-5



A sound wave propagating through a long air-filled cylinder. The average equilibrium position of a plane of air molecules is shown by a solid line, the average longitudinal displacement from equilibrium of a plane of air molecules by a dashed line intrinsic motion described by the temperature of the system and induced motion caused by the sound wave.

To derive the appropriate wave equation for sound waves, we can again use Newton's second law, as we did when deriving the wave equation for waves on a taut string. Consider a plane of air located (on average) at position x along the tube. The displacement due to a sound wave is described by $y(x,t)$, where the variable y describes a longitudinal and not transverse displacement from equilibrium. In the same way that we described the tension on a taut string, we can describe the equilibrium pressure on the plane of air by P (a constant). The change in pressure at that location at a given time will then be $\Delta P(x,t)$. In the same way that we described the linear mass density of a string, we can describe the equilibrium volume density of the air by a constant, ρ, and the density at a given location and time as $\rho(x,t) = \rho + \Delta\rho(x,t)$.

In this situation, Newton's second law states that the force exerted on a plane of air located at x consists of two parts. At a given time t, the magnitude of the force exerted by the air to the right of the plane is $F_{left} = A(P+\Delta P(x))$ and the magnitude of the force exerted by the air to the right of the plane is $F_{right} = A(P+\Delta P(x+dx))$. The net force is then equal to

$$F_{net} = A\left[\Delta P(x) - \Delta P(x+dx)\right] = -Adx\frac{\partial(\Delta P)}{\partial x}. \tag{2-6}$$

By Newton's second law, we know that the net force equals the mass times the acceleration of the gas. Since the mass of gas in a region dx is $A\rho dx$ and its acceleration is given by $\frac{d^2y}{dt^2}$, we have

$$\frac{\partial(\Delta P)}{\partial x} = -\rho\frac{\partial^2 y}{\partial t^2}. \tag{2-7}$$

To develop this equation further, we must apply concepts from thermodynamics. In a sound wave, we assume that the oscillation of the system is such that the temperature of the gas does not remain constant. Instead, we can state that the heat exchange of a region of gas with another region is zero, since all processes happen too quickly for heat exchange to occur. We can write an equation for the differential change in heat, dQ,

$$dQ = \left(\frac{\partial Q}{\partial V}\right)dV + \left(\frac{\partial Q}{\partial P}\right)dP = TC_V\frac{dP}{P} + TC_P\frac{dV}{V} \tag{2-8}$$

where the last part of the equation includes the definition of specific heat of an ideal gas at constant volume and at constant pressure.

In a tube with cross-sectional area A, the volume of air between two planes of air molecules separated by a distance dx will equal Adx. When a sound wave is propagating through the system, each plane will be displaced a different amount from equilibrium. The first plane will be displaced $y(x)$ and the second $y(x+dx)$. Thus, the volume of air between the two planes is equal to.

$$A\left[dx + y(x+dx) - y(x)\right] = A\left[dx + dx\frac{dy}{dx}\right] = V + dV \ . \qquad (2\text{-}9)$$

Setting $dQ = 0$ in equation 2-8, we can use equation 2-9 to write

$$dP = -P\frac{C_P}{C_V}\frac{dy}{dx} = -P\gamma\,\frac{dy}{dx} \qquad (2\text{-}10)$$

where the term γ has been introduced to describe the ratio of the specific heats.

Using equation 2-10 in equation 2-7 gives the wave equation for the propagating sound wave. We find

$$\frac{P\gamma}{\rho}\left(\frac{\partial^2 y}{\partial x^2}\right) = \frac{\partial^2 y}{\partial t^2} \ . \qquad (2\text{-}11)$$

Note that the only difference between equation 2-11 and equation 2-4 is in the variables that describe medium properties. Otherwise, the mathematical form is identical for waves on a taut string and sound waves, meaning that an analysis of the mathematics and physics for the two types of waves should be very similar.

Physical meaning of the wave equation

Local interactions on a global scale

Because the wave equation for a wave on a taut string arises from Newton's Second Law, we can interpret the physics in terms of concepts that students have learned in their previous mechanics course. For example, a correct interpretation of Newton's Second Law requires that only those forces acting directly on an object influence that object (this concept has been referred to as Newton's "Zeroth Law"[9]). Though this seems obvious, students have great difficulty with the idea when applied to free body diagrams of point particles.[10] The difficulties students have with this concept when applied to point particles should exist when applied to continuous systems, also.

In continuous systems, the additional difficulty exists that Newton's "Zeroth Law" must be applied to every point in the medium. Local interactions at all locations in the medium must be considered. The conceptual distinction between interactions on a local level and the analysis of these interactions everywhere (i.e. globally) requires an understanding of the relevant size of analysis of the system. Since this is often a new concept for students, we can expect them to have difficulties making the distinction between local and global analyses of the physics.

For a sound wave, the interpretation of the wave equation uncovers a subtlety with which many students may have problems. The fundamental idea is that one considers the pressure gradient across the region of air through which the wave propagates. For compression (high density) to be followed by rarefaction (low density), the pressure gradient across the region of air must be both positive and negative. A model describing sound waves in terms of the transfer of impulses from one region to another, *only* in the direction of wave propagation, would not account for the rarefaction process. An effective force in the direction opposite to the propagation direction must also be exerted. This effective force can only be described by considering the pressure gradient and not the pressure in the direction of wave propagation.

Solutions of the wave equation as propagating waves

Solutions to the partial differential wave equation depend in a conceptually difficult manner on two variables. When describing the one-dimensional spatial translation of particles whose location at time $t = 0$ is the origin, we can write $x = \pm vt$ or $x \pm vt = 0$. More generally, we can say that the function,

$$g(x,t) = x \pm vt \qquad (2\text{-}12)$$

describes the location of a particle that starts out at any position x and moves either to the left or the right with a velocity v. Waves are also spatially translated at constant speed, but they are spread out over a large region in space. As such, the displacement from equilibrium in the medium can be thought of as being translated independently. We can expect solutions to the wave equation to have some similar form.

If we carry out a coordinate transformation on the variables in the wave equation, we can find this form. By substituting $\eta = x+ct$ and $\xi = x-ct$ into equation 2-4 (where c is an as-yet undefined variable), we can rewrite equation 2-4 to say

$$\frac{\partial^2 y}{\partial \ \partial\eta} \quad \boldsymbol{0}$$

In the process of simplifying the equation, the variable c has been defined as $c^2=T/\mu$ and represents the speed with which the wave propagates through the medium. Thus, the solutions to the wave equation have the form of $x \pm ct$. The displacement of the medium from equilibrium is then a function of a single function which is a function of two variables,

$$y(g(x,t)) = y(x \pm ct)\ . \qquad (2\text{-}14)$$

Since the quantity y describes the displacement of the medium from equilibrium at all points (i.e. the shape of the wave), equation 2-14 describes the translation of this shape through the medium.

Plugging equation 2-14 into the wave equation shows that this functional dependence leads to correct solutions of the wave equation, as long as the velocity of the wave is set equal to physical quantities in the wave equation. (This will be discussed in more detail below.) Thus, the spatial translation of waves through a medium arises as a consequence of the local interactions between elements of the medium. Spatial translation is a consequence of the local interactions of the physical system.

The issue of the distinction between local and global descriptions of the system again may cause difficulties for students. The displacement of the medium is described by $y(x_0,t_0)$, where x_0 and t_0 are specific values of position and time. Thus, an equation of the form $y(x_0,t)$ describes the motion of a single point in the medium as a function of time, while $y(x,t_0)$ describes the displacement from equilibrium of the entire medium at one instant in time. But, the shape of the entire medium at all times is described by $y(x,t)$, where the specific functional dependence describes the translation of the entire shape to the left or the right (in one dimension). The global translation of the entire system is the most visible phenomena of visible wave systems (ocean waves, waves on a string or spring, etc.). Thus, we can expect students to focus on the global descriptions of waves rather than the local phenomena within the system that cause the spatial translation.

Wave velocity depends on medium properties

One consequence of equation 2-14 being a solution to equation 2-4 is that the wave velocity, *c*, is a function of properties of the medium. We find that

$$c = \sqrt{T/\mu} \tag{2-15}$$

for waves on a string or tightly coiled spring, while

$$c = \sqrt{p\gamma/\rho} \tag{2-16}$$

for the propagation of sound through air.

If we assume for the moment that the gas in which the sound is propagating is an ideal gas, we can relate *P* to the temperature, *T*, of the system using the ideal gas law, $PV=nRT$ (*R* is the Rydberg constant). Since $\rho=M/V$ (*M* is the mass of the gas in a volume *V*), the speed of sound in the system is equal to

$$c = \sqrt{\gamma RT/M}\ . \tag{2-17}$$

Further analysis is possible, but the basic conceptual meaning of equations 2-15 and 2-16 is that the speed of propagation of a wave through a medium depends on the properties of the medium and nothing else.

This concept may be difficult for students to understand. In all other instances, students have learned that some sort of force was necessary to cause a change in motion. (Note also that many students have felt that some sort of force was necessary to continue a motion, see chapter 2). In this situation, no external force is necessary for the propagation of a wave through a system (where there are obvious changes to the motion of elements of the system). We can again expect students to have difficulty distinguishing between the internal forces and the external observable elements of the system.

In the case of waves, ignoring the internal forces of the system and focusing only on the spatial translation of the waveshape may create a dilemma for some students. They may revert to the impetus physics or Aristotelian physics which Halloun and Hestenes describe. The description of wave propagation through a system can be thought of as a large domain description. An analysis of the internal forces that allow the wave to propagate can be thought of as a small domain description of waves. In other words, a failure to understand the relevance of different domain sizes of wave physics may push students toward incorrect reasoning.

Superposition

Since the wave equation is linear, any linear superposition of solutions of the form $y(x \pm ct)$ will also be a solution to the wave equation. Thus, a wave described by

$$y(x,t) = y_1(x - ct) + y_2(x + ct) \tag{2-18}$$

leads to two separate wave equations, one for y_1 and one for y_2.

Buried within the summation of these individual waves is the concept that the waves described by y_1 and y_2 can only add when their values of x and t are the same. Though this seems obvious from a mathematical point of view, it may not be so for students. The dependence on two variables again plays a role here, and the fact that both variables must be equal may be difficult to interpret physically and mathematically.

In addition, values of y_1 and y_2 are only added at a specific time, t, when the values of x are equal, but they are added for all x. The issue of local summation done on a global level (i.e. everywhere) shows that the fundamental conceptual distinction between local and global phenomena also plays a role in superposition.

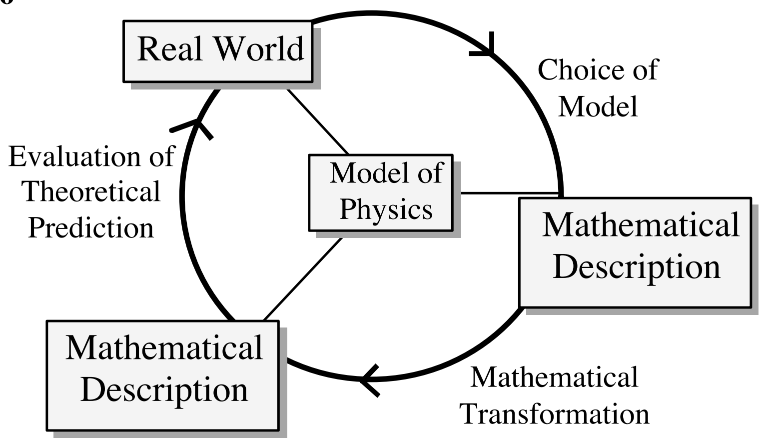
It is possible, through superposition, to have the model which led to the linear wave equation break down in certain situations. For example, two waves individually may still be within the small angle regime, but added together may fall outside the regime. Recall that there was a simplification in the derivation such that the sine terms describing the vertical components of the tension on the string element could be replaced by the slope of the string on either side of the string element. If two waves add in such a way that their sums no longer hold to the model because of large angle deviations from equilibrium, then an inconsistency of the model is uncovered.

The role of modeling

The possibility of the linear model breaking down raises an issue with respect to the way in which physical models are used in science. Based on observations, we develop or choose mathematical models to describe the physical world. These mathematical models can then be modified through mathematical transformations to either account for other observations or make predictions. Any predictions made by the model must then be compared to the physical world. A representation of the cycle that describes the relationship between observations of the real world, the choice of mathematical model and analysis, and the interpretation and comparison of the predictions with the real world is shown in Figure 2-6.

In the case of linear superposition breaking down due to the inapplicability of the small angle approximation, the difficulty lies in the use of the mathematical transformation to interpret the new physical situation. The choice of model for the system seems appropriate because each wave satisfies the small angle approximation. But, the choice of model is shown to be incomplete because it cannot adequately describe the phenomena that it claims to. The inconsistency between prediction and model choice is not found until mathematical predictions are compared to physical reality. In chapter 6, I describe an instructional setting where exactly this breakdown in the model of superposition occurs.

Figure 2-6



The interaction between the real world and a theoretical model which describes it and predicts its behavior. The choice of a model of physics affects the choice of mathematical model and how the model is mathematically transformed.

Initial conditions and boundary conditions

The wave equation describes only the manner in which the wave propagates through a system and how waves interact with each other but does not describe how the wave was created nor its behavior at boundaries to the medium. The speed of wave propagation (which enters into the wave equation) depends on medium properties. Linear superposition is a consequence of the wave equation. But the manner in which waves are created is determined by the initial conditions of the system (i.e. in terms of time dependent events at a specific location or possibly many locations in space). The manner in which waves interact with the boundaries of the medium are determined by the boundary conditions of the system.

To describe how a wave is created, we can discuss boundary or initial conditions that are either continuous disturbances to the medium at one location or disturbances that last a finite amount of time. Because the disturbance to the system propagates through the system, the former leads to a disturbance of finite length and duration (such as a wavepacket) while the latter leads to a continuous disturbance (such as a sine curve or sawtooth wave). In this dissertation, I will describe the the finite length waves as *wavepulses* and continuous waves as *wavetrains*. An example of each is shown in Figure 2-7. I will use the term waves to mean both wavepulses and wavetrains, i.e. all propagating disturbances to a system.

The boundary condition plays a role by driving the shape of the string at a given location in space. This type of boundary condition is a time dependent function for a point in space.[11] For example, the boundary condition can be given at some location x_0 by a function depending on time. For a sinusoidal wavetrain on a string which stretches from $x=0$ in the positive x direction, this equation may be of the form $y(x=0,t)=A\sin(2\pi ct/\lambda)$, where λ is the wavelength of the propagating sinusoidal wavetrain of amplitude A. For a wavepulse with a Gaussian shape, the boundary

Figure 2-7

Wavepulse: finite length

Wavetrain: endlessly repeating

The difference between a wavepulse and a wavetrain, illustrated with a finite length sawtooth shape and a repeating sawtooth pattern. Both shapes represent propagating disturbances to the equilibrium state of the system, but, for example, the propagation of the wave is more easily visible with a wavepulse than a wavetrain.

condition may be of the form $y(x = \boldsymbol{0},t) = Ae^{-(ct/b)^2}$, where b describes the width of the wavepulse.

Note that the creation of the wave does not determine the speed with which the wave moves. The velocity is determined by the medium through which the wave propagates. In both examples of initial conditions above, the velocity of the wave is therefore a given that determines the relationship between the duration of the motion and the width or wavelength of the propagating wave. For sinusoidal waves, this relationship is given by $\lambda = cT$, where T is the period of the wave. For a wavepulse created on a taut string by moving one's hand quickly back and forth, one can describe the width of the wave (at its base) by $W=cT$, where T is the amount of time the hand was in motion. Of course, the creation of the wave may affect the validity of the approximations we use to describe the system (for example, a large amplitude wave may lead to large angles which may make the linear wave equation inadequate as a description of the physical situation).

Students may have difficulty understanding wave motion without additional discussion of how waves are created. The interpretation of boundary conditions as the source of wave motion is rarely emphasized in physics textbooks.[12] Most commonly, portions of the medium (either a string or air, for sound) are shown with a propagating wave, without discussion of how that wave was created. We know, from previous PER, that students often have difficulty separating the cause of motion from the motion itself (for example, the impetus model described above shows this confusion). We also know that students often invent forces to account for motion (for example, Clement's results described above). We can expect students to invent causes or forces for the wave motion that they see.

Furthermore, it may be difficult for students to distinguish between the velocity as determined by the medium and the motion (described by boundary conditions) which causes the wave. Consider a person holding a long, taut spring lying on the ground and shaking it regularly back and forth (this is a common demonstration done in classrooms). The time it takes for the demonstrator to complete either a full period of a wavetrain or to create a wavepulse is determined by the speed with which the hand moves back and forth. If the hand moves faster over the same distance as in a previous demonstration, the effect is to create wavetrains and wavepulses that are narrower. The effect is not to make the wave move faster. The distinction between transverse velocity and propagation velocity may cause difficulties for students.

In order to describe the physical behavior of waves at the edge of the system in which they are propagating, we again must use boundary conditions. For example, a string on which a wave propagates can either be attached or free to move. In the case of sound waves, similar distinctions exist between regions where displacement from equilibrium is possible and where none is possible. The boundary conditions then describe the properties of reflection and transmission. They whether or not there can be a displacement and what sort of displacement can exist at the location of the boundary. For example, for a string fixed to a wall at location x_0, the boundary condition might be $y(x_0,t)=0$ for all times t.

Students might have problems with this idea for a variety of reasons. Rather than showing a distinction between spatially local or global domains, the issue of boundary conditions involves the distinction between constant situations (the boundary condition) and instantaneous events (the shape of the string at an instant in time). Previous PER has shown that students often have difficulties distinguishing between two events that occur at different times, and that students often integrate all times into a single description.[13] We can expect to find the same types of difficulties in wave physics.

Previous Research Into Student Difficulties with Waves

Very little previous research has been published on student difficulties with mechanical waves. Maurines[14] and Snir[15] studied student understanding of wave propagation, Grayson[16] (also with McDermott[17]) and Snir studied student understanding of the mathematical description of waves and the superposition of waves, and Linder[18] (also with Erickson[19]) studied student descriptions of sound.

In the discussion below, I will first describe the research setting and methods of each of these researchers. This will include a more complete description of the issues and the student populations they investigated. This brief discussion will be followed by descriptions of the observed student difficulties with the wave physics topics outlined above.

Research context and setting of previous research

The student populations investigated in previous research include pre-service teachers, engineering students, physics majors, high school students, and physics graduate students.

Maurines[14] asked 1300 French students questions which dealt with the topic of wave propagation and simple mathematical reasoning about waves. Of these, 700 students had no previous instruction on waves and were in secondary school (the age equivalent to American high schools) and 600 had previous instruction on wave physics. The latter group was a mixture of secondary school and university students.

The investigation consisted of eight written free response questions. The questions addressed the topic of wave motion through a medium, the relationship between the creation of the wave and its subsequent propagation, and the motion of an element of the medium due to the propagating wave.

Maurines points out that results within each of the two groups were so similar that "no distinction can be made between the different subgroups." Thus, Maurines uses representative data from subgroups of her study to describe student difficulties. There were differences between the students who had received instruction on waves and those who had not. Specific questions that Maurines asked will be discussed in more detail below.

Linder and Erickson's[18] work on student understanding of sound waves took place with ten Canadian physics majors who had graduated from college in the previous year and were enrolled in an education program to get certification in teaching physics. The ten interviewed students were enrolled in a one year course for teacher certification to teach at the high school (secondary) school level. Students were interviewed for 40 to 80 minutes. During this time, they answered a variety of questions dealing with their personal experiences with sound, descriptions of simple phenomena, interpretations of typical representations of sound waves, and predictions of how the speed of sound can be changed in a medium. Examples of student comments and reasoning will be given below. Data were gathered from an extensive analysis of interview transcripts. Data were analyzed by categorizing student interview explanations in terms of elements common to other explanations given by the same student and elements common to explanations given by other students.

Grayson and McDermott's work was done at the University of Washington, Seattle (UW), and Grayson continued this work at the University of Natal, South Africa (UNSA). The work done at UW consisted of investigations of the kinematics of the string elements for propagating and superposing waves. Student understanding of two-dimensional kinematics was investigated to help develop a computer program that would address student difficulties with the material. At UW, individual interviews were conducted with 18 students after they had instruction on waves and kinematics. (The questions will be described in more detail below.) Grayson continued this research at UNSA with two different student populations. The first consisted of in-service teachers taking a six week summer program that focused on the teaching of kinematics. Most teachers were not physics instructors, so this was their introduction to kinematics. They were asked the same types of questions as the UW students before, immediately after instruction, and then again on the final examination. In a third study, Grayson investigated the understanding of twelve introductory physics students who had studied kinematics but not waves. They were also asked the same types of questions as the other students before and after instruction. Instruction in both instances at UNSA consisted of students using a program designed to help students view the motion of string elements as waves travel along the string. Grayson made additional observations as the students used the programs, noting both difficulties with the program and conceptual difficulties with the material.

Like Grayson, Snir developed a computer program to help students develop their reasoning skills with waves. In the development of the program, he investigated the difficulties of Israeli students with wave propagation and superposition after they had completed instruction on waves. Studies were conducted with tenth grade students who were interviewed before and after instruction. The complete research protocol and results were never published.[20] The number of students and the types of questions asked are thus not known. Snir's results will be mentioned but not elaborated upon below, since they are consistent with those of Grayson and Maurines.

Student difficulties with the propagation of waves

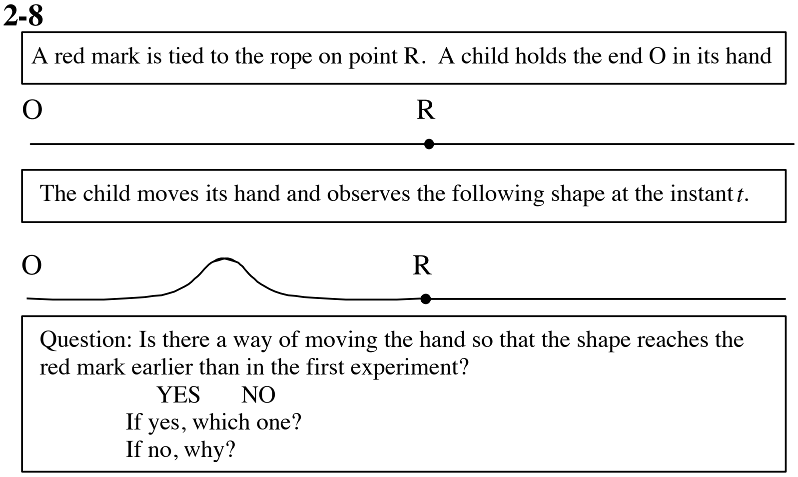
Maurines and Snir focus on the reasoning students use when describing wave propagation on a taut string. Linder (and Erickson) focus on student explanations of sound wave propagation. The similarities between some of the explanations indicate that students have similar difficulties with the material.

Propagation on a taut string or spring system

Two questions by Maurines show student difficulties with the relationship between wave creation and wave propagation. In the first, students were asked if it was possible to change the speed of a wavepulse by changing the motion of the hand that creates it (see Figure 2-8). In the second, Maurines describes the realistic scenario that the wavepulse amplitude decreases over time, and students are asked if the speed of the wavepulse changes as this occurs (see Figure 2-9).

Common student responses indicated that a majority of the students thought of wave propagation in terms of the forces exerted by the hand to create the wavepulse on the rope. For example, students stated, "the speed depends on the force given by the hand," or "the bump will move faster if the shake is sharp" (i.e. if the movement of the hand is faster). Maurines gives results from subsets of the secondary school and the

Figure 2-8



Question asked by Maurines to investigate how students viewed the relationship between the creation of the wave and the motion of the wave through the medium. A correct answer would be "no," because only medium properties affect wave speed. See reference 14 for further discussion.

university student population. (Recall that she said that results within each group were similar, implying that the statistics she gives for the subgroup are consistent with the statistics for the whole group.)

Very few students who had completed instruction gave the correct answer to the question in Figure 2-8, which states that there is no way to move the hand to create a faster wave. Of 42 secondary school students who had no instruction in waves (and 16 university students who did), 36% (25%) gave the correct answer. Of the students who gave incorrect responses, 60% (75%) stated that it was possible to change the wave speed through a different hand motion. For these students, 84% (67%) gave justifications that mentioned force, as indicated with the first quote above. Students seem to have profound difficulties separating the creation of a wavepulse (i.e. the initial conditions of the system) from its propagation through the system. The quotes given above, though brief, indicate that students are using an impetus-like model to describe the movement of a wavepulse through a medium. The wavepulse propagates due to the motion of the hand and a change in hand motion will affect wave speed.

Student responses to the question shown in Figure 2-9 also indicated that many students did not separate the initial conditions from the propagation properties of the wavepulse. A correct answer to the question would state that the speed of the wavepulse would not change while the amplitude decreased. Maurines quotes a student saying "The height decreases as the action of the hand gets weaker. The speed decreases also. If the bump disappears, it is because the force which caused it disappears as well. During that time, the speed decreases." This student's reasoning is indicative of the impetus model of mechanics, described above. The "force which caused" the wavepulse disappears as the amplitude disappears, and as the force is used up, "the speed decreases." Maurines states that of 56 secondary school students who had not received instruction in waves (and 42 university students who had), 30% (45%) gave the correct answer and 68% (55%) gave incorrect answers. Of the students giving incorrect answers, 58% (35%) used reasoning force-based similar to the student quote above. Again, the evidence indicates that students misinterpret the physics of the creation of the wave with its propagation.

Maurines interprets student descriptions in terms of students' notions of force and a quantity she calls "signal supply." This signal supply is a "mixture of force, speed, [and] energy." The impetus model often guides student reasoning with respect to the signal supply. Thus, the higher the signal (the more force is used to create the

Figure 2-9

This bump disappears before reaching the other end of the rope. Does the speed of the bump vary on the way?

YES NO

Why?

Question asked by Maurines to investigate how students interpreted damping in a wave system. A correct answer would be that the damping affects only the amplitude but not the propagation speed. See reference 14 for further discussion.

wave, the wavepulse), the faster the wave. Student comments are consistent with this interpretation. For example, some students state that the propagating wavepulse "is losing its initial power," and others state that "there is a [wavepulse] which is moving because of the force F" exerted by the hand. The latter student is confusing the force needed to create the wave with the forces internal to the medium that allow the original force to propagate through the medium. Thus, we see that students are unable to separate the creation of the wave from its propagation. Maurines states that many do not make the distinction between force and velocity.

Snir's[15] interpretation of student difficulties with the relationship between wave creation and propagation is similar to Maurines's, but he does not cite evidence for his result. He describes finding that students speak of a wave's "strength," or "energy," or "intensity," much like Maurines describes "signal supply." He also implies that students use impetus-like reasoning to say that waves with larger intensity (higher amplitude or frequency) have more strength and therefore move faster. Because he does not provide evidence for his interpretation (as described above), it is difficult to interpret his findings, but they seem to be consistent with Maurines's. In Chapter 3, I discuss similar results have found at UMd. In Chapter 5, I propose a more detailed explanation for student reasoning than the one used by Maurines or Snir.

Sound wave propagation

Linder[18] (also with Erickson)[19] found that students who had completed their undergraduate studies of physics (including wave physics and sound) have great difficulties understanding the propagation of sound waves through air. In one question, they asked students to describe what would happen to a candle flame located near the end of a tube when one clapped two pieces of wood together at the other end of the tube (see Figure 2-10). Student descriptions of the effect on the candle flame of the sound wave caused by the clap showed that students thought of sound using incorrect models and inapplicable analogies. Similar questions involved sound due to the popping of a balloon and sound caused by the vibration of a tuning fork.

Common student descriptions of sound waves in these settings involve the incorrect descriptions of the motion of air or air molecules to account for sound. For example, one student states that "sound creates a wave that is emitted and is focused on the tube - and so the wave travels down." The interviewer asks "Pushing air in front of it?" as a provocative question to elicit possible difficulties the student may have with

Figure 2-10

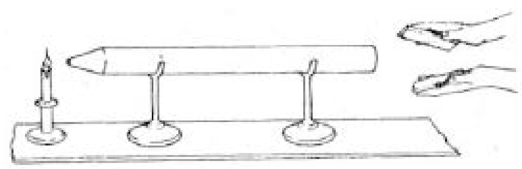


Figure given students in the Linder and Erickson interviews. Students were asked to describe how clapping two pieces of wood together would affect the candle flame located on the other end of a long tube from the location of the clap. See reference 19 for further discussion.

source to the ear that hears it. Linder and Erickson observe that students describe the motion of air as either the flow of large blocks of air from one point to another or as the motion of specific air molecules that transmit sound while all other molecules continue in their usual random motion.

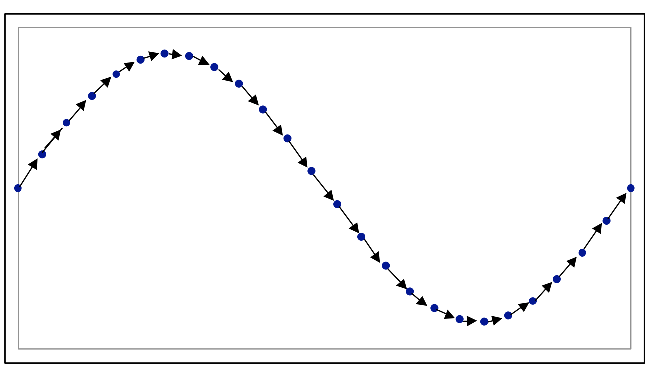
Another common model that Linder and Erickson describe involves the impulse transfer model, as if sound were transmitted linearly along a path of adjacent beads. Rather than describe sound waves in terms of a pressure gradient, one student speaks of forces only in the direction of wave propagation. He states,

> *[J]ust consider a row of beads sitting on the table. And you tap a bead at one end and you knock all the beads along and at the other end you have your finger and you can feel the tap. That would be analogous to a book dropping and creating the motion of all these smaller things in the air we call molecules which act the same as the beads and move this disturbance around until your finger at the other ends can feel it; in this case with the ear at the other end that is feeling it.*

This student is thinking of sound waves on a microscopic level of individual colliding air molecules, but avoids the very difficult idea that density and pressure propagation through air forms a sound wave. The problematic physics of the impulse transfer model of sound has been discussed above.

Linder observed an interesting variation of the impulse transfer model that allowed a student to account for the sinusoidal path of sound waves that is commonly drawn in textbooks. In textbooks, the sinusoidal path describes the longitudinal displacement of a region of air from its equilibrium position. Linder observes that a student who sketches a sinusoidal curve made up of colliding air molecules (seeFigure 2-11). Linder summarizes the student's model as: a sound wave consists of "molecules in the air colliding with each other in such a way that a transverse pathway" is created. As Linder states, "The molecular collisions are generally not 'head-on' but rather tend

Figure 2-11



Student sketch to show how sound propagates. Sound consists of glancing collisions between adjacent particles such that the recognizable sinusoidal shape is created. See reference 18 for further discussion.

to be ‘glancing’ in such a manner as to give rise to the ‘correct’ changes in direction to form a sinusoidally shaped collision-wave.” The student giving this response is mistaking the graph of displacement from equilibrium as a function of position for a picture that describes the interaction between elements of the medium through which the wave travels. This confusion of graphs and pictures has been investigated in more detail with respect to student interpretations of graphs in the kinematics.[21]

Linder and Erickson observe that some students think of sound as the motion of a quantity (like energy or impetus) that is transferred from molecule to molecule. This is similar to the idea of “signal supply” described by Maurines and the “strength” described by Snir. Linder has observed that many students believe “changing particle displacement, changing sound pressure, and changing molecular velocity all to be in phase with one another.” Thus, students do not distinguish between different variables that describe the system, much like the students observed by Maurines and Snir do not distinguish between velocity, frequency, power, and energy.

Other similarities also exist between Linder (and Erickson’s) findings and Maurines’s results in the overall confusion students have about propagation speed. Some students state that the speed of sound is determined by the physical obstruction of the medium (thus, a denser medium should have slower sound waves, the opposite of what actually occurs). This idea seems related to the concept that Maurines discusses, where students describe a wave exerting a force on the medium. The less force is exerted, the slower the wave. Similarly, the less resistance from the medium, the less force is needed to create a fast wave, and the faster a wave created with great force will move. The relationship to Maurines’s “signal supply” and Snir’s “strength” is supported by Linder’s comment that some students state that wave speed is a function of inertia reduction. Thus, we see that students seem to use the same descriptions of waves when describing mechanical waves on strings or springs and sound waves.

Linder presents an interesting result which has not been discussed by others who have investigated student understanding of wave physics. He observed that students have great difficulty with the idea of the equilibrium state of the air through which sound waves travel. As one student states (when describing the problem shown in Figure 2-10), “Equilibrium position will be a position of rest. Before you clap, all the [air] particles are in a position of rest and as you clap you are causing particles to move so particles start jumping all over the place; then they all return back because they try and return to equilibrium. Everything always tries to go to equilibrium.” The student is having difficulty distinguishing between the different scales of the system, air molecules or regions that are, on average, at equilibrium. If students have difficulty with understanding the equilibrium condition of a system through which waves propagate, their understanding of wave propagation may be much less robust than we would like. Similar difficulties have been observed and discussed in introductory mechanics by Minstrell with respect to force and motion and the at rest condition.[22]

The study of student understanding of sound waves is rich because it shows evidence of many of difficulties found in other areas of PER. Some students have difficulties with the representations used to describe sound and misinterpret graphs as pictures (as in the example of colliding air molecules traveling along a sinusoidal path).

Many students have difficulty with the equilibrium state of the system (as in the inability to distinguish between air molecules and a description of the medium based on density of a region of air). One should note that many of the student difficulties are specific to sound waves but also related to difficulties students have in other areas. This suggests that common descriptions can be found to account for a large variety of student difficulties with physics.

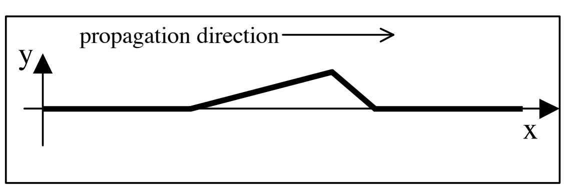
Student difficulties with the mathematical description of waves

Grayson's work investigates student use of two-dimensional kinematics to describe the propagation of waves and the motion of the medium through which the waves propagate. Students were asked questions of the following type: given a graph of y vs. x (vertical and horizontal position, respectively), of an asymmetrically shaped pulse,[23] graph y vs. t and x vs. t for a point, and v vs. x for the string (see Figure 2-12). The discussion below uses results from written responses and comments and quotes gathered by Grayson while observing students using a program to help them develop their conceptual understanding of the topic.

A correct understanding of kinematics and physics in these questions would include the idea that solutions of the form $y(x \pm ct)$ propagate without changing their shape (in an ideal, dispersionless medium). The motion of the medium is transverse to the motion of the wave. To describe the velocity of a piece of the medium (a small section of the string) over time, one can sketch the string at regular time intervals and use the definition of average velocity ($v=\Delta x/\Delta t$) to describe the velocity at different instants in time. To describe the velocity of the entire string at some instant in time, one can use the same method and find the velocity of each element of the string at one time.

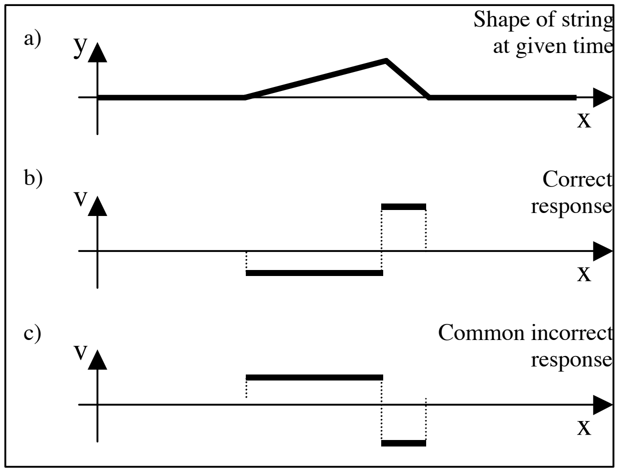
Grayson describes how students (both before and after instruction) approach certain ideas algorithmically when she describes how students attempt to find the shape of a v vs. x graph. She found that many took the slope of the y vs. x graph rather than thinking of the time development of the y vs. x graph and using a relevant procedure to find the velocity at each point along the string (see Figure 2-13). In other words, the students did not have an operational understanding of how to find v vs. x and used an incorrect algorithmic method instead.

Figure 2-12



Grayson presented students with a diagram like this one, indicating an asymmetric wavepulse propagating to the right on a long, taut string. Grayson then asked students to sketch graphs of the following quantities: y vs. t and v vs. t for a string element as the wave passes that string element, and v vs. x for the entire string at the instant in time shown in the diagram. See references 16 and 17 for further discussion.

Figure 2-13



Common student difficulty with a *v* vs. *x* graph of a string on which a wave is propagating. a) Given sketch of an asymmetric wavepulse propagating to the right, b) correct response, showing velocity of each string element based on the motion of the string, c) most common incorrect response, showing velocity of each string element based on the slope of the given *y* vs. *x* graph. See references 16 and 17 for further discussion.

Table 2-1

	Pretest	Posttest	Final Exam
UW phyiscs students (N=18)	--	22%	--
UNSA introductory physics students (N=12)	33%	75%	--
In-service teachers (N=19)	53%	84%	79%
In-service teachers (N=23)	26%	65%	

Percentage of correct responses for students sketching a *v* vs. *x* graph of an asymmetric wavepulse propagating along a string (see Figure 2-13). The UW students were interviewed, the other students answered written questions. Two different populations of in-service teachers were investigated in successive years. None of the incorrect responses by in-service teachers used the slope of the *y* vs. *x* graph to answer the question. See reference 16 for further discussion.

Data for the three student populations which Grayson investigated are shown in Table 2-1. The most common mistake students made was to take the slope of the *y* vs. *x* graph incorrectly, as described above. Grayson attributes the improved post-instruction performance of the in-service teachers and UNSA students (in comparison to the UW students) to the use of the computer program to help address student difficulties. She also notes that none of the in-service teachers took the slope of the *y* vs. *x* graph when answering the question after instruction.

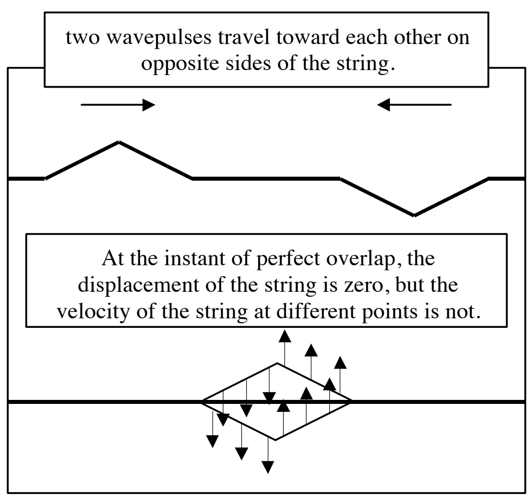
Student use of an inappropriate algorithmic method for finding answers to questions they are otherwise unable to answer suggests that students do not imagine the motion of the medium when the wave passes through it. A consideration of the motion of the medium would show that the incorrect response found by taking the slope of the *y* vs. *x* graph (shown in Figure 2-13c) is inconsistent with the motion of the medium. The leading edge of the wave is moving up, not down, as indicated on the graph. Thus, by investigating student understanding of the mathematics of wave motion through a medium, we find results similar to Linder's. Both Linder and Grayson observe that students have profound difficulties describing the motion of the medium as a result of the wave.

Student difficulties with superposition

Grayson and Snir address the issue of student understanding of wave superposition. Since Snir does not give data to support his conclusions, I will focus on Grayson's work in the following discussion. Grayson asked students to describe the shape of a string on which two identically shaped wavepulses were traveling toward each other on opposite sides of the string. She asked specifically for the shape of the string and the velocity of different elements of the string at the moment of maximum overlap. The situation and a correct response are shown in Figure 2-14.

Grayson finds that students consistently give the same incorrect responses. Students state that waves will collide, and either bounce off each other or cancel each other out and disappear permanently. Grayson states that "some students did not realize that pulses pass through each other. Instead, several students said that two pulses would bounce off each other and travel back towards where they came from."

Figure 2-14



Superposing wavepulses on opposite sides of a long, taut string. In the lower sketch, the individual wavepulses are shown together with velocity vectors indicating the direction of the motion of the string due to the wavepulse. Note that the string has zero displacement but that the velocity of the string is non-zero where the pulses overlap (except at the exact middle point). See references 16 and 17 for further discussion.

Grayson gives a possible explanation for the permanent cancellation of waves when describing student difficulties in distinguishing between the displacement and velocity of the medium through which the wave travels. Students are often unable to distinguish the two, causing problems in their description of continuing wave motion after waves have interacted. For example, students see a flat shape when two symmetric wavepulses of identical amplitude on opposite sides of a string add completely destructively (see the sketch of the correct response in Figure 2-14). Those who interpret the lack of displacement such that the velocity of the string is zero will then state that nothing will move anymore. Thus, an incorrect interpretation of the kinematics (i.e. the difference between displacement and velocity) may be used by students and lead to an incorrect physical interpretation of the situation. This interpretation may guide students to say that the wavepulses are permanently canceled in this situation.

Snir describes similar results. He also discusses how students will speak of waves that bounce, collide, or cancel each other permanently. Snir states that the idea is borrowed from mechanical collisions, but he does not elaborate how this may be the case.

Research as a Guide to Curriculum Development

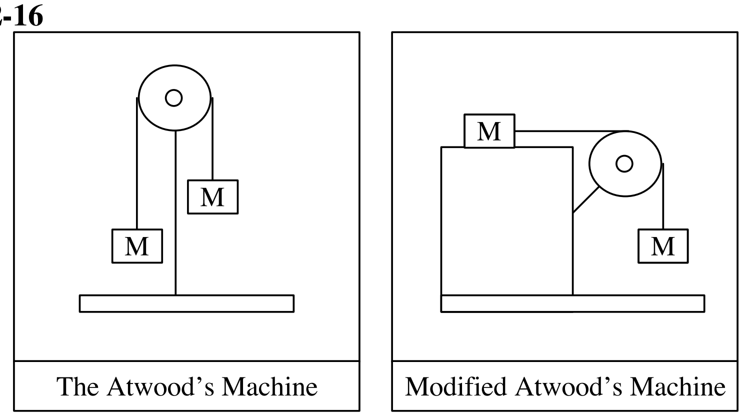
The discussion of previous research into student difficulties with waves serves as an example of PER done to come to a deeper understanding of how students approach physics and build a functional understanding of the material. Another aspect of PER involves the building of curriculum materials that address student needs as effectively as possible. The paradigm of instructional design used by PERG at UMd is based on that of the University of Washington, Seattle (UW) (see Figure 2-1). To show the background of the research-curriculum design paradigm, I will describe one example from UW in detail. Interested readers can find more information about the UW methods by following references in summarized papers and in other sources.[24] Research by UMd PERG has also shown that tutorials are more effective in helping students develop a deeper understanding of the physics.[25]

The development of instructional materials begins with the investigation of student difficulties. Researchers at UW investigated student understanding of tension in the context of the Atwood's and modified Atwood's machines (see Figure 2-16).[26] The apparatus consists of weights attached over an (ideally, frictionless) support by a string. Students often encounter this example in the classroom, solving problems from the textbook or seeing a demonstration done by a professor.

McDermott *et al.* found that a similar situation elicited nearly identical difficulties with the fundamental ideas of acceleration, force, and tension as the original Atwood's machine. Rather than having the force of gravity play a role in the physics, the UW question used an explicit external force to move two blocks (of mass m_A and m_B ($m_A<m_B$)) connected by strings (see Figure 2-15). One hundred students were asked the question in Figure 2-15. These students had previously had instruction on tension and the course included a laboratory that dealt explicitly with the Atwood's machine.

Students had fundamental problems with the concept of tension in this setting. They had the most difficulties when they were asked to compare the force exerted by string #1 on block A with the force exerted by string #2 on block B. (To have the same acceleration, the force of string #1 on block A must be greater, since the force exerted by string #2 on B is equal to the force exerted by string #2 on A, but in the opposite direction, and the sum of the forces must still be to the right for block A.) Only 40% stated that the force exerted by string #1 was greater than that exerted by string #2. The other two most common responses were to say that the tensions were equal and that the tension on string #2 was greater. Students who gave the latter response used the reasoning that the accelerations were equal, F = ma, and $m_A < m_B$ to say that the force exerted by string #2 was greater. As McDermott *et al.* state, "these students seemed to believe that the force exerted by each string depended only on the mass of the block to which it was directly attached and which it was pulling forward." Students who stated that the tensions were equal (20%) are quoted as saying "it is the same force," and "the force exerted on string 1 goes through [block A] onto string 2." This implies that students believed that the force exerted by string 1 was transmitted through block A to string 2. Further analysis of a similar question, not discussed here, showed that this thinking was robust in more advanced situations. Furthermore, graduate students asked the same question had similar difficulties (though only 40% were incorrect).

Figure 2-16



Atwood's machine and Modified Atwood's machine apparatus. In both cases, a string is stretched between two masses and the string hangs over a pulley. The UW research project involved an investigation of both apparatuses. See reference26 for further discussion.

Figure 2-15

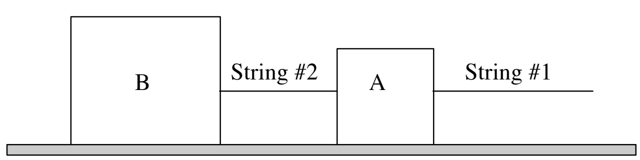


Diagram from the UW pretest. A hand was pulling to the right on string #1. Students were told to assume the strings were massless. They were asked to compare the acceleration of Blocks A and B and to compare the forces exerted on Blocks A and B. See reference 26 for further discussion.

In summary, the students answering the question incorrectly failed to isolate each block and identify the forces acting on it. Also, many failed to correctly analyze that string #2 was pulling on both blocks, not just block B. Thus, in applying Newton's second law to a situation like the Atwood's machine, they were unable to adequately describe which "F" and which "m" to use, even when most knew the "a" was the same for both masses.

To address these difficulties, McDermott *et al*. designed a tutorial to address student difficulties. *Tutorials*[27] are a research-based instructional method developed at UW which place students in small groups and get the students to actively think through the physics content of the worksheets they are completing. Tutorials replace traditional TA-led recitations. The worksheets are designed to challenge students and their understanding of a physical situation and the model they use to understand the situation. Students without a functional understanding of the material (i.e. unable to apply the conceptual ideas relevant to the situation to new and novel topics) will have difficulty with the material and will be helped to develop a functional understanding.

The premise of tutorials is *elicit-confront-resolve*.[26] First, tutorials are designed to *elicit* from students any difficulties they might have with the material by asking for a prediction of a physical situation that has been shown through research to be difficult for students. Then, questions asked in the worksheet or by the facilitator-TA *confront* students with observations or reasoning which contradict students' incorrect predictions. Finally, once students have been confronted with inadequacies (if any) in their understanding, they are led to a *resolution* that helps them gain a deeper understanding of the physics involved.

For the student, the tutorial cycle consists of four aspects. Students take a brief pretest during lecture every week. Pretests are conceptually based, non-graded quizzes which usually follow lecture discussion of a topic. Most commonly, pretests are given after students have completed homework problems dealing with the physical topic addressed in the pretest. After the pretest, students participate in tutorials (attendance is not mandatory, but at UMd, 85% to 100% of the students attends tutorial section). Students have tutorial-based homework which give them the opportunity to apply and develop the ideas they have learned in tutorial in order to further build their functional understanding of the material. Finally, on each examination, one question is based on tutorial materials. These examination questions also help evaluate student performance based on tutorial instruction.

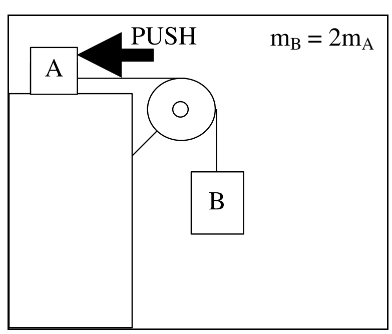
To provide students with an opportunity to develop their understanding of tension, the UW researchers developed a set of activities related to the question in Figure 2-15 and the Atwood's machine apparatuses shown in Figure 2-16. Students are asked to analyze situations where two blocks on a table are in contact with each other (a hand pushes one block which pushes another), where two blocks are connected by a massive string (a hand pushes the first which then pulls the second), and where two blocks are connected by a massless string (and a hand pushes the first block which pulls the second).

In the tutorial,[28] students are required to apply the concepts and skills they have learned in class, such as Newton's second law, free body diagrams, and Newton's third law to analyze the situation. Questions are designed to elicit difficulties that have been found through the analysis described above. Students analyze each situation in detail before moving on to the next, getting help from the TAs in the classroom as needed. For example, many students have difficulty making correct free body diagrams of the strings (both massive and massless strings). Also, many students have difficulty isolating each of the masses in their analyses. After a series of exercises, students extend their understanding by applying the concepts they have worked on to new situations. For example, they repeat the above analyses with friction between the blocks and the table. They also apply their developed reasoning to the actual Atwood's machine.

To investigate whether students who participated in tutorial instruction came to a deeper understanding of the material than students who did not, McDermott *et al.* asked identical examination questions of two different student populations. In one lecture-only class, students had four lectures a week, while in two tutorial classes, students had three lectures and one tutorial a week. As the authors state, "none of the tutorials had dealt with the particular systems involved." Also, all classes used identical textbooks.

Student understanding of tension, as measured by their performance on an examination problem (shown in Figure 2-17) was significantly better than before, though not as good as an instructor would hope. In the examination question, students consider a modified Atwood's machine. They are asked to compare the tension in a string when a force holding a mass in place is removed. The most common incorrect response students gave was to say that the tension would not change since only block A was affected by the removal of the force. In other words, the students were looking only at the local information about block A and not the entire system. In the non-tutorial class, only 25% of the students gave the correct response (that the tension was now less than the weight of block B, since the block would accelerate downward). In the tutorial classes, more than 50% gave this response. McDermott *et al.* point out that far fewer students treated the blocks and string as independent systems. Thus, students who had participated in tutorial were able to think of the global system more

Figure 2-17



Examination question asked at UW to investigate student understanding of tension after instruction. Students are told that masses A and B are originally at rest. Students were asked how the tension in the string would change when the force holding mass A in place was withdrawn. The question was answered by both tutorial and non-tutorial students. See reference 26 for further discussion.

clearly than students who had received traditional lecture instruction on the same material.

In addition, tutorial students were better able to use skills not specific to the situation but important for a detailed understanding of physics, such as "drawing free body diagrams… identifying third law force pairs, and … analyzing dynamical systems qualitatively." Also, while non-tutorial students gave primarily justifications based on algebraic formulas, the tutorial students applied dynamical arguments to the questions. The evidence suggests that tutorials, though only replacing one hour of instruction a week, give students the opportunity to develop their reasoning and skills in ways that traditional instruction does not.

The authors find that the Atwood's machine tutorial addresses student difficulties with tension in such a way that students gain the basic and fundamental skills they need in their study of physics. As they point out, "the emphasis on concept development that characterizes the tutorial materials is not intended to undermine the need for instruction on problem-solving procedures." Instead, the success of the tutorial lies in part with the idea that they do not teach by telling, but provide an opportunity for students to "integrate the counterintuitive ideas that they encounter in physics into a coherent framework" by giving students "multiple opportunities to apply the same concepts and reasoning in different contexts, to reflect upon these experiences, and to generalize from them."

Summary

In this chapter, I have presented evidence that PER can play an important role in helping instructors gain an understanding of student difficulties with physics. PER can also help instructors develop effective instructional materials that provide students with the opportunity to improve their understanding of physics. These materials can be investigated to measure their effectiveness, such that a recurring cycle of research, curriculum development, instruction, and research is put in place. The curriculum development described in this chapter dealt with issues in mechanics, but other areas of physics have also been investigated.

For example, investigations have shown that students have difficulties with some of the fundamental concepts of wave physics. Some of these concepts, such as the mathematics, the distinction between local and global phenomena, and the role of initial conditions, provide physics education researchers with an opportunity to investigate ideas that are important to an overall understanding of physics. To investigate student understanding of waves, one must first summarize the model that we would like our students to learn. By emphasizing the conceptual background in the model of wave physics taught in the introductory courses, we are able to focus our attention on the most fundamental ideas that we would like our students to learn in our courses. Published PER results on student difficulties with waves suggest that students have profound problems that hinder them from developing as deep an understanding of physics as we would like. Furthermore, many of the difficulties that have been described seem related to one another and to other PER results in areas such as kinematics and mechanics. This suggests that a detailed investigation of many areas of wave physics will give researchers a window into how students develop their understanding of physics.

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[1] For a detailed review of the needs and goals of PER, the reader is referred to the UMd dissertation of Jeffery M. Saul. Saul focused on student beliefs and attitudes toward physics and the role of these beliefs on student performance on conceptual and quantitative problems.

[2] The method described for the analysis of transcripts generally falls under the description of phenomenography. For more details, see Marton, F., "Phenomenography – A Research Approach to Investigating Different Understandings **21**:3 28-49 (1986).

[3] See reference 1 for a detailed discussion.

[4] For example, the work done here at UMd has focused on student difficulties with Newtonian physics with respect to the relationship between Force and velocity or Newton's third law; see Redish E. F., J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **65** 45-54 (1997).

[5] Clement, J., "Students' preconceptions in introductory mechanics," Am. J. Phys.**50**, 66-71 (1982).

[6] See both Halloun, I. A, and Hestenes, D. "The initial knowledge state of college **53**, 1043-1055 (1985); and Halloun, I. A, and Hestenes, D. "Common sense concepts about motion," Am. J. Phys.**53**, 1056-1065 (1985).

[7] McCloskey, M, "Naïve theories of motion," in *Mental Models*, edited by D. Gentner and A. Stevens (Lawrence Erlbaum, NJ 1983) 299-324.

[8] Similar results have been discussed in another context by Trowbridge and McDermott. See Trowbridge, D. E. and L. C. McDermott, "Investigations of student understanding of the concept of velocity in one dimension," Am. J. Phys.**48**, 1020 (1980); "Investigation of students' understanding of the concept of acceleration in one **49**, 242 (1981).

[9] Hestenes, D. "Modeling instruction in mechanics," Am. J. Phys.**55**, 440-454 (1987).

[10] For example, in my classroom experience, I find that students often include inappropriate forces, such as Third Law force pairs and forces exerted by the object rather than those exerted on the object. This result has been investigated in more detail by many researchers; see, reference 9 and references cited therein.

[11] A different possible initial condition may also describe the shape of the string at all locations for a specific instant in time, though the creation of a wave using this method is quite difficult. (But, it is a simple way to use the shape of a string at a given instant in time as an initial condition for all future events).

[12] For example, Alonso and Finn, *Physics*, Tipler, *Physics*, Wolfson and Pasachoff, *Physics Extended with Modern Physics*, and others…

[13] More details can be found in the Mel Sabella's dissertation research at the University of Maryland, College Park. Sabella has found that students often treat an extended period of time as if all events occurred at the same time. Sabella, Mel, Edward F. Redish, and Richard N. Steinberg, "Failing to Connect: Fragmented Knowledge in Student Understanding of Physics," *The Announcer* **28**:2 115 (1998).

[14] Maurines, L., "Spontaneous reasoning on the propagation of visible mechanical *Int. J. Sci. Ed.*, **14**:3, 279 (1992).

[15] Snir, J., "Making waves: A Simulation and Modeling Computer-Tool for Studying Wave Phenomena," Journal of Computers in Mathematics and Science Teaching, Summer 1989, 48 - 53.

[16] See Grayson, D. J., "Using education research to develop waves courseware," Comput. Phys. **10**:1, 30-37 (1996). Also, see Grayson, D. J., "Use of the Computer for Research on Instruction and Student Understanding in Physics," dissertation, University of Washington, Seattle, 1990.

[17] See both McDermott, L. C. "Research and computer-based instruction: Opportunity **58**, 452-462 (1990) and Grayson, D. J. and L. C. McDermott, "Use of the computer for research on student thinking," Am. J. Phys. **64**, 557-565 (1996).

[18] An overview of student conceptions of sound waves can be found in Linder, C. J., "Understanding sound: so what is the problem," Phys. Educ. **27**, 258-264 (1992).

[19] The original research is described in two papers: Linder, C. J., "University physics students' conceptualizations of factors affecting the speed of sound propagation," Int. J. Sci. Ed. **15**:6, 655-662 (1993) and Linder, C. J. and Erickson, G. L., "A study of tertiary physics students' conceptualizations of sound," Int. J. Sci. Ed. **11**, 491-501 (1989).

[20] Personal communication from J. Snir. The graduate student who had been conducting the research did not complete the project and no further findings were published.

[21] See reference 8, reference 17, and also Beichner, R. J. "Testing student interpretation of kinematics graphs," Am. J. Phys. **62** 750-762 (1994).

[22] Minstrell, Jim "Explaining the 'at rest' condition of an object," Phys. Teach. **20** 10-14 (1982).

[23] For a discussion on the usefulness of asymmetric pulses in studying student difficulties with waves, see the discussion in chapter 9 (specifically, p. 202) of Arons, A. B., *A Guide to Introductory Physics Teaching* (John Wiley & Sons Inc., New York NY, 1990).

[24] McDermott, L. C., P. S. Shaffer, and the Physics Education Group at the University of Washington, *Instructor's Guide for Tutorials in Introductory Physics* (Prentice Hall, New York NY, 1998).

[25] See reference 4 for more details. Also, see Steinberg, R. N., M. C. Wittmann, and E. F. Redish, "Mathematical Tutorials in Introductory Physics," AIP Conf. Proc. **399**, 1075-1092 (1997), for a description of materials discussed in more detail in chapter 6.

[26] McDermott, L. C., P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. **62**, 46-55 (1994).

[27] McDermott, L. C., P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice Hall, New York NY, 1998).

[28] Available as part of the materials in reference 27.

Chapter 3: Student Difficulties with Wave Physics

Introduction

From the Fall, 1995 (F95) to the present, I (together with other member of the Physics Education Research Group (PERG) at the University of Maryland (UMd)) carried out a series of investigations of student understanding of the physics and mathematical description of mechanical waves on a taut string or spring. (I will use notation "F95" or "S96" throughout the dissertation to describe Fall or Spring semesters and their years.) Student difficulties with the physics and the mathematical description of wave propagation and with the superposition of waves were investigated. From F96 onward, we also investigated student difficulties with sound waves and the propagation of waves through air. The research methods used in these investigations have been introduced in chapter 2.

We find that many students have profound and meaningful difficulties with fundamental ideas and concepts not just of wave physics but of the general ideas and approaches of physics which are often taken for granted in physics instruction yet which students must learn in our classes. For example, many students are unable to functionally describe the meaning of a disturbance to the equilibrium state of a system. Many are unable to adequately describe the concept of linear superposition, having great difficulty in considering many different points at once. The mathematics which describe wave propagation also cause trouble for students, and it seems that misinterpretations of the physics guide many students in their misinterpretation of the mathematics. We have also found evidence of the opposite, that students use misinterpretations of mathematics to guide their reasoning about the physics. These results are specific to the investigation of wave physics, but the manner in which we find students unable to build a coherent and functional understanding of the physics may cause problems for their study of physics in many other subjects.

Research setting

All data for this dissertation were collected from students in the Physics 262 class at the University of Maryland, College Park (UMd). Physics 262 is the second of a three semester, introductory, calculus-based physics course for engineers. Topics covered in the course include hydrostatics and hydrodynamics, oscillations, waves, heat and temperature, and electricity. Students are required to have taken physics 161 (or an equivalent course in Newtonian mechanics), and they are also required to be enrolled in a calculus II (or higher) course. Physics 262 has a required laboratory that meets once a week.

In the discussion section that accompanies the course, students participate in either a traditional TA-led recitation or a tutorial. In the TA-led recitation sections, a TA typically works through problems at the board. In some recitations, the TA leads a broader discussion in which some students might solve problems at the board, but the focus is still on a single person, and most of the students are not highly engaged in the discussion. Tutorials are a research-based instructional setting that replaces recitations, as has been described in chapter 2.

Students cover the topic of waves in a three or four week period (depending on the professor). Topics include wave propagation, superposition, intensity, power, wave harmonics, and usually the Doppler shift and other advanced topics. Since the more advanced ideas depend on an understanding of the basic ideas that students learn at the beginning of their study of waves, we have focused our research on the basic concepts and fundamental ideas of waves.

Chosen wave representations

To investigate student understanding, we often ask questions about the physics in an unfamiliar context that requires students to use what we hope is familiar physics. When studying waves, students often encounter only infinitely (or very) long waves which stretch (effectively) from negative to positive infinity.[1] As has been described in chapter 2, we have often chosen to investigate student understanding of wave physics by using wavepulses rather than wavetrains.[2] By a *wavepulse*, we mean a single localized disturbance that propagates along the string. By a *wavetrain*, we mean an infinitely (or very) long (e.g. sinusoidal) disturbance (see Figure 2-6).

One of the goals of PER is to see how students are able to carry over their understanding from one setting or topic to another. Our decision to investigate student understanding with wavepulses rather than wavetrains allowed us to see more clearly how students were thinking of the propagation of a wave. We could also see how students approached superposition. With a sinusoidal wavetrain, the mathematics to describe the wave is simpler than for a wavepulse, but it becomes difficult to visualize the motion of the medium due to the propagating disturbance. It also becomes difficult to interpret student responses (both sketches and descriptions) if a repeating pattern is used. Rather than simplifying the mathematics for students, we used wavepulses to find how students made sense of wave physics on a conceptual level.

Student Difficulties With Wave Propagation: Mechanical Waves

A wave is a propagating disturbance to a system. The medium of the system does not propagate with the wave and is not permanently displaced from equilibrium. Previous research (see chapter 2) has shown that students have difficulties separating the initial conditions of a system through which a wave propagates (i.e. the manner in which the wave is created) from the propagation of the wave itself. This point is often neglected when discussing wave physics, where it is possible to discuss relevant and important concepts contained in the wave equation without ever discussing the initial conditions of the system. We find that students are unable to distinguish between the manner in which a wave is created and the manner in which the wave propagates along a string.

Investigating student understanding

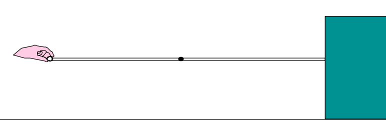
We chose to investigate how students view the relationship between how a wave is created and how the wave propagates through the system with a variety of instruments or probes. Interviews provided us with detailed descriptions of howa small number of students view the physics. With the understanding of possible student responses gained through an analysis of interviews, we can come to a better understanding of student written responses that we can give to much larger populations of students.[3]

The general question asked in all our interviews and written questions involved a taut elastic string held on one end by a hand and on the other end attached to a distant wall. A correct answer to the questions shown in Figure 3-1 and Figure 3-2 would indicate that the speed of a wave traveling along a taut string or spring depends only on the tension and mass density of the medium. The manner in which the disturbance is created does not affect the speed of the wave.

In the question shown in Figure 3-1, students were asked to describe what physical parameters could be changed to change the speed of the wave on a taut string. Even though the question asked for possible physical parameters that could affect the speed of the propagating wave (implying properties of the string on which the wave propagated, not the manner in which the wave was created), many students stated that the motion of the hand would play a role in the speed of the wave. The wording of the question may have lead more students to answer the answer the question correctly (tension and mass density are physical parameters), since some students might not consider the hand a physical parameter of the system.

Because the original wording of the question may have guided students away from their personal beliefs about the correct answer, we changed the wording of the question in later questions to the more open-ended wording shown in the free response (FR) question in Figure 3-2 (Version 1). The multiple-choice, multiple-response (MCMR) question (Version 2 of Figure 3-2) was developed in order to investigate the same student difficulties in a different fashion. In this type of question, students are asked to give all possible correct responses. They are offered a long list of possibly

Figure 3-1



A long string is attached to the wall as shown in the picture below. A red dot is painted along the string between the hand and the wall. A single pulse is created by the person holding the string and moving it up and down once. The string is firmly attached to the wall, and cannot move at that point.

When a pulse travels along a taut, elastic string, we can measure its velocity. What physical parameters could be changed to change the velocity of the pulse?

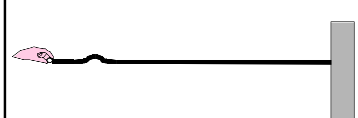
Wave propagation Question, Fall-1995, pre-instruction, N=182. Note the phrasing of the question, implying that only physical parameters can change the speed of the pulse.

correct responses. While reminding students of the correct answer, the offered responses could also remind students of many possible incorrect responses. Details of how the FR and MCMR question were asked at different times will be given when specific data are discussed.

Two sets of interviews dealt with student understanding of wave propagation concepts. In S96, nine students were asked an FR question nearly identical to the one shown in Figure 3-2 during an interview. In S97, 18 students first answered the FR question and then the MCMR question. The interviewer did not allow them to go back to change their answer on the FR question (an answer already captured on videotape). Two students interviewed during the same investigation answered only the MCMR question. The S97 interviews were part of a diagnostic test that will be described in greater detail in chapter 7.

Because of interview responses, we made two modifications to the original MCMR question. First, in the S96 interviews, we found that some students were giving an answer that we had not included in F95. They used the idea of the force needed to create the wave to explain changes in wave speed. They usually referred to the "force of the wave" when giving this explanation. This response will be described in more detail below. Second, we included the possibility of "none of the above" to give students the opportunity to describe their own model of wave propagation, even in the MCMR question. Responses *i*, *j*, and *k* were included in the MCMR question from S96 onward.

Figure 3-2



Version 1: Free-Response (FR) format: A taut string is attached to a distant wall. A pulse moving on the string towards the wall reaches the wall in time t_0 (see diagram). How would you decrease the time it takes for the pulse to reach the wall? Explain your reasoning.

Version 2: Multiple-Choice, Multiple-Response (MCMR) format:
A taut string is attached to a distant wall. A demonstrator moves her hand to create a pulse traveling toward the wall (see diagram). The demonstrator wants to produce a pulse that takes a <u>longer time</u> to reach the wall. Which of the actions *a–k* **taken by itself** will produce this result? More than one answer may be correct. If so, give them all. Explain your reasoning.

a. Move her hand more quickly (but still only up and down once by the same amount).
b. Move her hand more slowly (but still only up and down once by the same amount).
c. Move her hand a larger distance but up and down in the same amount of time.
d. Move her hand a smaller distance but up and down in the same amount of time.
e. Use a heavier string of the same length, under the same tension
f. Use a lighter string of the same length, under the same tension
g. Use a string of the same density, but decrease the tension.
h. Use a string of the same density, but increase the tension.
i. Put more force into the wave.
j. Put less force into the wave.
k. None of the above answers will cause the desired effect.

Free response (FR) and Multiple-choice, multiple-responses (MCMR) versions of the wave propagation question. Answers *e* and *g* are correct in the MCMR question, and we considered answers like *e* and *g* to be correct on the FR question.

Discussion of student difficulties

In this section, I will first describe student comments in interviews and then give a statistical overview of their responses to written questions.

After students have completed instruction on waves, many still use ideas of force and energy incorrectly when answering the free response (FR) question shown in Figure 3-2 Version 1. In both the S96 and S97 interviews, students had difficulties with the fundamental concepts of wave propagation. Some students used reasoning based on the force exerted by the hand to create the pulse. One student stated, "You flick [your hand] harder...you put a greater force in your hand, so it goes faster." Other students state that creating a wave with a larger amplitude takes greater force and thus the wave will move faster. Some students state that shaking your hand harder (in interviews, this was usually accompanied by a quick jerk of the hand) will "put more force in the wave." Another student used reasoning based on energy to describe the effect of a change in hand motion. He stated, "If we could make the initial pulse fast, if you flick [your hand], you flick it faster... It would put more energy in." This student is failing to distinguish between the velocity of the hand, which is associated with the transverse velocity of the string, and the longitudinal velocity of the pulse along the string.

To many students, the shape of the wavepulse also determines its speed. One student stated, "Make it [the pulse] wider, so that it covers more area, which will make it go faster." In follow-up comments, this student explained that it took more energy to create a larger pulse, and that the pulse would move faster because it had more energy. We have also found that some students state that <u>smaller</u> pulses will move faster. "Tinier, tighter hand movements" will allow the wave to slip more easily (thus, faster) through the medium.

Students rarely give only one kind of explanation in interviews. They can use both correct and incorrect reasoning to describe changes to wave propagation speed. One student described how to make a slower wave in the following way:

> *Well, I know that tension affects the wave speed. … [And] the amplitude would affect it {the student shows a hand motion with a larger displacement but same time length}. I think possibly, you see a slower pulse … if the force applied to the string is reduced … that is: the time through which the hand moves up and down [is reduced].*

Though the student starts with the correct response, he then describes a mixture of incorrect ideas: the "size" of the hand motion, the "force" applied to the string, and the speed of the hand motion. Of note is that there was a long pause between the correct response and the other, incorrect responses. During this time, the student was obviously thinking of the physics, so the interviewer remained quiet. Had the interviewer immediately asked a new question, the insight into the student's understanding would not have been as deep and the student's true understanding of the physics would not have been uncovered.

The initial conditions that determine the creation of the wave and its size play a large role in student explanations which use force and energy in their reasoning.

Though the properties of the physical system determine the wave speed and the initial conditions do not, many students believe the initial conditions play a role in propagation speed. Since a hand is used to create the wave, student explanations seem to make use of an active agent that creates the waves. This interpretation is consistent with previous findings by Maurines[4] and also with the Impetus model described in chapter 2.

Student use of multiple explanations was also observed on written questions. In F97, we asked both FR and MCMR questions on diagnostic tests at the beginning of the semester before all instruction and near the end of the semester after all instruction on waves had been completed. In the beginning of the semester, students first answered the FR question, turned it in, and were then handed the MCMR question. This ensured that they did not change their answers on the FR question as a result of seeing the list of MCMR options. During the semester, instruction consisted of lecture, textbook homework problems, and tutorials designed to address the issues discussed in this paper. (The instructional materials will be discussed in more detail in chapter 6.) The data from before and after instruction illustrate the difficulties students have even after working through specially designed research-based materials. After all instruction, students answered the FR and MCMR questions in successive weeks as a supplement to their weekly pretests given during lecture.

By comparing student responses on the FR and MCMR questions, we can probe the distribution of ideas used by students to understand the physics of wave speed. Table 3-1(a) shows how students answered both the FR and MCMR questions before instruction. Only those students who answered both FR and MCMR questions both before and after instruction are included (i.e. data are matched). Students' written explanations echo those given during interviews. By comparing student responses on the two question formats, we can see how consistently students think about wave speed.

At the beginning of the semester, very few students give only the correct answer, but most of them include it in the responses to one of the two questions. Almost all of the students answer that the hand motion will affect the wave speed.

Students predominantly use only one explanation when answering the FR question. The offered responses on the MCMR question appear to act as triggers that elicit additional explanations, especially from students who give the hand motion response on the FR question. Of the few students (9%) who answered the FR question using only correct reasoning, most answered the MCMR question consistently (78%). These students seem to have a robust understanding of the dependence of wave speed on medium properties. However, more than three-fourths of the students emphasize the incorrect hand motion response at the beginning of the semester (77% of the students give the hand motion response on the FR question).

Table 3-1(b) shows student responses at the end of the semester (after modified instruction, described in more detail in chapter 6). Student performance is somewhat improved, with more students giving completely correct explanations. Nearly all students (98%) recognize the correct answer on the MCMR question, but a majority of the class (58%) still believes that changes in hand motion play a role.

Table 3-1

(a)

	Speed changes due to change in:	Student responses on free response question: Only tension and density	both the medium and hand motion	the motion of the hand	other
Student responses On MCMR question	only tension and density	7%	1%	2%	1%
	both the medium and hand motion	1%	2%	60%	10%
	the motion of the hand	1%	1%	11%	3%

(a) Comparison of student pre-instruction responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions before all instruction.

(b)

	Student Response:	Student responses on free response question: Only tension and density	both the medium and hand motion	the motion of the hand	Other
Student responses On MCMR question	Only tension and density	40%	2%	2%	2%
	Both the medium and hand motion	8%	17%	20%	2%
	the motion of the hand	2%	1%	2%	0%

(b) Comparison of student post-instruction responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions after all instruction on waves.

In both the pre and post instruction tables, the most common off-diagonal elements of the tables show that students who answer the FR question using only hand motion explanations are triggered into additionally giving correct medium change responses on the MCMR question. Apparently, they recognize the correct answer, but do not recall it on their own in an FR question. Because fewer students are triggered in the other direction (from correct medium change explanations to additionally giving the hand motion response), we believe that the quality of understanding of those students who give the correct FR response is higher than those who are triggered to give multiple explanations. Nevertheless, it is noteworthy that so many of the students answer incorrectly even after explicit instruction on the topic. The issue of instruction will be discussed in more detail in chapter 6.

In summary, we find that students do not make a distinction between the initial conditions and the medium properties of the system. We see that most students give correct answers to describe changes to wave motion when offered the correct response, even before instruction, but they often do not think consistently about the physics, even after instruction. In a later part of the dissertation, I will discuss how individual students are able to give more than one response to describe a single physical situation, and how students use more than one model to think of waves.

Student Understanding of With Wave Propagation: Sound Waves

We have also investigated student understanding of the fundamental issues underlying a consistent physical picture of the nature of sound. Our findings show that the difficulties described in the previous section appear here as well. We find that students are unable to separate the medium from the wave, possibly because they are unable to interpret how they visualize the system in which the wave is traveling.

Investigating student understanding

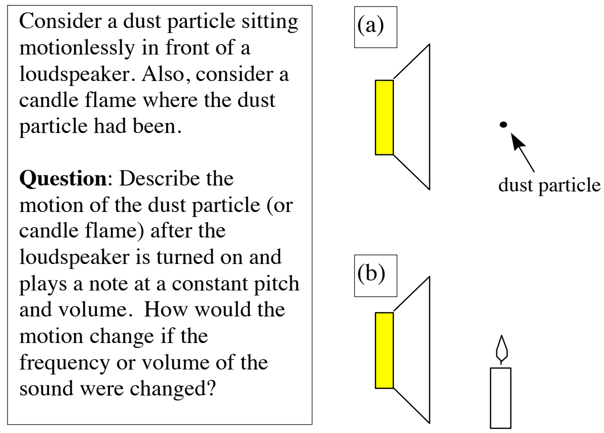
To investigate how students distinguish between the motion of the wave and the medium, we posed two different types of questions about sound waves (see Figure 3-3). We asked students to describe the motion of a dust particle sitting motionlessly in front of a previously silent loudspeaker after the speaker had been turned on (Figure 3-3(a)). In addition, we asked students to describe the motion of a candle flame placed in front of a loudspeaker (Figure 3-3(b)).

The physics of these two situations merits discussion. The dust particle, we told the students in interviews, is floating motionlessly in a room with no wind (i.e. no outside air currents). This is plausible, when considering that buoyancy can support a dust particle of the right density at the desired height. The lack of air currents is not plausible when considering the candle flame because the heat from the candle causes convection currents. These currents only occur in the near vicinity of the candle, though, and add little to the effective size (i.e. width) of the candle. For both systems and at the appropriate size and time scale, we can assume that the medium through which the sound waves travel is motionless except for the motion from equilibrium caused by the sound waves themselves.

In both questions, we asked about audible frequency sound waves, between 10 and roughly 5,000 Hz. Assuming a speed of sound in air of roughly 340 m/s, this gives a range of wavelengths between 7 cm and 34 m. All of these wavelengths are much greater than the size of either the dust particle or the candle flame (roughly 1/2 to 1 cm wide). The shortest wavelengths occur at a frequency that is already outside of the common frequencies heard on a daily basis in speech or in music. (The highest of these are usually around 2000 Hz, giving a wavelength of 17 cm.) Based on our choice of dust particle size and candle flame size, we can treat them as point particles which move in response to the motion of the medium in which they are embedded.

We expected students to point out that the dust particle and the candle flame would oscillate longitudinally from side to side due to the motion of the air. We expected that the detailed physics of the differences between the dust particle (or candle flame) and the medium of air discussed in the previous paragraph were beyond the level of all the students probed . None ever raised these issues. This is consistent with our use of interviews to help determine the state space of possible responses

Figure 3-3



Two different situations in which the sound wave question was asked, (a) the dust particle sound wave question, (b) the candle flame sound wave question. In interviews, students were not given a diagram, but had a loudspeaker and a candle and were asked to imagine the dust particle or the candle flame. In pretests and examination questions, students had a variety of diagrams, all equivalent to the ones shown.

students might give in a situation. Though we were prepared for the discussion, the students had difficulties with different fundamental issues.

Two sets of individual student interviews gave us insight into how students made sense of the physics. In the first (F96), 6 students answered questions related to the motion of both the dust particle and the candle flame. These students had completed lecture instruction on sound waves and most were above average (getting either an A or B in the course) according to their descriptions of their grades.[5] They were asked to describe the motion of the object, if any, once the loudspeaker was turned on. They were also asked how that motion would change if the frequency and the volume of the loudspeaker were changed (and the dust particle or candle flame began its motion in the same location as the original object). In the second set of interviews (S97), twenty students who had completed either traditional or tutorial instruction in waves answered the dust particle question. In these interviews, a multiple-choice, multiple-response (MCMR) format version of the dust particle question was given. Because of the interview setting, it was possible to probe their responses to the question in this format to see how they arrived at their answers and how they were using the offered responses to choose their own beliefs about the movement of the dust particle. Some students seemed to focus on only the first instant of motion of the dust particle away from the speaker and did not state that there was motion due to the rarefaction of air until asked to extend their first response in time. The question responses were subsequently rephrased to account for this possibility and to suggest to students that they consider more than just one instant in time. The final version of the MCMR question is shown in Figure 3-4.

Once we had used interviews to describe student difficulties with sound waves in detail, we administered a variety of written questions to gain an understanding of how common these difficulties were in the classroom during the course of the semester. We asked the dust particle questions in three different situations: before any instruction on waves, after traditional instruction on waves, and after traditional and tutorial instruction. In one semester (F97), we asked students both the FR and MCMR versions of the dust particle question after they had completed instruction on waves. We asked and collected the FR question first to ensure that students would not change their response based on the offered MCMR answers.

Discussion of student difficulties

We found that students' difficulties did not change during the course of the semester, but the frequency with which they occurred did change depending primarily on the type of instruction that students had on waves. The state space of responses that we had developed using the interviews was therefore productive in describing student difficulties at all times of instruction.

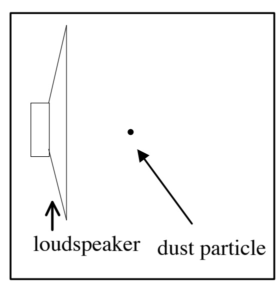
In the interviews carried out in F96, we found that most students had great difficulty separating the propagation of the sound wave from the motion of the medium through which it travels. One student's responses were representative of the reasoning used by 4 of the 6 students in the interviews.

"Alex" (names used are aliases chosen by the interviewed students), described how the dust particle would be pushed away by the sound wave. In the following quotes, the interviewer is referred to with "I" and Alex with "A."

> I: *The loudspeaker is turned on, and it plays a note at a constant pitch. Could you describe the behavior of the particle after the speaker is turned on?*

Figure 3-4

A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant (low) pitch. **Which choice or combination of the choices** *a–f* (listed below) can describe the motion of the dust particle after the loudspeaker is turned on? **Circle the correct letter or letters.** Explain.



Possible responses for question 2:
a) The dust particle will move up and down.
b) The dust particle will be pushed away from the speaker.
c) The dust particle will move side to side.
d) The dust particle will not move at all.
e) The dust particle will move in a circular path.
f) None of these answers is correct.

MCMR format sound wave question, F97, N=92 students (matched) answered this pre and post-instruction on waves.

> A: *It should move away because the sound vibration, the sound wave is going away from the speaker, especially if constant pitch means you have one wave going … It's going to move away from the center.*

Later, when asked the same question in a slightly different fashion, Alex stated:

> A: *It would move away from the speaker, pushed by the wave, pushed by the sound wave … I mean, sound waves spread through the air, which means the air is actually moving, so the dust particle should be moving with that air which is spreading away from the speaker.*
>
> I: *Okay, so the air moves away --*
>
> A: *It should carry the dust particle with it.*
>
> I: … *How does [the air] move to carry the dust particle with it?*
>
> A: *Should push it, I mean, how else is it going to move it? [Alex sketches a typical sine curve.] If you look at it, if the particle is here, and this first compression part of the wave hits it, it should move it through, and carry [the dust particle] with it.*

Here, Alex was describing the peak of the sine wave exerting a force on the dust particle, only in the direction of propagation.

Alex had a clear and complete description of the motion of the dust particle due to the sound wave. He believed that a sound wave consisted of air moving away from its source, and that the dust particle would therefore move with the air, away from the speaker. The sound wave provided the force which acted on the particle to make it move away from the speaker. Alex did not use the idea of rarefaction during the interview.

To see whether Alex used this description even when the physical situation changed, he was asked the following question:

> I: *We have the same loudspeaker, and we create the same situation as previously. We have the loudspeaker turned off, and you place a new piece of dust, exactly like the previous one, in the same location as before. Now you turn the speaker on, but rather than having the original pitch, the frequency of the note that is produced by the speaker has been doubled ... How would this change the answer that you've given?*
>
> A: *That would just change the rate at which the particle is moving. … The wave speed should be, it should double, too. … Yeah, speed should increase.*
>
> I: *How did you come to that answer?*

> A: *I was thinking that the frequency of the wave, a normal wave, shows us how many cycles per some period of time we have. … You can have twice as many cycles here in the same period of time. …*
>
> I: *And what effect does it have to go through one cycle versus to go through two cycles?*
>
> A: *If it goes through one cycle of the compression wave like this, then the first wave should hit it here [points to the peak of the sine curve that he had previously sketched]. And … the second wave which has frequency which is twice as big should hit it twice by then, which should make it go faster.*

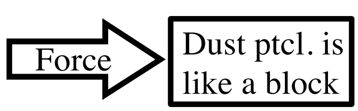
Due to a hand motion that he made repeatedly when referring to the "hit" on the particle, Alex was asked the following question:

> I: *So each compression wave has the effect of kicking the particle forward?*
>
> A: *Yeah.*
>
> I: *So when you've been kicked twice, you're moving twice as fast?*
>
> A: *Basically, yeah. Right, because the force … [referring to a sketch he drew, like the one in Figure 3-5] If you have a box, and you apply a force once, the acceleration is, force equals mass times acceleration, you can find the acceleration. Then, if you apply the same force a second time to the same object, you give it more, more, well, it just moves faster.*

I have given these lengthy interview excerpts to show the robustness with which Alex could describe his conceptual understanding of sound waves and to show the general difficulties Alex had with basic and essential physics concepts.

Alex's misinterpretation of frequency illustrates how students can use language correctly but misinterpret its meaning. He stated, "the frequency of the wave, a normal wave, shows us how many cycles per some period of time we have," but he was unable to use his definition when describing the physical behavior of the system. At another point in the interview, he indicated that he thought the wavelength of the sound wave would stay constant when the frequency changed. At no point during the interview did he state that the speed of the sound wave depended on the medium properties of the air through which it traveled. In an equation like $v = f\lambda$, he was free to choose one of the

Figure 3-5



Alex's sketch of the sound wave exerting a force on the dust particle. Alex described the wave exerting a force on the dust particle (and later candle flame) only in the direction of wave propagation.

variables to remain constant. He could not clearly explain why he believed the wavelength was constant.

Another point of interest is his confusion between acceleration and velocity (possibly the confusion between acceleration and impulse). The description that the sound wave exerts a force only in the direction of wave propagation shows that Alex thinks of the leading edge of the wave pushing everything in front of it away from the sound source, much like a surfer riding on an ocean wave. To explain the surfer analogy, he described the motion of a ring on a string on which a pulse is propagating. The effect of a pulse on a small ring placed on the string was to push it along. (Alex gave a partially correct answer for a ring that is on a string; the wavepulse will make the ring move in some longitudinal fashion that depends on the angle of the string as the wavepulse passes by.) He used the term "impulse" to describe the wavepulse <u>and</u> the effect of the wavepulse on its surroundings.

> A: *This impulse will hit the ring here, and … should go and make it move forward, the same way it should be with a dust particle in the air.*

When I asked him the effect of changing the volume of the sound produced, he stated the following:

> A: *I guess I'm not thinking physics too much. … [I'm thinking of a] stereo system at home, if you turn it up, you can feel the vibration from farther away from the speaker, so basically [the dust particle] should move, once again, it should move faster.*
>
> I: *What effect did changing the volume have on the compression wave?*
>
> A: *Increased the amplitude…*
>
> I: *And that has the effect of the compression wave moving faster?*
>
> A: *Not quite, it just hits the particle with more force. … If you kick the thing, instead of kicking it faster, you're just kicking it harder. It's going to move faster.*

Again, Alex described the effect of the wave exerting a force only in the direction of propagation to make the dust particle move forward. Of the 6 students who participated in the interviews, four gave similar descriptions of the effects of the sound wave on the dust particle.[6]

Of the other two students, one student gave responses which were inconsistent, stating the correct answer (horizontal oscillation) while also stating that the dust particle would not move. Even with continued questioning, the student was unable to provide a clear response, showing that a profound confusion lay behind the student's correct responses. It is possible that this student would perform very well on examinations where the student is aware of the correct answers that the instructor is seeking, but still not have an actual understanding of the physics of sound waves.

The last student interpreted the common sinusoidal graph used to describe sound waves (either pressure or displacement from equilibrium as a function of time or position) as a picture rather than a graph and used this misinterpretation to guide his reasoning. He described transverse motion by the dust particle (and no motion by the candle flame, since it was unable to move up and down due to its attachment to the wick). This student misinterpreted the common sinusoidal graph of sound waves (where the vertical axis describes *horizontal* displacement from equilibrium as a function of position or of time), and used this misinterpretation to guide his understanding of the motion of the medium. He described that the longitudinal compression of the sound wave would squeeze the dust particle to push it up or suck it back down (due to the "vacuum" caused by a sort of rarefaction between longitudinal waves). The longitudinal wave would cause transverse motion in the dust particle. The detailed physical explanation this student gave is indicative of how seemingly simple misunderstandings (reading a graph as a picture) can have a profound effect on how students come to make sense of the physics they learn.

When asking the dust particle question of many students, we have found that lecture instruction had little effect on student understanding of the relationship between the motion of sound waves and the motion of the medium through which they travel. Table 3-2 shows (unmatched) student responses from the beginning of S96 and the end of F95. Students answered a slightly different version of the question in which the loudspeaker was enclosed in walls to form a tube. A non-trivial number of students (roughly 10% in both cases, listed within the "other oscillation" category) sketched standing wave patterns (e.g. sinusoidal standing waves with the correct nodes and antinodes at the end of the tube) to describe the motion of the dust particle. The tube walls were removed in later questions to remove this source of student confusion, but the result is an important one. Students seem to pick the familiar details or surface features of a problem to guide their reasoning in their responses. The tubes triggered a response based on common diagrams with which they were familiar, but this response showed the difficulties that students have in understanding the material.

Table 3-3 shows student explanations from F97 to the dust particle question before instruction, after traditional instruction, and after modified instruction (described in more detail in chapter 6). We see that very few of the students enter our courses with a proper understanding of the nature of sound wave propagation. Before instruction, half the students state that the sound wave pushes the dust particle away from the speaker. Some, like Alex, describe the dust particle moving in a straight-line path. Others describe the dust particle moving along a sinusoidal path away from the speaker. The latter students seem to misinterpret the sinusoidal graph of displacement

Table 3-2

Time during semester: MM used:	Before all instruction S96 (%)	Post lecture F95 (%)
CM (longitudinal oscillation)	14	24
Other oscillation	17	22
PM (pushed away linearly or sinusoidally)	45	40
Other and blank	24	14

Comparison of student responses describing the motion of a dust particle due to a loudspeaker. Data are from F95 post instruction and S96 pre-instruction and are not matched (S96, N=104. F95, N=96).

Table 3-3

Time during semester: Explanation:	Before all instruction (%)	Post lecture (%)	Post lecture, post tutorial (%)
Longitudinal oscillation	9	26	45
Other oscillation	23	22	18
Pushed away linearly or sinusoidally	50	39	11
Other	7	12	6
Blank	11	2	21

Student performance on sound wave questions before, after traditional lecture, and after additional modified tutorial instruction. Data are matched (N=137 students). The large number of blank responses in the post-all instruction category is due to the number of students who did not complete the pretest on which the question was asked.

from equilibrium as a picture of the path of the particle. After specially designed instruction, student performance has improved greatly, but lingering difficulties remained. The curriculum materials and an analysis of their effectiveness will be described in chapter 6.

With sound as well as with mechanical waves, students have great difficulty distinguishing between the medium and the propagating disturbance to the medium. The difficulties we have found include:

- the use of surface features of the problem and misinterpretations of graphs to help (mis)guide reasoning about sound wave propagation, and
- the use of descriptions of force and pushing to describe the movement of the medium only in the direction of wave propagation (like a surfer riding a wave).

Student understanding of both mechanical and sound waves indicates that their functional understanding of the physics is not as robust as we would like. Their focus on surface features of the problem indicates that they are unsure of their understanding of the material and will try to make sense of the situation using inappropriate clues in the problem. Their focus on forces that are originally exerted on the system to create the wave and make it move forward indicates that they are not thinking correctly about the relationship between the creation and the propagation of waves.

Student Understanding of the Mathematics of Waves

One of the fundamental topics of wave physics when it is first introduced is the mathematical description of propagating waves. Students are confronted with functions of two variables, often for the first time. The difficulties they have with the mathematics of waves (hereafter referred to as wave-math) can have lasting effects on their understanding of such advanced topics as the wave equation (often not covered in the introductory sequence), the propagation of electromagnetic radiation, and the mathematical description of quantum mechanics. The difficulties that we observe should therefore indicate what sort of problems students might have with mathematics at later stages in their careers. We find that many students do not have a good understanding of how an equation can be used to describe a propagating wavepulse, and we find that some students have serious difficulties interpreting the meaning of the equation which describes the wavepulse at a given instant in time.

Investigating student understanding

We investigated students using both interviews and written questions on the following issues:

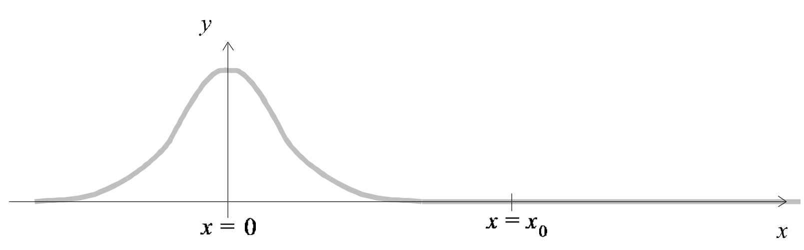
- the mathematical transformation that describes translation of a disturbance through a system, and
- the physical interpretation of the mathematics that describe propagating waves.

The question shown in Figure 3-6 presents students with an unfamiliar setting in which to describe wave motion. Students most commonly encounter sinusoidal shapes when discussing waves due to the ease of the mathematical interpretation and the usefulness of the sinusoidal description in physics. By presenting students with a Gaussian pulseshape, we are able to probe their understanding of the mathematics of wave propagation while ensuring that they are not responding by using partially recalled responses from previous questions. Part A of the question asked students to sketch the shape of a (Gaussian) wavepulse traveling to the right that had propagated a distance x_0 along a taut string. Part B asked students to write an equation to describe the shape of the string at all points once the wavepulse had traveled a distance x_0 from the origin.

Figure 3-6

Consider a pulse propagating along a long taut string in the $+x$-direction. The diagram below shows the shape of the pulse at $t = 0$ sec. Suppose the displacement of the string at this time at various values of x is given by

$$y(x) = Ae^{-\left(x/b\right)^2}$$



A. On the diagram above, sketch the shape of the string after it has traveled a distance x_0, where x_0 is shown in the figure. Explain why you sketched the shape as you did.

B. For the instant of time that you have sketched, find the displacement of the string as a function of x. Explain how you determined your answer.

Wave-math question answered by N=57 students in S96. The question has since been used in other semesters and in interviews with individual students.

We considered a response to part A to be correct when students showed the pulse displaced an amount x_0 and the amplitude essentially unchanged, as shown in Figure 3-7(a). We considered any answer to part B that replaced x with $x - x_0$ to be correct.

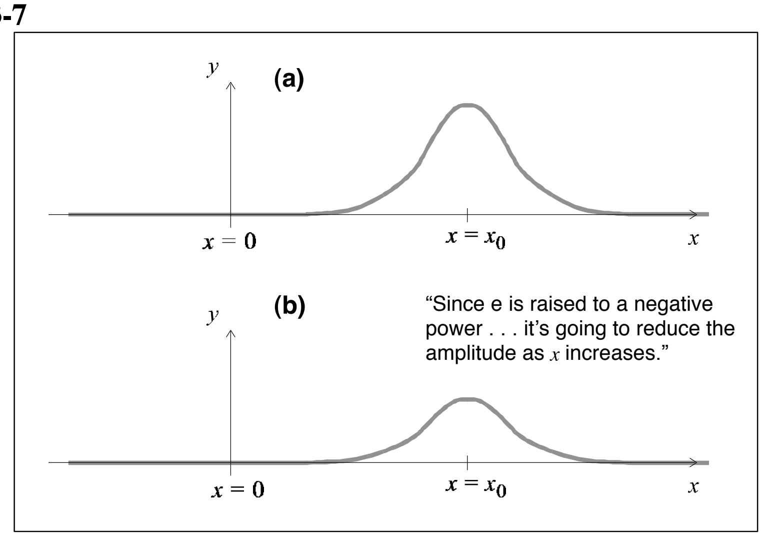
Three different student populations participated in individual interviews. In the first, students (N=9) had not seen the wave-math question before. In the second, four students (in a different semester) had seen the wave-math question in a post-lecture pretest (i.e. pre-tutorial quiz) they had taken within the previous 48 hours. We used these interviews to validate the students' pretest responses. In the third student population, ten students answered the wave-math question in a diagnostic test two months after they had traditional and tutorial instruction on waves.

In addition to the interviews, which provided the basis for our understanding of student difficulties, we asked the wave-math question in a number of written pretests. These pretests were given after students had traditional instruction but before they had tutorial instruction on the mathematics of waves. As in the other areas that have been investigated, the nature of difficulties did not change according to where the students were in their instruction, only the frequency with which a group of students had specific difficulties changed.

Discussion of student difficulties

Students often used misinterpretations of the mathematics to guide their reasoning in physics or they used misinterpretations of the physics to guide their understanding of the mathematics. Though most students describe the physical shape of the propagated wave correctly, those who do not have a consistent incorrect answer. This provides an opportunity for us to gain insight into the ways in which students

Figure 3-7



Correct and most common incorrect response to the wave-math problem in Figure 3-6. (a) A correct sketch of the shape of the pulse at a later time, showing the amplitude unchanged, (b) An apparently correct sketch of the shape of the pulse showing the amplitude decreased - but typically accompanied by incorrect reasoning.

arrive at an incorrect understanding of physics.

In S96, the correct answer was given by 44% of the students who were interviewed and 56% of the students who took the pretest. Most of the rest of the students (56% of the interview-students and 35% of the pretest-students) drew a pulse displaced an amount x_0, but with a decreased amplitude, as shown in Figure 3-7(a). On the surface this appears to be a reasonable response in that it is consistent with what would actually happen as a result of the physical phenomena (not mentioned in the problem) of friction with the imbedding medium and internal dissipation. However the explanations given by students suggest that they are not adding to the physics of the problem but are misinterpreting the mathematics. All of the interview students and many of the pretest students cited the equation describing the shape of the string at $t = 0$ as the reason for the decrease in the amplitude.[7] As one interviewed student[8] said, "Since e is raised to a negative power . . . it's going to reduce the amplitude as x increases."

But the exponential given in this problem represents a decrease in y in space (at $t = 0$) and not time. These students are failing to recognize that x corresponds to a variable which maps a second dimension of the problem, not the location of the peak of the pulse. Also, these students are interpreting the variable y as the peak amplitude of the wavepulse, not the displacement of the string at many locations of x at different times, t.

One student was explicitly misled by the mathematics of the Gaussian function, though he originally stated the correct response.

> *Okay. Umm … Let's see. "Sketch the shape of the spring after the pulse has traveled* (Mumbling as he rereads the problem) *… Okay. Over a long, taut spring, the friction or the loss of energy should not be significant; so the wave should be pretty much the exact same height, distance, -- everything. So, it should be about the same wave. If I could draw it the same. So, it's got the same height, just a different X value.*
>
> *No, wait. Okay, "… the displacement of* (More mumbling, quick reading) *… is given by" – B, I guess, is a constant, so – It doesn't say that Y varies with time, but it does say it varies with X. So – that was my first intuition – but then, looking at the function of Y … Let's see, that – it's actually going to be – I guess it'll be a lot smaller than the wave I drew because the first time – X is zero, which means A must be equal to whatever that value is, because E raised to the zero's going to be 1. So, that's what A is equal to. And then as X increases, this value, E raised to the negative, is going to get bigger as we go up. So, kind of depending on what V is… Okay. So, if X keeps on getting bigger, E raised to the negative of that is going to keep on getting smaller. So the – So the actual function's going to be a lot smaller. So, it should be about the same length, just a lot shorter in length.*

This student describes the physics (including relevant approximations and idealizations) correctly, but then revises his physical understanding to fit his misinterpretation of the mathematics. Here we see a clear example of the way (first discussed when comparing student response to the FR and MCMR question) that students can have two conflicting descriptions of the same situation. In this case, the mathematics triggers in the student a form of reasoning that contradicts the simple physical description the student originally used.

Part B of this problem asked about the mathematical form of the string at a later time. We considered any answer that replaced x with $x - x_0$ to be correct. However, none of the S96 interview students and fewer than 10% of the pretest students answered this way (see Table 3-4).

The most common incorrect response was to simply substitute x_0 for x in the given equation. These students write constant functions that have no x-dependence, $y(x) = Ae^{-\left(x_0/b\right)^2}$ or $y(x_0) = Ae^{-\left(x_0/b\right)^2}$. This response was given by 67% of the interview students and 44% of the pretest students. All students drew a string with different values of y at different values of x, yet many of them wrote an equation for that shape with no x dependence. There were other students who wrote a sinusoidal dependence for y, again in conflict with what they drew for the shape of the string. Even after instruction on waves, many students seemed to be answering the mathematical part of this problem independently of the way they were answering the physical part. In another class (where students had participated in traditional instruction on this subject), a modified version of this question was asked on a post-instruction midterm examination. In this case, 45% of the students gave a sinusoidal answer to mathematically describe the shape of a pulse.

In S97, the pretest was asked of a another class (which had the same modified instruction). The percentages of correct and incorrect responses were nearly exactly the same as for S96, as shown in Table 3-4. More detailed analysis of student responses showed that 2/3 of those students who drew a smaller amplitude displaced wave explicitly mentioned the exponential in the equation when explaining how they

Table 3-4

	Example(s)	**% of interview respondents (N=9)**	**% of pretest respondents (N=57)**
correct response	$y(x) = Ae^{-\left(\frac{x-x_0}{b}\right)^2}$	0%	7%
Constant function with no dependence	$y(x) = Ae^{-\left(x_0/b\right)^2}$ $y(x_0) = Ae^{-\left(x_0/b\right)^2}$	67%	44%
Sinusoidal	$y(x) = \sin(kx - \omega t)$	22%	2%
Other	$x = b\,\ln\left(\frac{y}{A}\right)$; $\frac{dy}{dx} = -\frac{2x}{a^2}Ae^{-\left(x/a\right)^2}$	11%	47%

Student use of functions to describe a propagating Gaussian pulseshape. Students were asked to write an equation to describe the shape of the string once the pulse had moved a distance x_0 from the origin.

arrived at their answer (i.e. 25% of the class used this reasoning). The other 1/3 of the students describing a smaller amplitude wavepulse gave many different reasons, the most common being that the variable "b" described a damping constant, so the amplitude must be smaller. These students are using a surface feature of the equation (the variable "b," used in their textbook to describe the damping constant in air resistance) to interpret the physics. Again, we see that students have difficulties interpreting the mathematics they are presented and use a variety of interpretations of the physics to guide their reasoning.

The difficulties described in this section include students failing

- to recognize the relationship between the physical situation and the associated equation,
- to understand the meaning of a function, and
- to treat a coordinate axis as a mapping of a dimension.

The interpretations that students give the mathematics focus only on the point of maximum displacement. The misinterpretation of a wavepulse as a single point of displacement rather than an extended area of displacement implies that students are thinking of waves differently from how physicists understand waves.

Student Understanding of Wave Superposition

For multiple mechanical waves traveling through a one-dimensional system, the concept of linear superposition describes the summation of the displacement due to each wave. As described in chapter 2, superposition occurs at each location in space (i.e. the sum of displacement occurs locally and due to local influences), but every location in the system must be considered (i.e. one must do the addition everywhere, or globally). The distinction between local and global phenomena is subtle in this situation, though not new to students who have used free body diagrams of extended bodies in their previous physics courses. Also, the topic is of great importance for later studies in physics. We find that students have difficulty understanding wave superposition to occur on a point-by-point basis, and some students have a "collision" model of wave superposition related more to particle mechanics than to wave physics. In wave "collisions," waves are treated much like objects that bounce off each other, such as carts or gliders on air tracks.

Investigating student understanding

In our investigations, we focused on three different elements of the physics of wave superposition where students might have difficulties. We investigated student understanding of superposition for

- the instant when the peaks of waves overlapped,
- the instant when the wave overlapped and the peaks of the waves did not, and
- an instant some time later, when the waves were no longer overlapping at all and had passed through each other.

Our questions used wavepulses rather than wavetrains so that we could clearly separate what students thought was happening.

We chose these three topics in superposition for three reasons. First, students often are asked about wave superposition in instances where sinusoidal waves overlap either perfectly constructively or perfectly destructively. By asking for a sketch when peaks are not overlapping, we are able to investigate whether students add displacements due to each wavepulse at all points along the string or only at the peaks. Second, by asking for the sketch when the peaks overlap exactly, we see how students sketch the shape of the entire pulse, and if they change the width of the pulse in addition to its amplitude. (Student comments in office hours led to this question.) Finally, students rarely consider what happens to superposed waves after they no longer have an effect on each other; wavetrains in problem sets never end, so the issue never arises. By asking for a sketch long after the peaks have passed through each other, we can investigate what ideas the students have about possible permanent effects of the wavepulses on each other. We have used the same three time periods in our questions, time limitations permitting.

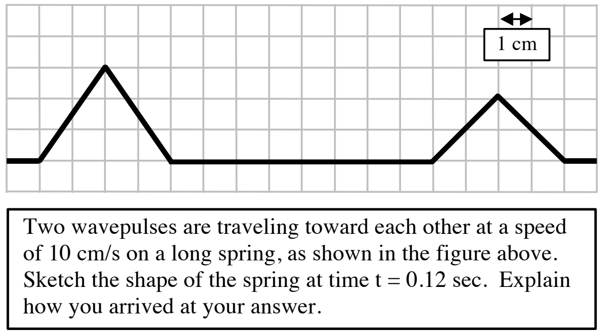
A variety of questions was used to investigate student understanding of superposition. Figure 3-8 shows two wavepulses on the same side of a string propagating toward each other. Figure 3-10 shows two wavepulses on opposite sides of a string propagating toward each other. In both cases, students were asked to sketch the shape of the string at the three times described above. Correct responses to the questions shown in Figure 3-8 and Figure 3-10 are shown in Figure 3-9 and Figure 3-11, respectively.

In each of the questions, a correct response would show point-by-point addition of the displacement due to each wavepulse at every point along the string. Furthermore, wavepulses that had superposed and then separated would look exactly as they did before interacting, without any sign of a permanent effect on each other. One of the reasons for the chosen representation of wavepulses was to facilitate the drawing of these sketches and to allow easier interpretation of student sketches.

Two sets of interviews on the topic of superposition were carried out. In S96, in a tutorial class, four volunteers answered the pretest question shown in Figure 3-12 in an interview that came after their lecture instruction on the material but before any tutorial instruction. This allowed us to validate the written responses we saw on pretests by comparing them with the more detailed verbal responses students given in interviews.

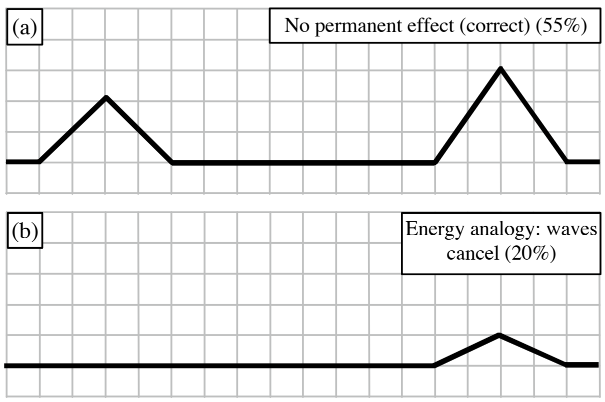
In diagnostic test interviews carried out in S97 with twenty students who had completed either traditional or tutorial instruction on waves, we asked a series of questions similar to the ones shown in Figure 3-8 and Figure 3-10. These were given in multiple-choice format, and students had a long list of possible responses from which to choose. Each response could be a possible correct answer for more than one question, and students were aware that they could use the same response more than once when answering up to five different questions. (This is a variation of a multiple-choice, multiple-response question, as described in the wave propagation section above.) Because these questions were asked during an interview, it was possible to

Figure 3-8



Wave superposition question from a diagnostic test given, Fall-1995 semester, N = 182 students. Students had no instruction on waves when they took this diagnostic.

Figure 3-9



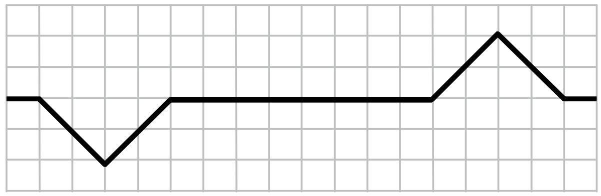
Common responses to diagnostic question from Fall-1995. (a) Correct response, (b) Most common incorrect response.

follow up on student responses and gain insight into the reasoning they used to explain their understanding of physics.

In the S96 semester, after we had developed a tutorial to address student difficulties with superposition, we asked a pretest question shown in Figure 3-12. This pretest followed lecture instruction on the basic concepts of waves (including superposition) but preceded the tutorial on wave superposition. Rather than using symmetric wavepulses of different amplitudes, we chose to use asymmetric wavepulses with the same amplitude. Though we had found interesting student ideas about the permanent effects of wavepulses meeting, we wanted to investigate in more detail how students did or did not use superposition when only parts of the pulses (but not the peaks) overlapped. The correct responses and the most common incorrect responses are shown in Figure 3-13.

During the F97 semester, we modified the pretest question from S96 and asked for an additional sketch of the string when the peaks overlapped but the bases of the pulses no longer perfectly overlapped. This question was asked in pre-instruction and post-instruction diagnostic tests.

Figure 3-10



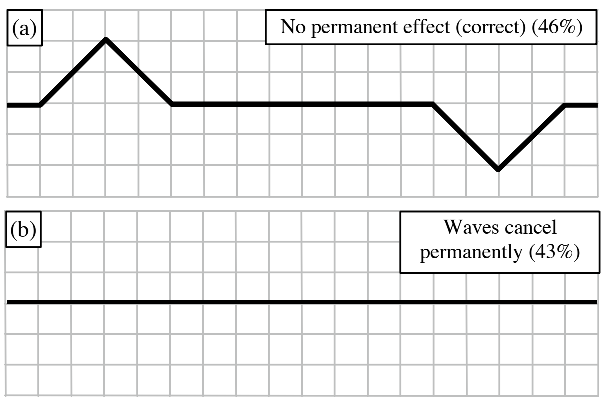
Two wavepulses are traveling toward each other on a long, taut string.

a. Sketch the shape of the string at the moment of maximum overlap. Explain.

b. Sketch the shape of the string a long time after the moment of maximum overlap. Explain.

Wave superposition question from a diagnostic test given, Fall-1995 semester, N = 182 students. Students had no instruction on waves when they took this diagnostic.

Figure 3-11



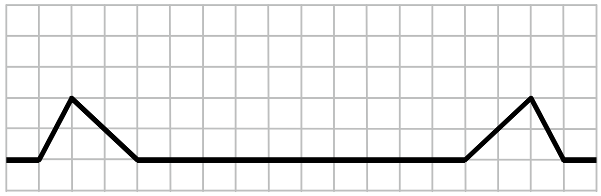
Common responses to part *b* of the diagnostic question in Figure 3-10, Fall-1995. (a) Correct response, (b) Most common incorrect response.. Note that response (b) is correct for part *a* of the question in Figure 3-10.

Discussion of student difficulties

Our results show that students have difficulties with each of the three areas of wave superposition investigated in our questions. As in the other areas of wave physics, a few student difficulties dominated the responses. These difficulties did not change during the course of instruction, but the frequency of their occurrence did. I will first discuss student descriptions of permanent effects of wavepulses on one another. Then I will describe the superposition of waves whose peaks do not overlap, and finally I will describe the superposition of waves whose peaks do overlap.

In the F95 pre-instruction diagnostic test, 182 students answered the question shown in Figure 3-8. A correct response to the question, given by 55% of the students (see Figure 3-9(a)), shows that the wavepulses pass through each other with no

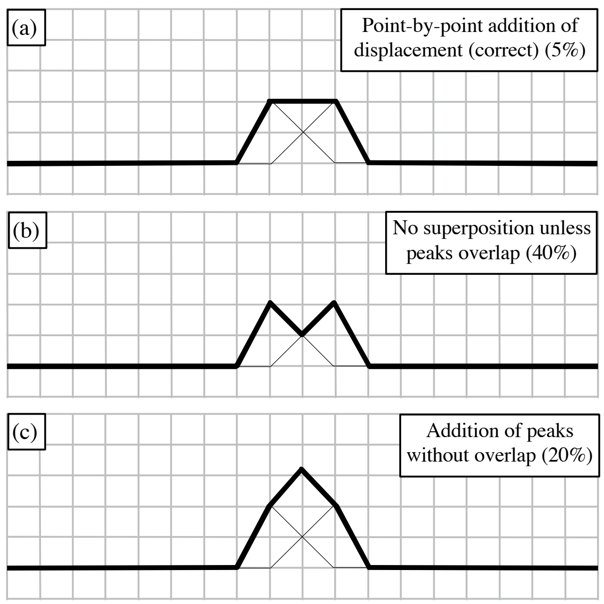
Figure 3-12



Two wavepulses are traveling toward each other at a speed of 10 cm/s on a long spring, as shown in the figure above. Sketch the shape of the spring at time t = 0.06 sec. Explain how you arrived at your

Wave superposition question from pretest given after traditional instruction, Spring-1996, N= 65. Students had completed lecture instruction on superposition.

Figure 3-13



Common responses to pretest question from Spring-1996. (a) Correct response, (b) common incorrect response, (c) common incorrect response. These responses were given on pretests and in interviews which followed lecture instruction on superposition and preceded tutorial instruction.

permanent effect on each other. One student summarized the most common incorrect response, given by 20% of the students (shown in Figure 3-9(b)), by saying "[Part of] the greater wave is canceled by the smaller one." A further 8% of the students state that the wavepulses bounce off each other.

In explanations, students implied that they were thinking of wave interaction as a collision. If we imagine two carts of unequal size moving toward each other at the same speed and colliding in a perfectly inelastic collision (imagine Velcro holding them together), then the unit of two carts would continue to move in the direction the larger was originally moving, but at a slower speed. The size of the pulse, in this situation, seems to be analogous to the momentum or energy of the pulse. One student's comment (given when answering a similar question in a later semester) supports this interpretation: "The smaller wave would move to the right, but at a slower speed." These students appear to be thinking of wavepulses as objects that collide with each other or cancel one another out.

Of the 182 students who answered the question on destructive interference in Figure 3-10 before instruction, 43% had difficulties with the question related to the ideas of bouncing or canceling waves. Of the other students, 10% did not answer the question, and 46% correctly indicated that the wavepulses continue in their original directions with their original shapes. The correct response and the most common incorrect responses are shown in Figure 3-11. We did not further investigate student understanding of destructive interference because their difficulties were similar to (though usually more common than) the difficulties students had with constructive interference. Students described the waves canceling out or bouncing off of each other much like they did with unequal amplitude waves interfering constructively. We believe that the students who described the waves bouncing off each other interpreted the shapes of the waves such that the wavepulses had equal strength or size. Like in a perfectly elastic collision between billiard balls, the wavepulses would bounce off one another, rather than cancel each other out completely and permanently.

When investigating student understanding of superposition when waves overlap but their peaks do not, we find that many students have a different type of difficulty than thinking of the waves as colliding. Very few students were able to answer this question correctly on the pretest (only 5% sketched Figure 3-13(a)). Of the students who said that there was no superposition unless the peaks of the pulses overlapped (40% of the students sketched Figure 3-13(b)), a common explanation was that "the waves only add when the amplitudes meet."

We have found that students giving this explanation use the word "amplitude" to describe only the point of maximum displacement, and they ignore all other displaced points in their descriptions. These students view superposition as the addition of the maximum displacement point only and not as the addition of displacement at <u>all</u> locations.

Other students also had difficulty with the process of wave addition. One-fifth of them sketched Figure 3-13(c) and stated that the points of maximum displacement would add even though they weren't at the same location on the string. This question was also asked in an interview setting. One interviewed student who used the word "amplitude" incorrectly, as described above, explained, "Because the [bases of the] waves are on top of each another, the amplitudes add." This student uses the base of the wave (its longitudinal width) rather than the (transverse) displacement of a point on the wave to guide his reasoning about superposition.

In investigating student difficulties with wave propagation, we found that students were using more than one explanation to guide their reasoning. We find similar results in our investigations of student difficulties with superposition. One student who answered the question in Figure 3-12 drew a sketch like the one shown in Figure 3-13(c). He explained,

> *[the pulses] are both colliding, and as they collide … if two of the same amplitude were to collide, it would double their amplitude. And so I believe this amplitude would get higher… They would just … come together.*

This student was using the idea of a collision between waves to explain how the amplitudes (inappropriately) add up to make a larger wave. He did not use the collision analogy to describe the waves canceling each other out, though, and gave the correct response for the shape of the string after the wavepulses had passed each other. Rather than showing an explicitly incorrect prediction on his part, his comments give evidence of the analogies he used to guide his reasoning. (As previously noted, students using the collision analogy often state that waves of equal size bounce off each other and do not cancel out, so their shapes will be the same once the waves have "passed through each other," which, in the case of a bounce, they have not done.)

In summary we observe that students have the following difficulties in understanding the physics of wave superposition:

- Waves are described as if they were solid objects which can collide with each other, bounce off each other, or permanently affect each other in some way.
- A wavepulse is described only by its peak point, and no other displaced parts of the system are superposed. When peaks do not overlap, the highest point due to a wave is chosen rather than the sum of displacements due to each wave. When the peaks of wavepulses do overlap such that the waves then add, only the peaks add.

In general, we find that students show difficulty with the concept of locality and uniqueness of spatial location. Students describe wavepulses with single points rather than as extended regions which are displaced from equilibrium, much like they did when answering the wave-math problem.

Summary of Specific Student Difficulties with Waves

In this chapter, I have described student difficulties with wave physics in the context of the propagation of mechanical waves on a taut string or spring, the propagation of sound waves, the mathematics used to describe waves, and superposition. In each case, the context has been used to uncover more fundamental difficulties with wave physics.

From the research into student understanding of wave propagation speed, we see that students have difficulty differentiating between the manner in which a wave is created and the manner in which it propagates through a medium. Many do not understand the fundamental idea of a wave as a propagating disturbance. Instead, as is suggested by the results from student descriptions of sound waves, some students believe that the wave actually exerts a continuous force in the direction of motion. Many students seem to have difficulty with the idea of the equilibrium state of a system. Student difficulties with mathematics indicate that the inability to understand a wavepulse as a disturbance to the medium plays a role in how students interpret the mathematics of waves. Student descriptions of superposition indicate that students also have difficulty describing the interaction between two waves and do not think of a wave as an extended region of displacement from equilibrium. The concept of a propagating disturbance, its cause, its effects, and the manner of its interaction with its surroundings are all difficult for students.

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[1] For example, look at the textbooks by Tipler, Serway, or Halliday and Resnick, where few problems involve wave phenomena that deal with finite length disturbances from equilibrium.

[2] Arons, A. B., *A Guide to Introductory Physics Teaching* (John Wiley & Sons Inc., New York NY, 1990). 202-218.

[3] The investigation of student difficulties with the relationship between the creation of waves and their propagation through the system is similar to the research previously done by Maurines. See chapter 2 for a discussion of her findings.

[4] Maurines, L. “Spontaneous reasoning on the propagation of visible mechanical **14**:3, 279-293 (1992).

[5] We did not have access to the students’ grades, so we relied on their comments for this statement. We have found that students are usually accurate in their knowledge of their grades and are often more pessimistic than necessary about their future grade.

[6] Our results are consistent with those observed by Linder and Erickson, as described in chapter 2. While Linder and Erickson have focused on issues of what students mean by sound and how they think of the medium, our focus has been on student use of force to guide their reasoning on this topic. Many of Linder and Erickson’s interpretations apply to our observations, as our interpretations also apply to their observations.

[7] In the interview format, we had the opportunity to obtain an explanation from all of the students. In the pretests, not all of the students give explanations, but those who do cite the exponential as the reason for the decay.

[8] This student was among the best in his class, and finished the course with the highest grade of all students.

Chapter 4: A Proposed Model of Student Learning

Introduction

One goal of physics education research is to go beyond the discovery and recitation of difficulties that students have with a specific topic in physics. By trying to organize how we see students approaching the material, we have the opportunity to gain deeper insight into how students come to make sense of the physics they are taught in our classrooms. We can then use our organization of student difficulties (and strengths) to help develop curriculum materials that more effectively address sometimes subtle and counter-intuitive student needs. This chapter presents a brief discussion of a possible organization of student difficulties according to a model of learning that will be used in chapter 5 to analyze the data presented in chapter 3.

To describe how students learn in our classroom, we need to develop a meaningful language that lets us describe, organize, and systematically discuss our observations of student reasoning.[1] Other fields, generally organized under the name cognitive studies, can provide a source of understanding and suggest models that help us make sense of student learning in physics. Their validity in the physics education research often lies in the suggestions these models make rather than in their exact details, but these suggestions can play a profound role in the manner in which we approach our classrooms.[2] Those readers less interested in the details of this learning theory are asked to read the conclusions of this chapter and Table 4-1 for a summary of the ideas contained in it.

Reasoning Primitives

Consider a simple action that is common and repeated often enough that it is not even consciously considered, e.g. pushing an object previously at rest across a surface. An effort must be exerted to get it moving. Similarly, when delegating work to another person, it is often necessary to motivate this person so that the work is begun. Though the two situations have little to do with each other, both are examples of the need for an "actuating agency" to set events (or objects or people) in motion.[2] The actuating agency can be thought of as a reasoning primitive common to many different settings.

In this sense, a primitive is a common and small logical building block that lets us describe basic elements of common events in many different situations. A suitable analogy can be made to the way physicists and chemists think of the atom. In many settings, the atom is the smallest relevant description of nature. One atom (the primitive) can be part of many different types of molecules (the situation). Of course, the substructure of the atom is of great interest, but not always relevant to the specific model one is considering. In the same way, one can discuss elements of primitives and how they develop, but the primitive itself is a relevant grain size (as discussed in chapter 2) for discussion. We can think of primitives as the building blocks with which people build their thinking.[3] Primitives can help simplify both everyday and physics reasoning situations.

For example, the common use of *actuating agency* can help explain some of the results described in chapter 2. In Clement's coin toss problem, students describe the effort needed to throw the coin in the air and speak of this "force" remaining with the coin as it rises. In this description, students use the actuating agency primitive when talking about the force exerted to set the coin in motion, but additionally assume that the force stays with the coin after it is released from the hand. Thus, students make sense of the physics of the coin toss problem by incorrectly over-applying an otherwise useful abstract idea that helps simplify our predictions about what happens when an object should be set in motion.

The most productive and relevant discussion of the use of primitives in physics has been carried out by diSessa[4,5] and by Minstrell.[6] diSessa's work has focused on very general reasoning elements used in a variety of situations including physics, such as the actuating agency described above,[7] while Minstrell's work has focused on how students apply primitives specifically in their reasoning in physics.

Table 4-1

Primitive	Definition	Example (mechanics related)
Force as mover	"A directed impetus acts in a burst on an object. Result is displacement and/or speed in the same direction."	Clement's coin toss problem as describe in chapter 2.
Working harder	"More effort or cues to more effort may be interpreted as if in an effort to compensate for more resistance."	To make a box begin to move across the floor, a larger force needs to be exerted than to keep it moving.
Smaller objects naturally go faster	Larger objects take more effort to create, see Intrinsic Resistance (to which it is related). Also related to "Bigger is Slower."	The same impulse delivered to a small object (coin) as to a large object (brick) will make the smaller one travel faster than the large one.
Intrinsic Resistance	"Especially heavy or large things resist motion."	Heavier boxes are harder to start moving across a floor (or lift up) than are lighter boxes.
Ohm's p-prim	"An agent or causal impetus acts through a resistance or interference to produce a result. It cues and justifies a set of proportionalities, such as 'increased effort or intensity of impetus leads to more result'; 'increased resistance leads to less result.' These effects can compensate each other; for example, increased effort and increased resistance may leave the result unchanged."	The speed of a coin tossed in the air depends on its mass and the force exerted on it to throw it in the air (see Force as Mover example).

Table 4-1 (continued)

Primitive	Definition	Example (mechanics related)
Dying away	"All motion, especially impulsively or violently caused, gradually dies away."	A coin tossed in the air slowly loses speed and stops (related to an impetus theory, that it has "used up" the ability to move, see chapter 2).
Guiding	"A determined path directly causes an object to move along it."	A ball traveling a circular path (guided by a wall, for example) will continue on a curved path even after the wall is no longer there (see FCI question…)
Canceling	"An influence may be undone by an opposite influence."	An object will move after one kick (see Force as Mover) and stop after another in the opposite direction.
Bouncing	"An object comes into impingement with a big or otherwise immobile other object, and the impinger recoils." (see Overcoming below.)	An small object will bounce off a large one, or two equal sized objects will bounce off each other.
Overcoming	"One force or influence overpowers another"	To get a box moving along a rough floor, the exerted effort must be larger than the resistance of the object (related to Ohm's in terms of competing proportionalities).

Primitives as defined by diSessa in his monograph (see reference4). For each primitive, a general definition is given, and an example (if possible, taken from the discussion in the chapter) is included.

General Reasoning Primitives

diSessa has developed a description of student use of primitives through observations of students' interpretations and generalizations of the everyday phenomena around them and their use of these interpretations to guide their reasoning in physics. Even though he draws his conclusions mainly from extensive investigations of student difficulties in the field of mechanics, he emphasizes the general nature of student primitives.

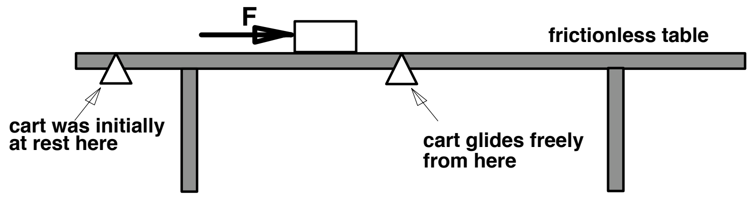
To illustrate how diSessa discusses student use of primitives, let us consider one example in detail (for a complete list of the primitives discussed in this chapter, see Table 4-1). The actuating agency primitive has already been introduced. A refinement of this primitive comes when one considers how different objects with different properties (such as different masses) are to be brought into motion. Consider two boxes with different masses resting on the same rough surface. The goal is to set them in motion. More effort will be needed to move a larger box. The physics of the situation is complicated, requiring an understanding of normal forces, friction (both the threshold nature of the friction force and difference between static and kinetic friction), and Newton's Second Law. A simpler way to think of the situation is to use the reasoning that "more requires more" (mass and effort, respectively) or "less requires less." In the simple linear reasoning that we often use, it is possible to say that the larger effort is then proportional to the resistance afforded by the larger mass such that the two boxes are set in motion in the same fashion.

diSessa refers to the compensatory reasoning based on resistance as the Ohm's primitive. The name comes from the correct physics reasoning found in Ohm's law, $V = IR$. If voltage changes, the current depends on the resistance of the circuit. We often see students use the reasoning "bigger mass requires bigger force" in our classroom interactions. This is not necessarily incorrect, but it is often overly simplistic. A more refined use of the Ohm's primitive than the example of setting a box in motion is the analysis of the acceleration of an object due to a force exerted on it, as described by Newton's Second Law, $F = ma$. In this case, the net force on the box and the acceleration of the box after the exerted force is larger than the maximum possible friction force can be compared. The effect of the force is not simply motion, as is implied by the simplistic application of the Ohm's primitive, but acceleration of the box. As illustrated by this situation, the use of the Ohm's primitive may be correct and appropriate, correct but overly simplistic, or even incorrect.

Student use of the Ohm's primitive can be seen in other, more difficult settings that are discussed at the introductory physics level. In research done at the University of Washington, students were asked to compare the change in kinetic energy and the change in momentum of two objects with unequal mass which start from rest and are moved a fixed distance by a constant force (see Figure 4-1).[8] A correct answer would

say that the change in kinetic energy was equal for the two but the change in momentum was unequal. By the work-energy theorem (Net work equals the change in kinetic energy, $\int \vec{F} \bullet d\vec{r} = \Delta KE$), both objects are moved the same distance by the same force, so their change in kinetic energy is the same. But the same force exerted on the two objects leads to a different acceleration for the two and the lighter object will have the force exerted on it for a shorter time. By the impulse-momentum theorem (i.e. the definition of force, rewritten as Impulse equals the change in momentum, $\vec{F}\Delta t = \Delta\vec{p}$), the object in motion for less time has a smaller change in momentum. We often encounter students who state that *both* the change in kinetic energy and the change in momentum should be equal. In the first case, they state that the mass is higher but the velocity is less and therefore the kinetic energy, $KE = 1/2\ mv^2$, is equal for the two objects. These students are getting the correct answer while using inexact reasoning that does not sufficiently analyze the physics. In the second case, these students again state that the higher mass and lower velocity compensate each other such that the change in momentum ($\vec{p} = m\vec{v}$) for the two objects is equal. Obviously, both cannot be true since the exponent on the velocity differs in the two equations. But we see that students are applying the Ohm's primitive incorrectly to both questions. In one case,

Figure 4-1



Two carts, A and B, are initially at rest on a frictionless, horizontal table. They move along parallel tracks (only one cart is shown in the figure above). The same constant force, F, is exerted on each cart, in turn, as it travels between the two marks on the table. The carts are then allowed to glide freely. The carts are *not* identical. Cart A appears larger than cart B and reaches the second mark <u>before</u> cart B.

Compare the *momentum* of cart A to the *momentum* of cart B after the carts have passed the second mark. Explain your reasoning.

Compare the *kinetic energy* of cart A to the *kinetic energy* of cart B after the carts have passed the second mark. Explain your reasoning.

Question asked to compare student understanding of momentum and kinetic energy. A correct answer to the first question would state that cart B spent more time being accelerated by the force, so its change in momentum (from rest) was larger. A correct answer to the second question would state that both carts had equal forces exerted over equal distances, so the change in kinetic energy (from rest) was equal for the two carts. Student responses to the question can be interpreted by means of common discrete reasoning elements, called primitives that students apply inappropriately to the situation.

though it is not linear, they get the right answer, while in the linear case, they give an incorrect response.

The Ohm's primitive involves proportional, compensatory reasoning and involves the recognition of different elements of the system. This makes it one of the more complicated primitives that diSessa describes. Rather than show how each of the primitives described by diSessa was developed and how it is used, I will describe those which will play a role in this dissertation and give examples of student reasoning which can be interpreted as using these primitives.[9] The primitives relevant to this dissertation fall into two categories, those related to force and motion and those related to collisions between objects.

Force and Motion Primitives

Three primitives effectively describe how students approach reasoning about force and motion in a way that will be important in later parts of this dissertation. These are the working harder, smaller is faster, and dying away primitives.

The working harder primitive describes the "more is more" or "less is less" element of the Ohm's primitive. This primitive describes reasoning where there is a simple linear relation between different objects and the idea of resistance is not included. Examples of the common reasoning using the working harder primitive include people who work more and get better grades or objects that have larger forces exerted on them move faster. This primitive seems very reasonable in some settings but can be easily misapplied. Force is proportional to acceleration, not velocity, for example.

The smaller is faster primitive describes how a small object is more easily made to go fast than a larger object. This is closely connected to the bigger is slower primitive. (Elephants seem slower than mice, though they usually aren't.[10]) This primitive makes sense, as long as one assumes that the same force is exerted on the light and the heavy objects (while again assuming that force is proportional to velocity and not acceleration). In terms of common sense reasoning, it is harder to move a large object than a small object (See chapter 2 for a discussion of common sense physics related to force and motion.)

Finally, the dying away primitive can be related to our existence in a frictional world. Every motion we experience eventually comes to an end. Many students generalize this inappropriately to situations such as Clement's coin toss example, given in chapter 2, where the dying away primitive plays a role in the impetus theory explanations given by students. The force that is "used up" as the coin is thrown into the air can be thought of as having "died away" in the process. In this example, we see how multiple primitives can play a role in the reasoning about a single physical situation.

Primitives Describing Collision

The collision primitives will also play a role in our descriptions of student difficulties with wave physics. These primitives include canceling, bouncing, and overcoming.

The canceling primitive is directly related to collisions and describes that motion stops when two objects collide with each other (thus, their motions have been canceled). Another example of reasoning using this primitive is the description that a box that is brought into motion by a force will be stopped by an equivalent force in the opposite direction. These forces can then be said to cancel out (even though the actual physics of the situation is more complex than such a simple description). This example illustrates how students applying primitives may ignore various elements of the problem to come up with a (in this case correct) answer through the use of overly simple reasoning.

The bouncing primitive describes the common sense reasoning used to describe a ball hitting a wall, for example. While ignoring the detailed physics of collisions, one can use the idea that objects simply bounce off of other objects that are in the way and immovable. This same reasoning (the object is in the way) plays a role in some student's descriptions of normal forces for objects lying on a surface, though the element of collisions is missing in the case of normal forces.

Finally, the overcoming primitive gives a less phenomenological and more analytical description for the same bouncing phenomena. For example, the force of the wall overpowers the force of the ball and sends the ball back from whence it came. This reasoning is very similar to the impetus theory described in chapter 2 in the sense that the moving ball has an intrinsic force that is overcome by the larger force of the wall. The confusion lies in describing force as an object or quantity specific to an object rather than the interaction between objects.[11] (This same confusion seems to play a role when students use the dying away primitive in Clement's coin toss problem.) Incorrect use of the overcoming primitive may be caused by students trying to make sense of their experiences in the language of the physics classroom rather than the real world description of the bouncing primitive (where balls just bounce off walls because that's what they do).

Facets of Knowledge: Context-Specific Interpretation of Primitives

diSessa is not the only physics education researcher to investigate the usefulness of using common elements to describe student difficulties with physics. Minstrell developed the idea of "facets" to describe the common elements of student reasoning that he found in his work as a high school teacher in Washington state.[12] Minstrell's facets are similar to diSessa's primitives in that they describe small observable relevant pieces of student reasoning. Minstrell chooses to look at specific observable elements of student reasoning, which, he states, is only possible by choosing a "grain size" of reasoning that is small enough to contain general ideas which can be applied in a great variety of situations. In the process, he focuses on the student's reasoning and not the correct physics (Compare this to the description of Halloun and Hestenes's work in chapter 2.)

As an example of the use of facets when describing student reasoning about force and motion in the classroom, Minstrell describes a set of facets commonly found in classroom discussions of the physics of motion (see Table 4-2). The Goal Facet is the desired explanation that an instructor would like to see. The others are examples of explanations that students give. The Mental Model Facet gives a broad description that links together many facets that can be applied incorrectly to a given physical situation. Note that none of the facets are always incorrect. Instead, all but the Goal Facet are often inapplicable in certain situations and are not general enough to be used in all situations.

Minstrell describes an example of the application of facets in student reasoning that comes in response to a question describing two students leaning (motionlessly) against each other, where one student (Sam) is "stronger and heavier" than the other (Shirley). Students are asked to compare the forces Sam and Shirley exert on each other. Students are offered a series of choices: Sam exerts a greater force, they exert equal forces on each other, Shirley exerts a greater force, or neither exerts a force on the other. The correct answer would be to say that they are exerting equal forces on each other (by Newton's third law). Some students state that Sam is bigger and must therefore exert a larger force (facets 475 and/or 478), but others state that they are motionless because Sam is hard to move and Shirley must be pushing, so she exerts a larger force. In a similar question, some students use the facet that "Passive objects don't exert forces." Thus, since Sam and Shirley are not moving, neither exerts a force on the other. Minstrell shows that these types of reasoning are consistently used to describe forces relating to motionless objects, moving objects, and forces caused by many different objects such as magnetic, gravitational, or pushing forces.

Student facets can be discussed as applications of diSessa's primitives to a specific setting. The answer stating that Sam is bigger and exerts a larger force is consistent with the overcoming and the Ohm's primitives (he has less resistance and therefore exerts a larger force). But the idea that Shirley must be pushing harder is also consistent with the Ohm's primitive. Thus, the same primitive can lead to contradictory facets and answers. We see that the Ohm's primitive can be considered the source primitive for facets 475 through 478 in Table 4-2.

Another example of facets as applications of primitives in a specific setting comes from the description that Sam and Shirley are exerting no forces because they are not moving. This is consistent with the actuating agency primitive, because (in this primitive) forces only occur when there is motion.

Neither diSessa nor Minstrell discuss how students come to apply specific primitives in their reasoning, nor do they discuss how students choose and use specific facets in a given setting. A variety of questions remain. How do students choose to use one or another primitive when answering specific questions about specific physical situations? How do their choices manifest themselves in the facets that we observe? And are students consistent in their use of facets? These questions play a large role in the dissertation. In later chapters, I will discuss how students come to choose specific

Table 4-2

470	Goal facet: All interactions involve equal magnitude and oppositely directed action and reaction forces that are acting on separate, interacting bodies.
472	Action and reaction forces are equal and opposite forces on the same object
475	The stronger/firmer/harder object will exert the larger force
476	The object moving the fastest will exert the greater force
477	The more active/energetic object will exert the greater force
478	The bigger/heavier object will exert the larger force
479	Mental Model facet: in an interaction between objects the one with more of a particular perceptually salient characteristic will exert the larger force.

Common facets described by Minstrell that relate to collisions between objects. Note that the xx0 facet is the "goal facet" that we would like students to have in our classrooms, while the xx9 facet is the "mental model facet" that is the organizing theme for incorrect student facets.

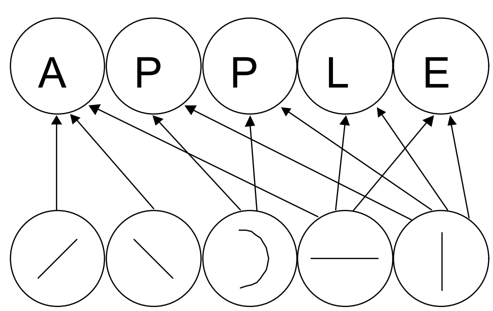
facets in a specific setting. We find that students can be described as using guiding analogies in their reasoning as they approach a specific physics setting. These analogies help determine which of the many (possibly contradictory) facets which could be applied to a situation actually are. This idea will be discussed in more detail in the section describing mental models, below.

Parallel Data Processing

In some of the examples described above, students could be described as using more than a single primitive (or facet) in their reasoning. For example, in Clement's coin toss problem, it was possible to describe some students as using both the actuating agency and dying away primitives. In order to describe the manner in which multiple primitives are used by students, we can ask how students connect primitives in their reasoning.

Consider reading the word APPLE. To perceive the individual letters in the word, one can break each letter into its simplest shapes. This creates a set of vertical, horizontal, and diagonal lines along with half circles (see Figure 4-2). Experienced readers do not read each letter based on its parts and then piece together the word from its constituent letters. Instead, the entire word is perceived at the same time. Researchers have effectively described the process of visual perception of entire words by focusing on how the individual elements of the words are perceived and interpreted in connection to each other.[13] For example, the combination of diagonal lines and a horizontal line in the right configuration creates an "A." The combination of vertical and three horizontal lines when connected correctly creates an "E." By assuming that the lineshapes are all interpreted and connected to each other at the same time (i.e. in parallel), one can describe how a finite set of symbols can form a single word. Because of the way in which many small elements are connected simultaneously to present one word to the reader, the theory of perception described in this example is called parallel data processing, or connectionism.[14] The latter term is used to emphasize the connections between different "nodes" of information. In this section, I will describe how children's learning of torque was modeled by using a connectionist model.

Figure 4-2



The word APPLE and the simple line shapes that can be combined to form all the letters in the word. According to connectionist theory, as the entire word APPLE is perceived, each letter is interpreted as the conjunction of different line shapes; all lineshapes are interpreted at the same time.

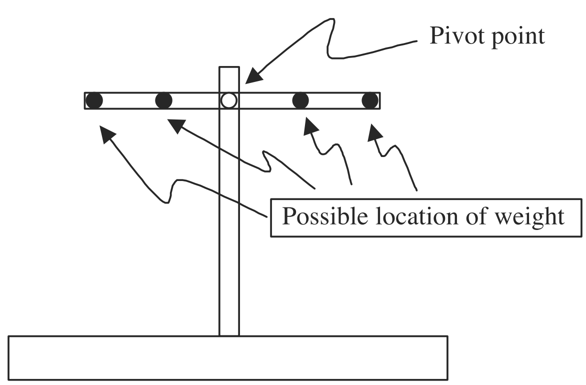
The APPLE example shows how a description in terms of parallel data processing involves taking individual, basic building blocks of perception and combining them into much more complicated structures like words. Most research into the use of parallel data processing has taken place in perception or linguistics, where the basic building blocks of perception (or grammar) are possibly quite different from those in physics. The purpose of this section is to show that the structure of parallel data processing can be helpful for understanding how students apply primitives to their reasoning.

Children investigated for their understanding of balance were asked to describe whether a set of weights placed a certain distance from a pivot point would balance the beam on which they hung (see Figure 4-3).[14] Pegs were placed at equal distances on equal-length arms of a balance beam. Small weights all of equal mass were placed at different locations on the beam while the beam was held in place. Subjects were asked to predict how, if at all, the balance beam would rotate if released. A correct answer would explain that the number of weights (proportional to the mass and therefore the force of gravity at that point) times the distance from the pivot point was the relevant measure (i.e. the torque is proportional to force and distance by $\tau = Fd$ in this simple situation). The beam will rotate in the direction of the side of the beam with the largest torque.

Observations show that children slowly come to realize that the relevant variables are weight of the object and distance from the pivot point.[15] Furthermore, observations show that, over time, children develop four different levels or patterns of reasoning with which they answer the question of how to balance the beam on which weights are already hanging.

The first and simplest pattern involves counting the number of weights hanging from each side of the balance beam. In the second pattern, children still look for the number of weights first, but if these are equal, then distance from the pivot is included in children's reasoning. In the third pattern, distance and weight are both always

Figure 4-3



Sketch of the torque balance task. Pegs are located at equal distances along equal-length arms of a balance beam. Small equal-sized weights were placed at different locations on the beam while the beam was held in place. Subjects were required to predict how, if at all, the balance beam would rotate if released.

considered, but with a special emphasis on equality. If one is equal, the other determines imbalance. If both weight and distance are greater for one side, the child states that side will drop. If one side has greater weight and the other has greater distance, the child using this model is unable to resolve the inconsistency. Finally, in the fourth pattern of reasoning, children learn to make a full explanation based on the sum of the products of weight and distance. Here, students are using both the weight and the distance from the pivot point in their reasoning. Evidence shows that children progress through these four patterns of reasoning as they gain experience, and that even college students are unable to consistently use the fourth pattern at all times in their reasoning.[16]

In describing the four patterns of reasoning that students use, the working harder primitive was applied in two different fashions to lead to two facets that the subjects appear to use in their reasoning. The weight facet seems to involve counting how many weights are being hung from each end of the balance beam. In the first pattern, if there is more weight, the balance beam will tilt in that direction. The distance facet involves the simple operational measurement of distance from the pivot point. In the second pattern, if the weights are equal but the distances from the pivot are unequal, the balance beam will tilt in that direction. In the second pattern, the distance facet is less important and its use dependent on an inability to apply the weight facet. In the third pattern, students use a refined version of the second model. Now, the distance facet is isomorphic in its reasoning utility with the weight facet. Balance is determined by a combination of the two, but without a refined description of what happens if they vary covariationally (i.e. one variable goes up while the other goes down). In the fourth pattern, the two facets are linked together to create a quantity (torque) which determines balance. One can describe the students using the fourth pattern as applying the Ohm's primitive, since they are now able to reason with three variables, two of which compensate for each other covariationally. Only when the two facets are correctly linked together is the concept of torque fully operationally understood.

Further research into student understanding of the physics of this situation has shown that students more easily answer the question (i.e. use a better model) when the weights or distances are very distinct, rather than nearly equal to each other.[17] When the weights or distances are distinct, it is possible to use only one facet to guide one's reasoning to the correct answer. This suggests that it is more difficult to use two facets at the same time than one.

Patterns of Association, Guiding Analogies, and Mental Models

In the previous sections, specific student difficulties were described as inappropriate applications of sometimes useful facets of knowledge or reasoning primitives. Primitives are too general, though, to be of much use by themselves. They are too general and can lead to contradictory responses. The organizational structure of primitives seems critical when we discuss how students make sense of the physics through the use of primitives. We use the idea of a "guiding executive" that guides students to use and interpret particular primitives in particular ways to particular situations. In general, we refer to this guiding executive as a pattern of association, or a mental model when it is highly structured, complex, and coherent.

When students consistently use a set of primitives inappropriately in a given setting, we can say that they have a pattern of association with which they approach the physics. The term is used to describe the semi-structured manner in which students bring a large body of knowledge to a situation. Some of this knowledge is applicable, while other pieces of what the student believes may be problematic.[18] Where primitives are single, individual, prototypical units of reasoning, a pattern of association can be thought of as a linked web of primitives and facets associated with a topic. Note, though, that analyzing student responses in terms of patterns of association can be helpful in trying to make sense of what we observe but does not imply that students have a specific fixed model in mind when they approach a situation. Patterns of association are more fluid and less precise than a physical model.

The term "model" has very specific meaning in physics. Patterns of association and even mental models are not physical models. They have certain traits that possibly make them problematic when used by students. Student patterns of association are often incomplete, self-contradictory, and inconsistent with experimental data. Based on the description of patterns of association as linked sets of primitives which students often use incorrectly, this should be no surprise. Note that incompleteness, self-contradiction, and inconsistency are possible traits of physical models, too. We may refer to an accepted physical model, determined through theoretical and experimental work and the agreement of the research community to be valid in certain physical realms with certain limitations, as a Community Consensus Model (CM). For example, the model of waves that we present to students in the introductory level is only the linear model, which is technically incomplete and sometimes inconsistent with the experimental data. Furthermore, the simple linear model of waves is sometimes not self-consistent. For example, as described in chapter 2, two superposing waves may create a situation that violates the small angle approximation in some part of the medium. But, a trained physicist is able to know the limits of the given CM, while students usually do not know the limits of validity of a given pattern of association.

Due to the accepted and understood limitations of the CM of waves, we can describe it as a mental model. Physicists agree on certain common elements to the model and are aware of shortcomings of the model, but use it to guide their general reasoning about a large number of wave phenomena. The terminology represents the distinction between the accepted and understood limitations of a mental model (as a reasonably complex, coherent, but partially contradictory model) and the looser form of a pattern of association.

Analyzing student reasoning in terms of patterns of association can be highly productive in trying to make sense of student reasoning about advanced topics in physics. In a paper which organizes research into student difficulties with light and optics, Igal Galili uses patterns of association to describe how students develop their understanding.[19] The paper builds on previous investigations of student understanding of light and optics, many of which have been used to develop curriculum designed to help students overcome their difficulties.[20]

Galili goes beyond a description of student difficulties and tries to explain the cognitive structure of student thinking in order to better develop curriculum that can address student needs. A comparison can be made to the way in which Halloun and Hestenes go beyond Clement's investigations, as described in chapter 2. A difference, as will be pointed out, is that Galili focuses on students' responses and does not categorize students according to the correct model. As Galili says, "Students' views are certainly organized. However, their organization is different from that employed in scientific knowledge." He cites Minstrell's facets as basic building blocks of knowledge, and writes, "clusters of facets, connected by causal links, are ... appropriate to describe mental images and represent operational models." As an example, Galili discusses three conceptual topics: understanding of light sources, image formation by a converging lens, and image formation by a plane mirror. In each case, he distinguishes between the

- naïve (pre-instructional),
- novice (post-instructional), and
- appropriate formal (or community consensus)

facets of knowledge. The novice facet of knowledge in each case is a hybrid between the naïve and the formal facet.

In the case of conceptual understanding of light sources, the naïve facet of knowledge is the "static light model." Some students, previous research has shown, believe that light fills space, i.e. like a gas filling a room. Researchers often find that after instruction students state that light emanates only in radial directions from the light source, with a preferred direction being toward the observer. (Galili calls this the "flashlight model.") This novice facet seems to be a hybrid between the naïve view and the formal facet, which states that light emanates in all directions from all sources. As Galili points out, the "flashlight model" can be the source of many reported student difficulties in unique settings (such as pinholes, lenses, mirrors, etc.) and more advanced settings.[20]

In the case of conceptual understanding of real image formation by a converging lens, Galili describes the difference between what he calls the holistic (naïve), the image projection (novice), and the point-to-point mapping (formal) facets of knowledge. In the naïve conceptualization of image formation, the full image moves to the lens, is inverted by the lens, and moves to the screen, where it can be seen.[21] The novice facet of knowledge is a modified version of the naïve facet, containing the idea of a light ray but with the idea of unique rays which are more important than others. Furthermore, Galili states, in this facet "each ray carries structural information about the point of origin," meaning that physical significance is attached to each ray in a way that is inconsistent with the formal, point-to-point mapping of object to image. In the formal facet of knowledge, light flux emerges in all directions from all points of the object. Some light rays interact with the lens and converge to an image point of each individual point. The role of the screen is not to create the image but to scatter light in all directions for observers who are not in the region where light diverging from the image source would reach them.

In the case of image formation from a plane mirror, Galili again describes holistic (naïve), image projection (novice), and point-to-point mapping (formal) facets of knowledge. Students using the naïve view state that the image of the object is "on the mirror," where it can then be observed. Galili describes two versions of the novice image projection conceptualization. In the first, light rays move first to the mirror in the shortest possible path, and then reflect to the observer. This reasoning violates the law of reflection (angle of incidence equals angle of reflection for a light ray). In the second novice conceptualization, the law of reflection is used correctly but students still use only single, specific, individual rays to show where the image is. They do not think of light emanating from all points of the source in all directions, a concept which is part of the formal conceptualization. Again, the novice facet of knowledge seems to be a mixture of the naïve and the formal conceptualization.

Table 4-3 summarizes Galili's description of the different patterns of association held by students. Using the language we have introduced, the formal pattern of association can also be referred to as the community consensus model. Research has shown that the novice mental model is often the one with which our students leave our courses.[20] Students use light rays, but rarely consider a full set of them.[22] Students describe the laws of reflection and refraction correctly, but only use special rays in their reasoning. This leads to difficulties where students believe that blocking one of the special rays leads to an incomplete image being formed.[19] Also, a screen is necessary for images to be observed (even in an area where the light which forms the image can be observed), since image formation and image observation are two distinct things in the novice model.

Galili also discusses how the hybrid model might come into being due to classroom instruction. He describes possible conceptual change where students move from a naïve, holistic mental model to the image projection mental model by "the transformation of certain naïve facets of knowledge into other facets which often implement the [image projection mental model]." The idea of conceptual change will be discussed in more detail below.

Certain issues and questions remain. Galili describes three different primitives (facets) that students use in the three patterns of association, but seems to assume that students can be described by a single pattern of association at any given time in their learning. Galili does not discuss the possibility that students might use facets

Table 4-3

Pattern of Association Physical Topic:	**Naïve**	**Novice**	**Formal**
Understanding of Light Sources	Static light fills space	Special Flashlight rays	Light emanates in all directions
What a Lens Acts on to Create an Image	Full images that travel through space	Special rays with physical significance	All rays (some then form an image)
What a Mirror Acts on to Create an Image	Full image (located on the mirror)	Special rays, not necessarily with law of reflection	All rays (some then form an image)

Galili's description of the facets students use in three different patterns of association.

inconsistently in different physical situations. Students might have more than one association pattern for a situation and they might use different patterns of association depending on which pattern the question brought up in the student. In such a situation, each association pattern might act as a guideline for student reasoning but not lead to firm rules of use. The pattern of association would act as a guiding executive in helping students choose which primitives to apply to a situation. I will discuss this idea of patterns of association as guiding executives of student reasoning later in the dissertation.

Models of Conceptual Change

A fundamental goal of education is to change the way that students look at the world around them. Previous research has shown that students do not enter our classrooms as blank slates, but that they bring a body of knowledge to the lecture halls and classrooms in which we teach them. In the previous section, students after instruction were described as possibly having a hybrid novice pattern of association containing aspects of both the naïve and the community consensus (or formal) model. For those students who are using the novice pattern of association, both the naïve pattern of association and the formal mental model seem to contain reasonable and useful elements. In a teaching situation, one can create a situation where students might apply the naïve pattern of association while also being aware of the formal and correct response. Thus, a situation of "cognitive conflict" may arise in the student though an awareness of the inconsistency of one's own beliefs. This provides an opportunity to help the student determine whether the elements of the naïve reasoning are valid in a given situation.

To describe the process by which students change their ideas about the world around them, we need a description that accounts for the development of student understanding. Such a model, referred to as the Conceptual Change Model (CCM) has been proposed and developed by Hewson and others.[23–26] As stated by Demastes et al.,[26] the process by which a student's conceptual model changes can be described in two different fashions. In the first type of conceptual change, a gradual change can occur, where "competing conceptions remain but eventually only one is consistently applied by the learner." Also possible are wholesale changes, which are not evolutionary in nature but instead can be described as complete, relatively sudden changes. The distinctions between gradual and wholesale change of knowledge play a fundamental role in this dissertation.[27]

Hewson and Hennessey have used the CCM to investigate student understanding of force and motion. The task involved a book placed on a table. Students in sixth grade were asked to choose which free-body diagram from a set of offered responses best represented the book. They were then asked to justify their response with both written and verbal explanations. The paper details how the understanding of a single student, Alma, changed during instruction.

Alma began the semester by stating that only a downward force was needed to keep the book on the table. She spoke of how her response was consistent with other responses she had given, and how the response was useful in her reasoning. Thus, her original conception was satisfactory to her needs. But, the authors point out, she was not very committed to it. In other words, though she gave an incorrect response, she did not explain in detail how she arrived at the response.

At the midpoint of the semester, Alma says, "My theory has definitely changed... I think that there are equal forces... because the book isn't moving... The two forces are equal." She has obviously changed her conception of forces from one in which a single force is required to hold the book down to one in which equal forces keep the book from accelerating from its present resting state (though she does not use these terms). She adds "I can now see why I picked [the previous answer], and I don't really believe this reason anymore." Alma has left a previous conception behind and has shifted into a new understanding of force and motion.

By the end of the semester, Alma has not only correctly described the forces on a book at rest, but she has been able to describe the need for these forces. Hewson and Hennessey refer to the process by which her ability to justify and explain the need for her response as "conceptual capture." To her conception of force, she added the idea that the table must be exerting an upward force. In her own words, she now believes that the table can exert a force (something she did not believe at the beginning of the semester).

We can describe Alma's learning during the semester in terms of facets and patterns of association. Alma's description matches difficulties that Minstrell has described.[28] In terms of the primitives that Alma uses, only one is needed at first. Gravity pulls down. She seems to be using the primitive (actuating agency) that only moving objects exert forces (i.e. the table is not exerting a force on the book). As the semester progresses, she learns to think not in terms of motion alone, but in terms of sums of forces. While dropping the actuating agency primitive, she must now account for the book not moving. To do so, she seems to add another facet to her reasoning: the table can exert a force on the book. Thus, the at-rest condition of the book can now be described by the link between two facets, and her association pattern of motion has changed from a simple to a more complex one. In terms of the use of multiple facets that must be linked together for a complete understanding of the physics, Alma's learning is similar to the development of children's learning about torque and the balance beam, as described above in the section on parallel data processing.

Demastes et al.[26] have pointed out that students in biology do not necessarily switch conceptions (or patterns of association, or mental models) in a wholesale fashion. Instead, Demastes et al. expand Hewson's description to say that students can go through different patterns of conceptual change which they describe as, "(a) cascade, (b) wholesale, (c) incremental, and (d) dual constructions." Since their paper does not deal with physics, I will not emphasize details here, but I will summarize their most interesting findings. They point out that "students are often not as logical or exclusive in their cognitive restructuring as researchers assume." Demastes et al. state that students do not necessarily rebuild or exchange their conceptual understanding when confronted with evidence that shows that their previous understanding is incorrect or insufficient. Instead, students may build a completely new and separate conceptual model that accounts for the new observations. The authors give an example where students have dual, conflicting conceptions, are aware of the conflict, and still say, “I have no problem with that.”

The CCM, as described by Hewson and others and expanded by Demastes et al., describes how students come to develop an understanding of class content. The model provides insight into events that happen within our students in our classrooms, and it provides predictions about student performance in our research. As illustrated in the research by Hewson and Hennessey, the CCM model is consistent with the idea that a shift in student understanding involves a change in the patterns of association used by students to describe a physical situation. Furthermore, the shift seems to function at the level of new primitives being introduced to the association pattern. But, as pointed out by Demastes et al., we should not expect our students to completely change their conception of a physical situation. They may be learning the material while still holding on to their previous beliefs about the applicability of specific facets of knowledge to settings outside of our classrooms.

Summary

In this chapter, I have described a description of student understanding and a model that may be used to describe student learning of physics. This model has been developed to serve as a productive simplification of the different elements of student reasoning that occur in the classroom.

We have chosen to describe student reasoning in terms of basic logical elements that are common to many areas of reasoning, not just physics. These reasoning elements are helpful in making sense of the world around us and are applicable in many different situations. For example, the notion that it takes effort to bring an object into motion is similar to the idea that it takes effort to motivate a lazy person. For both phenomena, an actuating agency is needed to cause a movement from rest. We refer to logical building blocks like the actuating agency as primitives. Primitives can be applied to a specific context in a variety of ways, so that the same primitive may lead to different interpretations of the situation. We refer to each such interpretation of a primitive in a context as a facet of knowledge. It is possible to have a single primitive lead to different and contradictory facets.

Students seem to use a variety of primitives (and facets) in connection with each other to describe certain sets of phenomena. We call these systems of primitives (or facets) patterns of association, or , when they are coherent and consistent, mental models. Often, students are guided in their choice of facets by the association patterns that they already have of what are deemed similar situations. Patterns of association can effectively describe analogies that students use to guide their reasoning. Thus, a researcher can use the idea of a pattern of association in two ways. In the first, a pattern of association describes the incomplete and possibly inconsistent knowledge that students bring to a physics problem in terms of the facets applied in their reasoning. In the second, it describes the knowledge that they believe should apply to the situation, and this knowledge they use as a guiding analogy to help guide their choice of facets in their solution of the problem.

In the context of student use of patterns of association and mental models, it is possible to describe student learning in terms of the facets that students use to guide their reasoning at different points of instruction. Students may re-interpret old primitives, learn new facets, or stop using certain primitives when they no longer apply to the physical situation. Also, depending on the domain size of analysis with which one approaches student difficulties with the physics, one can say that an individual student may use multiple patterns of association or mental models simultaneously. This can be interpreted at the level of facets, where students have different, non-overlapping sets of facets, and at the level of mental models, where students use different guiding analogies to develop their understanding of a given situation.

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[1] See, for example, Redish, E. F. "Implications of Cognitive Studies for Teaching Physics" Am. J. Phys. **62**, 796-803 (1994) and Hestenes, D., "Wherefore a science of teaching?" Phys. Teach. **17**, 235-242 (1979).

[2] An excellent discussion of this approach can be found in: Hammer, David "More than misconceptions: Multiple perspectives on student knowledge and reasoning and an appropriate role for education research," Am. J. Phys. **64**, 1316-1325 (1996).

[3] The discussion of primitives is closely related to many ideas of schema theory. For the most concise definition of schema theory, see Alba, Joseph W. And Lynn Hasher, "Is Memory Schematic," Psych. Bull., **93**, 203 (1983). The authors critique a large amount of the schema theory literature while not denying the existence of schemas (or primitives) in everyday, common reasoning patterns. Since we are concerned with the use of everyday reasoning patterns in the classroom, a schema theory is still applicable to this analysis.

[4] diSessa, A. A., "Towards an epistemology of physics," Cognit. and Instruct. **10**, 105-225 (1993).

[5] In reference 4, diSessa refers to his units of basic reasoning as "phenomenological primitives" (or "p-prims" for short), but we have found that "p-prims" are essentially the same as the schemas referred to as prototype theories in the cognitive studies literature. In order to keep the number of terms introduced in this chapter to a minimum, we will refer to an individual p-prim as a specific primitive while still using the classifications given by diSessa.

[6] Minstrell, J. "Facets of students' knowledge and relevant instruction," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 110-128.

[7] The term "actuating agency" has been proposed by David Hammer to more accurately describe diSessa's phrase "Force as Mover." See Hammer, D., "Misconceptions or p-prims, How might alternative perspectives of cognitive structures influence instructional perceptions and intentions?" J. Learn. Sci. **5**:2, 97-127 (1996) for more details.

[8] For more detailed description of the research which included the described experiment, see Pride, T. E. O'Brien, S. Vokos, and L. C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," Am. J. Phys. **68**, 147-157 (1998) and references cited therein.

[9] For a more complete description, see reference 4.

[10] The hardware store, Hechinger's, recently ran television advertisements for a "Big and Fast" sale, stating that things in nature were never big AND fast. To illustrate this, they showed a variety of small but fast objects such as a mouse and large and slow objects such as an elephant. They then stated that sometimes objects could be large and fast. To illustrate, they showed a Saturn V rocket at the beginning of the take-off sequence (when it is actually moving very, very slowly).

[11] We can also interpret this confusion as an example of the failure to distinguish between a quantity ($\vec{p}$) and its rate of change ($\vec{F} = {}^{d\vec{p}}\!/_{dt}$).

[12] See reference 6, p. 92.

[13] For example, people can still read words where parts of certain letters have been covered up; the parsing process seems to include the ability to fill in a partially complete pattern using the context in which it appears (i.e. the other letters).

[14] See Klahr, D. and B. MacWhinney, "Information Processing." In *The Handbook of Child Psychology*, *Vol.2, Cognition, perceprtion, and action*, edited by W. Damon (Wiley, New York, 1998) 631-678.

[15] Klahr, D. and R. S. Siegler, "The representation of children's knowledge," in *Developmental psychology: An advanced textbook* (3rd ed.), edited by M. H. Bornstein and M. E. Lamb (Erlbaum, Hillsdale, NJ, 1992).

[16] See, for example, Ortiz, L. G., P. R. L. Heron, P. S. Shaffer, and L. C. McDermott, "Identifying and Addressing Student Difficulties with the Static Equilibrium of Rigd Bodies," *The Announcer* **28**:2 114 (1998).

[17] See reference 14 for more details.

[18] Norman, D. A. "Some Observations on Mental Models" In *Mental Models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum Associates, Hillsdale NJ, 1983) 7-14.

[19] Galili uses the term mental model for what we have called a pattern of association. For more details, see Galili, I., "Students' conceptual change in geometrical optics," Int. J. Sci. Educ. **18**:7, 847-868 (1996).

[20] For a summary of research into student understanding of geometrical optics, see reference 19 and references cited therein. For an example of how research into student difficulties with light and optics leads to curriculum development, see Wosilait, K., P.

R. L. Heron, P. S. Shaffer, and L. C. McDermott, "Development and assessment of a research-based tutorial on light and shadow," Am. J. Phys. **66**:10, 906-913 (1998) and references cited therein.

[21] For a detailed discussion of student descriptions of this conceptualization, see Galili, I., S. Bendall, and F. M. Goldberg, "The effects of prior knowledge and instruction on understanding image formation," J. Res. Sci. Teach. **30**:3, 271-301 (1993) and references cited therein.

[22] Bruce Sherwood, of Carnegie-Mellon University, has proposed that all textbooks follow a certain theorem: light attracts glass. In other words, only those rays oflight which leave the source and pass through a lens or are reflected by a mirror are shown, and all other rays are left off the diagram. The result is that students who use the novice, hybrid mental model may be reaffirmed in their belief that only some specific and special rays are important to the physics. This will often lead them to the correct answer while using incomplete reasoning.

[23] Hewson, P. W. And M. G. A'B. Hewson. "The role of conceptual conflict in conceptual change and the design of science instruction," Instr. Sci. **13**, 1-13 (1984); "The status of students' conceptions," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 59-73.

[24] Hewson, P. W. And M. G. Hennesy. "Making status explicit: A case study of conceptual change," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 176-187.

[25] Posner, G. J., K. A. Strike, P. W. Hewson, and W. A. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Educ. **66**:2, 211-227 (1982).

[26] Demastes, Sherry S., Ronald G. Good, and Patsye Peebles, "Patterns of Conceptual Change in Evolution." J. Res. Sci. Teach. **33**, 407-431 (1996).

[27] The different processes are usually referred to as "assimilation" and "accommodation." See reference 25 for a brief review and references therein for more detailed descriptions.

[28] Minstrell, J. "Explaining the 'at rest' condition of an object," Phys. Teach. **20** 10-14 (1982).

Chapter 5: The Particle Pulses Mental Model

Introduction

In chapter 3, I describe specific student difficulties with physics in the context of waves. These topics included:

- a failure to distinguish between a disturbance to a medium and the manner of the propagation of the disturbance in the medium through which it travels,
- the inability to consistently describe the condition of an equilibrium state of the medium,
- the interpretation of the mathematics of waves in overly simplified terms that often show no functional dependence on variables that describe changes in both space and time, and
- the failure to adequately describe the interaction between two waves both as they meet and after they have met.

In each topic of investigation, the specific difficulties are indicative of more fundamental questions, such as how students understand and make sense of physics. In this chapter, I will use the context of student difficulties with wave physics to propose a model with which we can organize the observed student difficulties.

Although I have described student difficulties with wave physics on a topic by topic basis, there are certain similarities in student reasoning we can use in each case. In chapter 4, I described a model of learning that helps describe and organize the difficulties we see students having. This model is built from the idea that students use basic reasoning elements called primitives that are reasonable in one context but may be applied inappropriately or incompletely in another. We can describe a set of primitives and the rules that tell students when to use them as a pattern of associations that guides student reasoning in unfamiliar situations. A pattern of associations is possibly incomplete, incoherent, and self-contradictory, and serves as an example of the type of guiding structure that students might have when dealing with unfamiliar material. When a pattern of association has a reasonable level of completeness and coherence, we can refer to it as a mental model. Our analysis of the manner in which students organize primitives into patterns of association can help us understand the manner in which student beliefs about wave physics change over the course of instruction. This can serve as an example of how students come to make sense of physics in general, not just wave physics.

In the first part of this chapter, I discuss the common primitives that students use when describing wave physics. I introduce a new primitive not previously described in the literature, the *object as point* primitive. As with other primitives, it is often useful and helpful in simplifying reasoning in some areas, but problematic when misapplied in wave physics. Then, I summarize extensive interviews with four students who answered questions on a large number of wave physics topics. The interviews illustrate how certain primitives are regularly but incorrectly applied to wave physics.

Research results have been gathered using techniques and investigations previously described in chapters 2 and 3.

In the second part of the chapter, I use student responses to describe the idea of a pattern of association that we refer to as the Particle Pulses Pattern of Association (which will be loosely referred to as the Particle Model or PM of waves). This pattern of associations describes the analogies that students use to guide their use of the specific primitives. In the second part of this chapter, I will discuss how the PM is used by students to guide their reasoning. In this case, the PM has not so much predictive as productive powers, helping the student choose which primitive (or facet) to apply to a given situation. I will also compare how students use the PM in comparison to reasoning based on the correct model of wave physics, as described in chapter 2.

Some of the interview or examination quotes have been given in the previous chapter but will be repeated here for further discussion. In some interviews, we see that students use more than one guiding analogy in their reasoning. This is consistent with the results described in chapter 3, where we saw students using more than one form of reasoning to describe a single physical situation.

Student Use of Primitives in Wave Physics

In chapter 4, I discuss a variety of primitives that have been studied mostly in connection to student reasoning in mechanics. In this section, I describe the common primitives used by students who show difficulties with wave physics. In addition to those primitives describe in chapter 4, we find that in their reasoning about wave physics, students seems to use at least one additional primitive not previously included in the literature. First, I use results from chapter 3 to illustrate student use of the "object as point" primitive. Then, I give a more detailed discussion of other commonly occurring primitives in the context of interviews with four students who had difficulties with many of the topics described in chapter 3.

The object as point primitive

The object as point primitive (henceforth called the point primitive) is based on observations of student descriptions of waves, but has a more general applicability. The point primitive plays a central role in this dissertation, being the focus of the mental model which I will describe later in the chapter. Before more rigorously defining the point primitive, I will motivate why we believe it exists by quoting from interviews used in the previous chapter.

In interviews in which students described their understanding of the mathematics which describe waves (what I have called the wave-math problem), students were presented with equation 5-1 in a situation in which they were asked to describe the shape of a propagating wave.

$$y(x) = Ae^{-\left(x_0/b\right)^2} \qquad (5\text{-}1)$$

We observed students' inabilities to properly describe the variables in the equation. Most notably, students could not adequately describe and use the variables x

and y. Most students who sketched a pulse whose amplitude had decreased gave the explanation that the exponent value would decrease as the value of x increased. They were using the variable x to describe the location of the peak (originally at x = 0), and then were interpreting the variable y to describe the peak amplitude of the wave, not the displacement of the string at all points. Many students were effectively interpreting the entire wavepulse, an extended region of displacement from equilibrium, as a single point.

Student use of the point primitive when answering the wave-math problem shows how a primitive that may seem appropriate is actually inappropriate when applied to a particular situation. First, student descriptions of decreasing amplitude are consistent with their observations of wavepulses whose amplitude decreases due to friction with the floor while propagating on springs on the floor, as shown during demonstrations in the classroom. When working with students on this material in the classroom, I have had some students state that the mathematics should be consistent with their observations (though we find that many students are unable to operationally carry out this general principle). The correct application of the deep principle that the mathematics and physics should be consistent (which we should encourage students to develop) may lead to student difficulties in this situation. Thus, the interpretation students use is strengthened by the fact that they consider the result to be obvious, i.e. consistent with their observations.

Second, students do not describe the possible physical reasons for the decreased amplitude in their explanations. Instead, they often cite the equation and the effect of a change in x on the exponent. Students fit the mathematics to the situation they observe by using the archetypal example based on a classroom demonstration, and in the process, they give (nonphysics) explanations which incorrectly use the mathematics. We observe that students are trying to interpret the equation and make sense of the equation (again, a skill which we should encourage them to develop), but that they have difficulties knowing how to make sense of the mathematics.

Evidence from other areas of wave physics show that students seem to be applying this primitive to more than the wave-math problem. In wave superposition questions, students who were asked to sketch the shape of the string when two asymmetric wavepulses partially overlapped often sketched the shape of each individual pulse without adding displacements at the appropriate points. (See, for example, Figure 4-13c). Those students again appeared to be simplifying an extended region of displacement down to one point. Students often use of the word “amplitude” to describe this point. A student who drew a sketch like the one in Figure 4-13c explained, “The waves only add when the amplitudes meet.” Unless the two points of the wavepulses which the student considers relevant overlap, these students assume there is no summation of displacements (superposition) in the region where the wavepulses do overlap. Interviewed students who gave an explanation like the one just described merely asserted that the shape was just as they had sketched it. They were unable to give a more detailed explanation, other than to say that there was no addition until the peaks overlapped. When asked about the other displaced regions, students often had no explanation as to how they would interact. Many students often are unable to explain through more than an assertion. It seemed that the assertion itself was sufficient as an explanation for these students. This "non-dissociability" of an explanation is a common characteristic of cognitive primitives.[1]

We also see an application of the simplification of a wavepulse to a single point in student descriptions of how to change wave propagation speed. A more detailed description of student explanations for changes to wave propagation speed will be given below. At this point, it is sufficient to say that students seem to make an analogy between the wavepulse and an object like a ball. By thinking of the wavepulse as a single point, students can apply ideas to wave propagation based on analogies to the motion of a point particle. Furthermore, a student who states "You flick [your hand] harder...you put a greater force in your hand, so it goes faster," gives an example of the heuristic principle which states that students will use simple body motions as part of their explanations. In interviews, students often make the hand motion of flicking their wrist up and down slowly to describe slow pulses and quickly to describe fast pulses. They use their body to help describe the base vocabulary of their reasoning, again consistent with diSessa's heuristic principles.

Another example of student simplification of waves to single points comes from the research into student understanding of sound waves. In the interview quoted at length in chapter 3, Alex described the sound wave as exerting a force on the particle. He sketched the wave as a series of pulses and described the pulse exerting a force on the dust particle as a "kick" or a "hit." During the interview, he had simplified the repeating sinusoidal wave to a succession of pulses, and then described each pulse as a point which could exert a force, kick, or hit the dust particle which it encountered in only one direction.

The point primitive is characterized by the description of a large, global object or wave in terms of a single point. In the case of wave physics, it seems to function as an interface between the shape of a wave (in some given or assumed representation) and the manner in which the wave can be influenced or influences its surroundings. We have frequently encountered the point primitive in student responses to questions in all areas of wave physics investigated for this dissertation.

Beyond the difficulties discussed in chapter 3, we have found additional evidence of its use in student descriptions of wave reflection. Students drawing a wavepulse on a string attached to a wall state that the wavepulse will not be reflected until the peak of the pulse has reached the wall. These students have difficulties in deciding on the shape of the string or pulse when the front of the wavepulse has reached the wall but the peak hasn't; they want to preserve pulse shape, but they also know that the string remains attached at all times.

The point primitive is not necessarily problematic. Instead, it is a perfectly reasonable and useful reasoning method when quickly analyzing certain physics problems. For example, when solving simple trajectory problems in Newtonian physics, the community consensus is to immediately simplify the object traveling along the trajectory to a point particle. Especially in situations where the rotation of an object is unimportant and there are no collisions, we treat the center of mass as this point, and ignore all other points. The analysis by which the point primitive is applied to the rigid body can be quite complicated. Finding the center of mass of a non-symmetric body involves complex integration and is today typically only briefly discussed in upper-division graduate mechanics or advanced engineering courses.[2] The source of difficulties in the use of the point primitive lies in how it is used in wave physics, not to its existence in the student's repertoire of reasoning tricks.

Common primitives in wave physics

Rather than again describing each of diSessa's primitives (see chapter 4) as they apply to wave physics, I describe how they are used in the context of four student's difficulties. Table 5-1 gives a summary of the primitives used by each student. Table 5-2 gives a brief description of each of the primitives first described in chapter 4 and the wave physics topics in which students applied it. (Note that Table 5-2 has been split into three sections due to its size.) For Table 5-1, some categories of Table 5-2 have been combined into one due to their similar nature in the context of waves. The reader is asked to refer to these tables during the discussion below.

In S96, we carried out a set of pretest interviews with four students over the course of several weeks. Each week, students were interviewed about their responses to questions asked on the pretest given in preparation for that week's tutorial.[3] Five weeks of interviews were carried out, where three addressed issues discussed in this dissertation. The four students were asked to answer the pretest questions while an interviewer probed their responses and investigated whether their written and interview comments were similar. Certain issues were probed more deeply during the interview than had been possible in the pretest.

In S97, 20 students from two different instructional settings participated in a diagnostic interview. Fifteen students had participated in early versions of tutorials designed to address student difficulties with wave physics. Five students participated after traditional instruction in a class with recitations. Most of the 20 students answered 18 questions which dealt with wave propagation speed, superposition, the physics of sound, wave reflection, and wave mathematics. Two subjects, wave propagation and wave mathematics, were investigated with both FR and MCMR questions. The sound question was asked in MCMR format only. A copy of the final version of the S97 interview diagnostic test is given in Appendix D-1. It is discussed in more detail in chapter 7.

Table 5-1

Students **Primitive*s***	**Ford**	**David**	**Kyle**	**Ted**
Object as point	X	X	X	X
Force and Motion (Ohm's, actuating agency, smaller is faster, working harder)	X	X	X	X
Collision (bouncing, Canceling, overcoming)	X	X	X	X
Dying away (possible interpretation of point **primitive)**		X		
Guiding			X	X

Brief summary of primitives used by students. For a more complete description of each primitive, see Table 4-1, chapter 4.

Not all students answered all questions for a variety of reasons. Due to time limitations, some students did not finish the diagnostic test. Also, during the course of the interviews, I made changes in the protocol based on student feedback and responses. Therefore, during the course of the 20 interviews, some questions were rephrased, some dropped, others added. The development of a diagnostic test to investigate student understanding of wave physics will be described in more detail in chapter 7.

Table 5-2a)

Primitive	Definition (wave physics specific)	Context (specific wave topic)
Object as point	An object (i.e. a wavepulse) is characterized by a single point (e.g. of maximum displacement from equilibrium); a large object is simplified by referring to just one piece of it, like the C of M …	**propagation speed:** ball-toss analogy **superposition:** a) no peak addition b) "global" addition **mathematics** a) x as peak location b) plugging in x_0 for eqn. **Sound** wave=pulse=point (exerts force)

Table 5-2b)

Collision Primitives		
Primitive	**Definition (wave physics specific)**	**Context (specific wave topic)**
canceling	A wave can be permanently cancelled by another wave (see dying away for opposing forces).	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none
bouncing	A wave (as an entire unit rather than a region of displaced elements of the medium) will hit another wave and bounce off it.	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none
overcoming	Two competing forces (i.e. waves) will interact such that one overpowers the other (see Canceling but for unequal amplitude waves).	**propagation speed:** none **superposition:** rules for partial cancellation of pulses **mathematics** none **sound** none

Table 5-2c)

	Force and Motion Primitives	
Primitive	**Definition (wave physics specific)**	**Context (specific wave topic)**
Actuating Agency	A directed impetus (force) on the medium causes a displacement of the medium and/or a wave speed in the direction of the impetus.	**propagation speed:** more force into wave = faster **superposition:** none **mathematics** none **sound** force on medium in propagation direction
Ohm's primitive	An agent or impetus (force) causes an action whose effect depends on the resistance of the medium or the effort of the agent (i.e. strength of the force). Effort and resistance are in a covariant relationship with each other.	**Propagation speed:** bigger pulse feels resistance and is slow, so wave needs greater force to go faster **superposition:** rules for superposition cancellation **mathematics** none **sound** increased loudness evens out with decreased frequency.
Working harder	The mechanism by which the Ohm's primitive plays a role in the actuating agency primitive, i.e. to make pulse go faster, you have to put more effort into it (hence, larger pulses took more effort to create, and will move faster).	**propagation speed:** bigger pulse needs more force **superposition:** bigger pulse has bigger force **mathematics** none **sound** force increases with volume
Smaller objects naturally go faster	Larger waves take more effort to create because there is more resistance to their creation; i.e. smaller "quicker" pulses are faster since less force is used to overcome the resistance of the medium.	**propagation speed:** tinier, tighter pulses go faster **superposition:** none **mathematics** none **sound** high f waves move faster
Dying away	The motion of the wave must eventually die away, i.e. the amplitude of the wave decreases not due to physical reasons but due to unscientific observations. By taking away the internal "force" from an object, it loses some of its amplitude.	**propagation speed:** none **superposition:** partial cancellation of pulses **mathematics** x increases, y decreases, and pulse shrinks **sound** none
Guiding	A piece of the medium must move along the track determined by the wave, like the dust particle moving on a sinusoidal path.	**propagation speed:** none **superposition:** partial cancellation of pulses **mathematics** none **sound** none

Common primitives and their use in wave physics.

We found that some students had consistent difficulties across a broad range of wave physics topics. Many of the tutorial students did quite well on the diagnostic, indicating the effectiveness of the specially designed curriculum. (These results will be discussed in more detail in chapter 6.) Those students who had not participated in tutorials showed more profound difficulties.

By looking at how the students who performed poorly answered questions in each topic, we find suggestions that they are using a common set of primitives in conjunction with each other. Three students, "David," "Kyle," and "Ted," stand out as having similar difficulties on many of the topics in the S97 diagnostic test.[4] One student, "Ford," stands out on the S96 pretest interviews. David and Kyle had completed tutorials on wave physics roughly two months prior to the interviews. Ted did not participate in any tutorial instruction. Ford had not yet participated in tutorials at the time of the interview.

Ford

In the S96 interviews, Ford showed difficulties with the topics of wave propagation and superposition. He had no difficulty with the wave-math pretest.

Ford's most interesting comments came while describing how to change wave propagation speed and while describing the collision of overlapping, superposing pulses. We began the interview by asking him how to create a single wavepulse on a taut string. He described a quick up and down hand motion and described the force needed to create the wave, an indication of the *actuating agency* primitive. When asked how to change the speed of a wavepulse, Ford responded (Ford's comments are indicated with "F," the interviewer's with "I"):

> *F: There are two scenarios that I have to think about, and since you want me to say right now... I'd send a quicker one (He draws a much smaller pulse, both in width and in amplitude, and he does a much quicker hand motion to describe how this pulse would be created) ...*
>
> I: *By quicker you mean, you did your hand motion like this?* (repeats the motion)
>
> F: *shorter, I wouldn't go* (makes large hand motion) *I'd try to make a shorter hand motion* (makes a quick flick of the wrist) … *It would get there faster.*
>
> *Or, I would send a huge pulse, where maybe [the pulse] could cover the whole thing* (i.e. the entire string) *in one pulse and maybe get there as fast as I put my wrist back down.* (does long build-up during this, and then slams hand down hard at the end) *The odd idea is that I don't know which one would work better.*

Ford indicated two different reasoning methods in this excerpt. On the one hand, he showed with his body how to make a smaller pulse and indicated that the smaller pulse would move faster. He did not distinguish between transverse velocity and longitudinal, propagation velocity. Note that his explanation makes use of a physical motion. This suggests that students are using a primitive in their reasoning. Ford's first explanation gives evidence of the smaller is faster primitive, since he spoke of a smaller wavepulse moving faster. At the same time, upon further reflection, Ford indicated that possibly a larger pulse would move faster. This explanation seems strongly connected to his description that a larger pulse would take more area, and a pulse with more area would move more quickly. His body motion is indicative of the working harder primitives, where a larger force is needed to create a larger pulse and a faster pulse is the result. Note also that the emphasis on the downstroke of the hand motion implies that he thought specifically of the speed and force of the hand motion at this time (showing difficulties with the idea that the leading edge of the wave would already be propagating forward). Ford's difficulty and inability to resolve these two descriptions caused a conflict in his thinking. Even after further questioning during the interview, he maintained that both explanations could cause a faster pulse, but he knew that one of them must be wrong.

The following week, Ford answered questions about wave superposition. (Before the interview began, he mentioned that the previous week's tutorial had helped him resolve his dilemma; he now knew that neither of his explanations was correct and was able to correctly describe the medium changes which caused a faster pulse.) We gave him two questions concerning wave superposition, the first shown in Figure 3-12 and the second shown in Figure 3-8.

In his descriptions of superposition, Ford spoke of the wavepulses as single units which collided with each other, and, as they collided, their amplitudes would begin to add up even though the highest points of the wave were not at the same point. "When they collide," Ford stated in the interview, "they have the same base, and I just added their amplitudes." His reason for adding the points of largest displacement, even when they do not overlap, suggests that even the point primitive does not necessarily restrict an object to one point. Instead, Ford seems to use it to make the entire object into one large point, where elements of the object then overlap and interact. But full interaction does not occur until the wavepulses completely overlap. As Ford stated when describing why the amplitude was not doubled during partial coincidence of the wavepulses, "they haven't fully combined yet, they haven't fully interacted yet."

Ford used the point, actuating agency, and working harder primitives to describe changes in wave propagation, and he used the point primitive when describing superposition. The combination of point primitive and the force primitives implies that Ford had a picture of waves that was at odds with the material being taught in the class. He did not describe waves as disturbances to the medium, and did not have a clear description of how waves interacted with each other.

Though Ford showed difficulties with only two of the physics topics, his use of many primitives suggests the manner in which inappropriately applied primitives can lead students to an incorrect understanding of the physics. He used conflicting primitives but did not accept that both would be valid, suggesting that the structure of Ford's understanding involved guidelines for reasoning that are not necessarily rigid, fixed rules and that he was uncomfortable with this flexibility.

David

David participated in the S97 diagnostic test interviews approximately two months after completing tutorial instruction on waves. The details of the instruction will be given in chapter 7 when the diagnostic test is described in more detail. David had no problems with the questions dealing with sound waves and did not answer questions about wave reflection due to time limitations during the interview. His difficulties with the other topics on the diagnostic test are consistent with those described in chapter 3.

David described how one could change the speed of a wavepulse on a string by saying,

> D: *Well, I know that tension affects the wave speed, that is … the rate the pulse moves down the string. And… the amplitude would affect it.* (He shows a hand motion with a larger displacement while saying the last sentence.) *I think possibly, you see a slower … pulse if the force applied to the string is reduced, that is: the time through which the hand moves up and down [is reduced].*

In his comments, David uses the actuating agency primitive as the basis for the incorrect part of his description. His description is consistent with thinking of the wavepulse as a point particle that moves faster due to a larger force (i.e. the point and working harder primitives seem to play a role). The reader should note that he begins with the correct answer before giving other explanations.

In the question dealing with the mathematical description of waves, David describes a decaying propagating wavepulse. He states that the shape would stay the same, but the amplitude would change, and says "this is a decay function" while pointing to equation 5-1. This shows clear evidence of the dying away primitive (and again the point primitive, since he is letting the single maximum amplitude point guide his description of the whole wavepulse). David uses the variable, x, to describe the position of the peak of the pulse. Thus, he seems to use the point primitive to interpret the mathematics with which he has difficulties.

David had two difficulties with superposition. In one question, he described two pulses meeting on a string as bouncing off each other, reversing sides on the string, and returning from whence they came. The collision analogy came quickly and spontaneously. With equal sized pulses, nothing cancelled out, and the pulses bounced off each other. The idea of one pulse overcoming another was probably not considered because the pulse sizes and shapes were equal. As soon as the interviewer asked him to explain how he arrived at the answer, he changed his mind, and gave a different description. Rather than using the bouncing primitive, he suddenly gave the correct answer that the wavepulses would pass through each other with no permanent effect. David's easy change of mind is consistent with our research results that, after any form of instruction, very few students give explanations which are consistent with the collision analogy.[5] David's changing explanation also supports diSessa's comment that many primitives related to collisions (overcoming, bouncing, and collision) are generally weakly supported by students.[6]

When describing superposition of two pulses whose peaks had not yet coincided but whose bases overlapped, David also showed difficulties consistent with the use of the point primitive. He gave answers which described no superposition in the region between the peaks of the pulses. The highest point of the string due to either pulse determined the shape of the string.

David showed evidence of a variety of primitives in his explanations during the diagnostic interview. As with Ford, we see evidence for three classes of primitives. David used the point primitive, some of the force primitives (actuating agency, working harder), and one of the collision primitives (bouncing). Furthermore, he applied the point primitive to the mathematics of waves and made use of the dying away primitive in the process. David's interpretation of wave physics seems based on reasoning that is inconsistent with actual observations. Instead, it seems to rely on analogies and reasoning based on previous studies in mechanics.

Kyle

Like David, Kyle participated in the S97 diagnostic test interviews two months after having completed tutorial instruction in waves. The details of the instruction will be given in chapter 7 when the diagnostic test is described in more detail. During the interview, Kyle inappropriately used many of the primitives described above. Only on the wave-math problem did he not use one of the previously described primitives, but he was unable to write an accurate equation for the wavepulse at a later time. Instead, he gave the same equation as the original one, when the pulse's peak is located at the origin. Though a profound difficulty, I had no further evidence to indicate the source of Kyle's answer. The lack of a temporal element to his equation possibly implies that Kyle was using the given equation to describe the entire wavepulse shape rather than the mapping of coordinates into each other (this would imply use of the point primitive to interpret the mathematics), but I am unable to support or refute this speculation from statements made in the interview.

When describing changes to wave propagation speed on the FR question, Kyle gave a variety of answers, not including the correct one. Kyle stated that one could make a slower pulse by "moving your hand slower," or "[moving] your hand higher, increasing the amplitude." In this description, a larger wavepulse would move more slowly through the medium, as if there were more resistance to its motion. This answer is indicative of the Ohm's primitive. Also, his response shows evidence of the smaller is faster response (since larger must be slower).

Later, when answering the same question a second time (in its MCMR format, rather than its FR format), Kyle seemed to reverse his reasoning. He gave the correct responses in addition to the responses he had given earlier. Both Kyle and David used inconsistent reasoning depending on the question asked of them. In this case, Kyle gave a correct response and stated that a lower tension would cause a slower wave. But, Kyle used faulty reasoning to arrive at his answer. He stated that a lower tension would create a smaller wavepulse, "the wave will be smaller, because you have less tension for it, [and that will make it slower]." Note that his response contradicts his earlier explanation that a larger pulse would be slower. This provides an excellent example for the student variation in student responses on material they have not mastered.

When describing wave superposition, Kyle spoke of colliding waves, and he showed no indication of wave superposition unless the peaks of the wavepulses overlapped. Much like Ford, Kyle spoke of collisions but never indicated that the pulses would cancel permanently. Describing the moment before the wave peaks coincided and the wavepulses partially overlapped, he said that the waves "were about to add," suggesting that he thought of the wave as only the peak of the pulse and not the entire displaced region. This shows evidence of the point primitive.

Kyle also had difficulties with the physics of waves reflecting from a fixed boundary (in the case of a wavepulse on a string firmly attached to a pole reaching the pole). He spoke of the pulse being absorbed into the pole to which the string was firmly attached, though he later changed his answer to state correctly that the pulse would be reflected and no energy would be lost in an ideal situation. As with Ford's description of wave propagation and David's description of wave superposition, Kyle switched from one explanation to another. He gave no clear reason for the switch. My asking him to explain how he arrived at his answer may have led to an evaluation of his answer which he did not verbalize. Also, my questions may have led to a change in the type of reasoning that he used, from a "gut instinct" to a "classroom style" answer.

Kyle later described reflection from a free end in terms of energy absorption (the pole would absorb the energy that was in the wavepulse). He said that the ability of the end of the string to move along the pulse meant that nothing would be reflected back, stating that "the movement of the string takes away the movement of the pulse." Though he did not explicitly use reasoning based on the overcoming primitive, we can describe his answers in those terms. With this primitive, one could say the pole is unable to be moved and must therefore exert a large force on the incoming wavepulse. That large force then manages to cancel out the incoming pulse.

On the sound questions, Kyle described sound waves pushing dust particles in a sinusoidal path away from a loudspeaker. As he stated, (his words are indicated by a "K" while the interviewer's words are indicated by an "I")

> K: *The dust particle will move up and down and the dust particle will be pushed away from the speaker.*
>
> I: *So the dust particle is going to move in a path away from the speaker* (indicates a sinusoidal path with a hand motion in front of Kyle)
>
> K: *Yeah.*
>
> I: *Why is that? If you could explain…*
>
> K: *Since the speaker is* (mumble, incomprehensible) *it will push the dust particle sideways. Since the dust particle is affected by the frequency, as long as the frequency is constant, it will move in a constant path.*

Later, when asked the effect of a change in the frequency, Kyle explained that

> K: *[The dust particle] will move faster because the frequency is higher which means that – since frequency is the one that affects the dust particle, if the frequency increases, the speed of the particle should increase, also.*

Though on the surface this indicates a correct response (a higher frequency will cause a faster average speed, since the particle is oscillating more rapidly), Kyle seemed to refer to the motion of the particle away from the speaker. Kyle often used the phrase "the frequency affects the dust particle," and when asked what he meant by the phrase, he stated

> K: *Frequency produces a sound wave, and the sound wave, somehow it will…* (He did not finish the statement, even with prodding from the interviewer).

In this description, Kyle is not using the term frequency to describe a property of the sound wave, rather, he states that the frequency *causes* the sound wave. This seems to be consistent with a description where the effect of a higher frequency is to make the particle move faster. Thus, Kyle's explanation seems consistent with the explanation given by Alex which was described in chapter 3.

In Kyle's explanation, the sound wave seems to guide the particle along the sinusoidal path. Kyle showed evidence of the guiding primitive (where the dust particle must move along the path determined by the sinusoidal sound waves) and the Ohm's primitive in his explanations of sound waves (where higher frequency has a proportional effect on wave speed).

Consistently, we see Kyle having difficulties with wave physics in nearly all areas that were investigated. He made consistent use of the point primitive, either directly as in wave superposition or implicitly as in wave propagation or sound waves (where the point primitive is a necessary step for the actuating agency primitive). He also made use of force primitives and collision primitives. Based on Kyle's responses, it is possible to interpret the guiding primitive as related to the force primitives, since it describes the relationship between force and motion.

Ted

Unlike David and Kyle, Ted did not have any tutorial instruction during the S97 semester. His class had traditional, TA-led recitations. In the diagnostic test interview, Ted answered questions from an early version of the diagnostic test which did not include any questions on wave-math. Ted showed profound difficulties with all other areas investigated in the diagnostic.

On the wave propagation question, Ted first stated on the FR question that only a larger amplitude would slow the pulse down. The larger pulse would move with more difficulty, he implied, because the pulse would "have to move more distance in the same amount of time."[7] The movement of the wavepulse <u>along</u> the curved string (i.e. with a larger curve, the length of the path to be traveled increases) implies the existence of the guiding primitive in his reasoning. It may also imply the existence of the smaller is faster (since larger is slower) primitive. When Ted came to the MCMR question on the diagnostic, he used far more explanations in his reasoning. He kept the larger amplitude response, but added that a slower hand motion would create a slower pulse. The slower hand motion would put less force into the wave (an example of working harder or Ohm's primitives, as explained above with respect to Kyle). In addition, Ted stated that changes in the medium, both tension and mass density, would affect the speed of the wavepulse.

Ted's description of superposition did not include the addition of displacement between peaks when the peaks did not overlap. For a situation where the waves (but not their peaks) overlapped, he stated that "the pulses haven't quite overlapped, so there's no reason [the amplitude] should jump up until they meet." He seemed to use the point primitive to consider only the peak of the pulse.

In his description of a wave reflecting from a boundary, Ted stated that the pulse would not be reflected from a free end. He spoke of the energy being absorbed into the pole, so that "nothing is left." This idea of absorption is consistent with the *overcoming* primitive, as described with Kyle above. The energy of the wavepulse is not sufficient to affect the pole, so the pole absorbs energy and does not transmit any back to the string. Thus, the wave does not return.

Ted's description of sound waves was also similar to Kyle's. He spoke of the dust particle moving away from the loudspeaker along a sinusoidal path, suggesting the use of the *guiding* primitive in this context (recall his previous use of it in the context of wave propagation). Furthermore, when the frequency of the wave changed, the speed of the dust particle being pushed away from the speaker changed. Thus, Ted seemed to use the *Ohm's* primitive in which proportional changes in frequency and speed occurred. Frequency was associated with the force of the wave on the dust particle, as Alex described in the interview excerpts given in chapter 3. Ted differed from Alex in that he explained that the volume of the sound wave affected its amplitude and not the force that it exerted on the dust particle. He maintained consistency with his description of transverse motion while moving away from the loudspeaker by stating that the speed of the particle away from the speaker would be the same as with a lower volume, but that the amplitude of its motion would change.

Ted had profound difficulties with all the topics of wave physics investigated in the version of the diagnostic which he took. He used both the point and *guiding* primitives in more than one context. Additionally, he used many of the force and collision primitives that we also found evidence for in the other students.

Summary of Common Student Used Primitives in Wave Physics

These four students consistently used a small set of primitives when incorrectly describing parts of wave physics. The four students all use the point primitive to simplify the shape of a wavepulse to a single point. In addition, they all make use of at least one of the force primitives. The collision primitives are also common to all students.

The primitives that these students applied to wave physics seem to be more strongly connected to Newtonian particle mechanics than they are to wave physics. The force primitives all seem related to difficulties students have in understanding the relationship between force and motion in classical mechanics (see, for example, the discussion in chapter 2 and references cited there). Since the collision primitives all seem related to collisions between hard objects (such as billiard balls), the students' difficulties appear to come from the incorrect application of ideas that may be appropriate in other areas of physics. In addition, some of the students used other primitives which suggest that they are not thinking of waves when they answer questions, or that when they think of waves, they have a model unlike the one that we would like them to have.

The Particle Pulses Pattern of Association

When students have consistent difficulties in one area of physics, it gives us the opportunity to organize their difficulties in a way that is productive and relevant for the development of instruction and investigations in other areas of physics. The four students described above all used a common set of primitives to describe wave physics. The primitives are often closely related to each other, such as the bouncing and overcoming primitives in descriptions of superposition or reflection from a boundary or the force primitives when describing sound waves and the physics of wave propagation. Because students use these primitives in conjunction with each other, we conclude that students are associating certain primitives in the context of wave physics. We find a consistent inappropriate pattern of association in the students' responses. We also find that students are not coherent in their use of this pattern of association and they are not consistent in its application. Thus, we cannot necessarily say that they are using a mental model in their reasoning. The topic of wave physics can serve as a context in which we show the value of using primitives, patterns of association, and mental models to describe many different student difficulties with physics.

In chapter 4, I discussed the idea of patterns of association and mental models. Both can be thought of as sets of primitives that are consistently applied to a situation and may serve as guiding principles for reasoning. One can describe student reasoning as if suggested rules or analogies to guide spontaneous reasoning.[8] Thus, patterns of association and mental models serve as a type of reasoning guideline for students, but are not necessarily the only guideline. Patterns of association describe looser constructions of student reasoning that are not as coherent, rigorous, or robust as the term "model" implies.

I should note that physical models often have the same limitations as the pattern of associations and mental models I am describing here. We refer to an accepted physical model, determined through theoretical and experimental work and the agreement of the research community to be valid in certain physical realms with certain limitations as a *Community Consensus Model* (or CM). An example of a CM would be the model of wave physics that students learn in the introductory physics sequence (as described in chapter 2). Within the limitations of the small angle, non-dispersive media approximation, we can use the wave equation and certain simple rules to analyze most physics problems. In more complex situations, this model no longer holds. Furthermore, when trying to apply the model, it may lead to results inconsistent with the model. For example, two small amplitude waves that superpose may add in such a way that the small angle approximation no longer holds. Thus, the introductory physics waves CM has the limitation of being, incomplete, and inconsistent with experimental data in certain situations. The difference between a typical weakly organized naïve student pattern of association and a CM is that a physicist is (usually) aware of the limitations of the CM and knows the shortcomings of the model while students often are unaware of the consequences of the contradictions in their reasoning.

When trying to organize student difficulties described in the previous section, we find it convenient to propose the existence of a *Particle Pulses Pattern of Association* (loosely referred to as the Particle Model, or PM) of waves. Table 5-3 summarizes the different aspects of the two mental models. Typical reasoning patterns involve the incorrect use of force or energy arguments and an inability to look at local characteristics of the wave. The PM can be described as the set of common primitives discussed in the previous section. These primitives include the pointprimitive, some or all of the force primitives applied incorrectly, and the use of the collision primitives. Student use of these primitives has been described in the previous section. In the remainder of this section, I will summarize common student responses when using the PM to guide their reasoning in terms of analogies with Newtonian particle mechanics rather than with separate incorrectly applied primitives.

The analogy to the ball toss seems to guide student reasoning in many topics of wave physics. Exerting more force when throwing a ball makes the ball travel faster upon release. Many students can be described as if they make an analogy to the

Table 5-3

Particle Model	Particle Pulses Pattern of Association	Community Consensus Model
A harder throw implies a faster particle.	A harder flick of the wrist implies a faster wavepulse.	Wave speed depends only on medium response to disturbance.
Smaller objects are more easily thrown faster.	Smaller pulses can be created that move faster.	Size of pulse and manner of wave creation do not affect wave speed.
An object's center of mass is considered when describing its motion (e.g. trajectory).	Only the peak of the wave-pulse is considered when describing superposition.	It is necessary to consider the entire shape of a wave to describe its properties (e.g. in superposition).
Objects collide with each other and their motion changes	Wavepulses collide with each other and they cancel or bounce off each other.	Waves pass through each other with no permanent effect.
Large objects traveling on a trajectory are described as points.	Propagating wavepulses are mathematically described only by the displacement of the highest point.	The mathematics of waves describes the displacement of every point of the medium.

Newtonian particle physics analogies of the Particle Pulses Pattern of Association and correct wave physics of the Community Consensus Model. Many students use both when answering questions containing wave physics.

amount of force required to create a pulse on a taut string; greater force in the hand motion creates a greater speed. In our investigations of the physics of sound waves, we have found evidence that many of students think that waves exert a force on the medium through which they travel and push the medium in front of them like a surfer on a wave. In chapter 3, Alex gives a description of the "surfer" description when explaining the interaction of sound waves and air. He also explains that sinusoidal waves are like a succession of pulses, each exerting a force (or "kick") in only one direction on the medium through which they travel.

Other students seem to make the ball-toss analogy when using energy arguments to describe how to change wave speed. A ball with a larger kinetic energy whose mass remains constant moves faster. Similarly, a pulse with more energy whose size stays constant must move faster. The explanation that a smaller mass will move faster is consistent with this explanation, too, because a smaller mass, with energy held constant, has a larger velocity. Though students do not explicitly state the analogy between their descriptions of wave speed and a thrown ball, their descriptions in interviews are often consistent with the idea that students' patterns of associations make use of this analogy.

Students often give point-particle descriptions of wavepulses. The ball-toss analogy gives an example of this reasoning in wave propagation. Similarly, in superposition, many students give the response that the wavepulses do not add until the points of maximum displacement overlap. They treat each wavepulse as a single point and ignore all other points. Other students describe the entire wavepulse as a single point, not ignoring the non-peak displaced points, but lumping all displaced points together into one.

Many students use a collision-like description of wave superposition to describe the interaction of wavepulses. Superposing wavepulses collide with each other and either bounce off each other or cancel out, depending on the situation. The remnant wavepulse possibly moves slower, having lost energy during the collision. Here we see a clear example of the way the pattern of association we use to organize student reasoning leads to descriptions different from a physical model of the physics. We have not found that students will <u>explicitly</u> state that the waves act like colliding carts, but we find that they often give descriptions consistent with the physics of cart collisions. Because of the similar explanations for the two situations, we believe students have an associative pattern which guides their understanding of wave interactions.

The lack of explicit evidence for incorrect student patterns of association should not keep us from looking for evidence of their existence. Two reasons exist for using the PM when trying to understand student difficulties with mechanical waves. First, by trying to organize student responses in terms of a model based on analogies, we account for the manner in which students approach our classes. By building a description that accounts for a majority of student responses, we may be able to make better sense of the classroom environment in which we teach. This may provide us with the opportunity to better diagnose what individual students are doing in our classroom, and help them overcome their difficulties. Second, by gaining insight into student use of patterns of association, we may learn an approach that could aid us in our attempts to understand student difficulties in other fields of physics.

Although we might expect student reasoning in the classroom to include many weakly organized components, we have found that student responses can be categorized with only two which are reasonably coherent. We can describe students as if they used elements of either the correct community consensus model of waves or a pattern of associations based on an over-application of ideas from Newtonian particle physics. Because we see only two reasonably coherent student descriptions of the physics, we have the unique opportunity to examine and evaluate the dynamic evolution of the mix. In chapter 7, I will discuss the evolution of student reasoning in the context of the modified curriculum developed by the Physics Education Research Group. This curriculum will be described in the following chapter.

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[1] See diSessa's comments in reference 5 in chapter 4 and Hammer's comments in references 2 and 7 in chapter 4 for a more detailed discussion.

[2] For example, in my graduate mechanics course, we spent a week or two discussing moments of inertia and rotations, but did not emphasize the subject greatly. In my advanced undergraduate mechanics course, only highly symmetric objects were considered and then only briefly.

[3] For more details on the use of pretests and the implementation of tutorials, see Chapter 7.

[4] I will refer to each student by the code name given and used during the interview. Code names were chosen by the student and correctly reflect the student's gender.

[5] The data supporting this statement can be found in chapter 6, when I discuss changes in instruction that had an effect on student performance on questions such as the one David answered.

[6] See reference 5 in chapter 4.

[7] Due to an error in videotaping, Ted's original interview videotape is unavailable. Ted's comments in this section are taken from extensive notes made during the interview. Though this allows us to gain insight into his reasoning, it prevents me from presenting lengthy excerpts of Ted's own words from the interview.

[8] Redish, E. F. "Implications of Cognitive Studies for Teaching Physics" Am. J. Phys. **62**, 796-803 (1994).

Chapter 6: Development, Implementation, and Evaluation of Tutorials

Introduction

Once student difficulties have been found and described in detail, PER can serve as a guide for developing effective curriculum. These materials can aid students develop the difficult concepts that they will need to understand in their future studies in physics and other fields.

Many different types of research-based instructional curricula have been developed and evaluated for their effectiveness in teaching students relevant physics.[1] At the University of Maryland (UMd), the Physics Education Research Group (PERG) has introduced *tutorials*, a teaching method created and designed at the University of Washington, Seattle, by Lillian McDermott, Peter Shaffer, and the Physics Education Group.[2,3]

In this chapter, I will use the area of wave physics to illustrate how research-based curriculum development can create a productive learning environment for our students. The tutorials described in this chapter have been developed through an iterative process of research, curriculum development, implementation, and evaluation for a period of one to three years. Working in collaboration with other members of the Physics Education Research Group (PERG), I have developed a set of tutorials in wave physics which are designed to help students learn certain fundamental ideas in physics. The three tutorials discuss the physics of

- wave propagation and superposition
- the mathematical description of waves, and
- sound waves (propagation and mathematical description)

(Copies of the final versions of the tutorials can be found in Appendix A, B, and C. Unless otherwise noted, I describe the most recent version of each tutorial.) Throughout the tutorials, students discuss and develop the ideas of equilibrium, disturbances from equilibrium, propagation of a disturbance through a medium, effects of two disturbances meeting each other, and the mathematical description of a physical system through the choice of an appropriate model. These are skills which, to a certain extent, are illustrated best in wave physics but whose ideas are important in other areas of physics and in the students' subsequent studies.

In the sections describing each tutorial, I will discuss the research basis of each tutorial. In addition, I will discuss research results that suggest that tutorials are more effective than traditional instruction in teaching students the fundamental topics of wave physics. Many results come from a diagnostic test that is discussed in more detail in chapter 7.

One general point should be made when discussing the effectiveness of the tutorials with respect to student understanding of the material. Though the descriptions below imply that students receive an hour of instruction on the material being discussed, note that they are not receiving traditional recitation instruction. The time spent on the physics is roughly equivalent in the two settings, but the manner in which students interact and learn in the classroom is different. In the discussion below

I will emphasize what students do in the classroom and evaluate their performance based on the tutorial activities.

Creating Video Materials For Classroom Use

A central piece of each of the tutorials involves students viewing digitized videos of waves propagating on a long, taut spring. These videos were created in the Summer, 1995 workshop on Teaching Introductory Physics Using Interactive Methods and Computers, held at Dickinson College. As part of the workshop, John Lello and I carried out a project in which we created the videos and developed preliminary versions of some of the curriculum materials presented below. In this section, I will discuss how the videos were created.

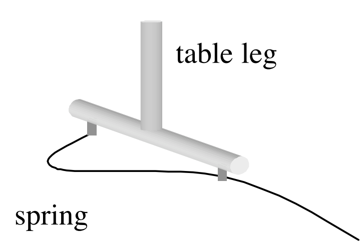
We stretched two long snake springs between two tables. A snake spring is a tightly coiled spring of roughly 1.5 cm diameter. The unstretched length of the springs was nearly 2 m. We stretched them to a tension of 10 N to 15 N and lengths of 4 m to 6 m, depending on the situation. On each end of the snake spring were loops (i.e., the last few coils of the spring, bent 90°). Each loop was fastened to the leg of a table by screws which are usually used to adjust the height of the tables. By keeping the table motionless (their weight held the springs firmly in place), we were able to keep the springs at a constant length and tension. Note that the spring was attached to the far leg of the table. The snake spring was free to move on only one side of the second table leg. See Figure 6-1 for a sketch of the set-up.

The waves used in the videos were created by pulling the spring through the gap between the table legs and releasing it. For example, by pulling the spring at exactly the midpoint of the gap between the two legs, we could create a triangular shaped pulse (see Figure 6-2).

The propagating waves were videotaped from a ceiling mounted camera whose signal was fed directly into a computer. The computer digitized the video signal immediately. This digitized video was then edited to include only the frames during which the wave was visible on the screen. Due to the design of the system, the speed of a propagating wave was approximately 8 m/s. Since the view field of the camera was roughly 2.5 m, the wave was visible on screen for roughly one quarter of a second. Since videotape is filmed at 30 frames per second, the wavepulses are visible in the videos for roughly 8 frames.

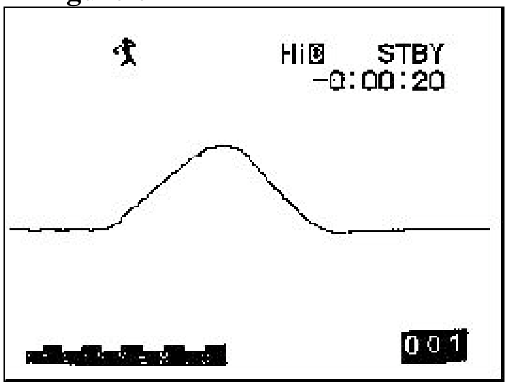
To videotape the wave propagating along the spring, a variety of problems had to be addressed. For example, the floor on which the spring rested was made up of

Figure 6-1



Sketch of set-up for creating wavepulses on a stretched snake spring. The spring is attached at one table leg and is pulled back by a hand (not shown). The spring is free to move along the second table leg.

Figure 6-2



Screen capture of "triangle.mov." The wavepulse moves from left to right along a stretched spring. The video is commercially available in the VideoPoint™ software package.[5]

tiles, creating a low friction surface that prevented excessive energy loss to the system. Unfortunately, the tile floor also reflected the light from the room very strongly, making the silver snake spring difficult to see on the video. After much experimenting with different colors and materials, we found that a dark blue felt cloth created the best backdrop on which to see a moving silver colored spring. The higher friction from the new material did not seem to affect wave propagation or wave shape in any appreciable way. We were unable to compare the decrease in amplitude (equivalent to the loss in energy) between the two designs because we were unable to see the spring clearly when filmed on a plain tile floor. Using analysis techniques described below, we were able to show that the wavepulse on the felt floor lost roughly 10% of its amplitude over the course of the 8 frames on screen. This effect was considered unavoidable and small enough to be acceptable for our needs (mainly because the effect was very difficult to see on screen).

Another problem we had consisted of finding the right shutter speed for the video recorder. The wavepulses we created were roughly 50 cm in width at their base with an amplitude of roughly 50 cm. (Note that there are dispersive effects due to the large amplitude of the waves, but in the time scale we were observing, we could ignore these effects). With waves moving at 8 m/s, it would take a point on the spring 1/32 s to move from equilibrium to maximum displacement. With a frame rate of 30 frames per second, the slowest possible shutter speed was 1/60 s. (Video cameras use an interleaving technique such that the slowest shutter speed equals half the frame rate.) During this time, a piece of spring could move from equilibrium to maximum displacement. The slow shutter speed would create a blurred image on screen. As a result, we set the shutter speed of the videotape as high as the camera allowed (1/1000 s). We can estimate the speed of the piece of spring to be relatively constant for most of its motion away from the vertices of the triangular pulse shape, since we notice that the slope of the wavepulse is relatively constant in Figure 6-2. Thus, the piece of spring moves 1/2 m in 1/16 s, making an estimated speed of 8 m/s. During 0.001 s, the spring moves a distance 0.008 m, or just under 1 cm, which is slightly less than the diameter of the snake spring. This creates some smudging in the video, but only a negligible amount.

Two issues were problematic when making the superposition videos. In some of our videos, we show superposition of wavepulses moving toward each other from opposite ends of the spring. To create these videos, we had to make sure that the wavepulses met as close to the center of the videotaped area as possible. This meant that the people holding the spring on either of its ends had to release their hold within 1/30 s of each other. We came up with a rather elaborate counting scheme and rhythm to allow for this. In our first attempt at illustrating destructive interference, we managed to have the wavepulses meet within 10 cm of the center of the video screen. The one point on the spring that never moved was nearly perfectly located in the center of screen. (In destructive interference, there is an instant when the entire spring is at equilibrium, but only one piece of the spring is never in motion.) In the case of constructive interference, we were never able to have the point of maximum displacement closer than 1 m from the center of the screen.

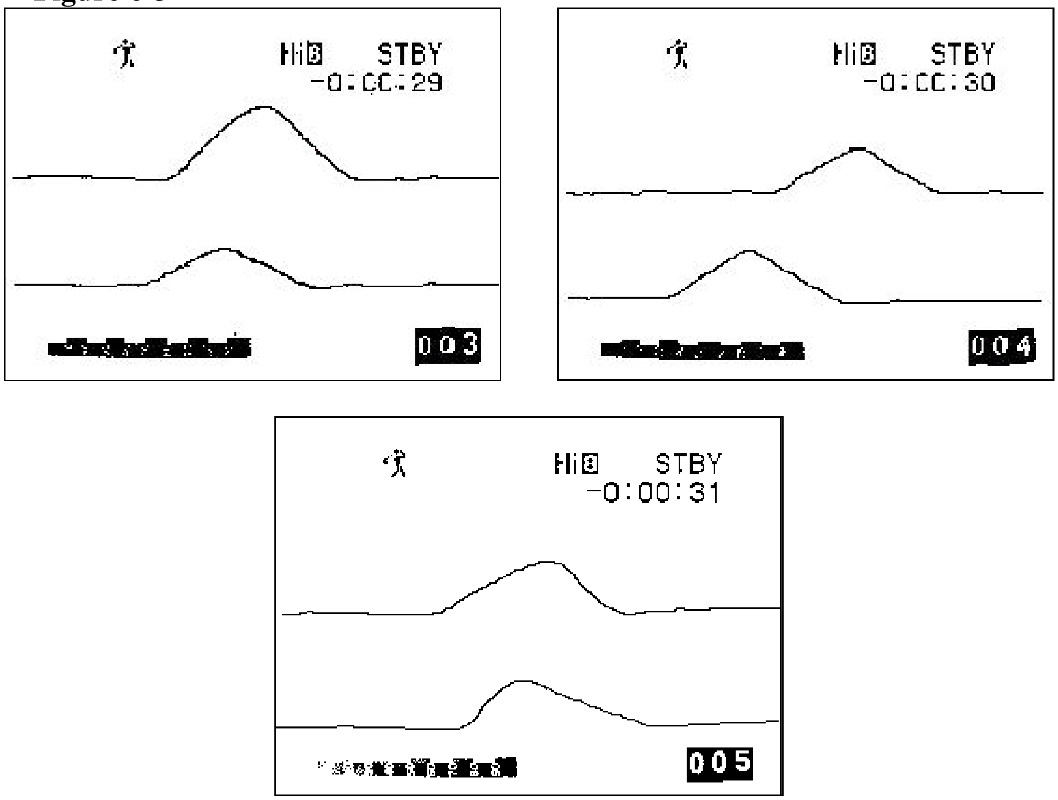
The second issue in creating the superposition videos involved the gross deformations to which we subjected the spring. When the two large amplitude wavepulses overlapped, the spring was stretched to an even larger amplitude. In the process, the dynamics of the system changed. Instead of the waves simply passing through each other at a constant speed, the time of the interaction was much longer than expected. Also, in the case of constructive superposition the moment of nearly perfect overlap of the peaks was captured on film. During this time, the amplitudes did not add up perfectly. We explain both of these phenomena by noting that the deformation to which the spring was subjected was much larger than the spring was designed for. In other words, the spring was simply unable to stretch enough. Later, when we attempted to stretch the spring to a similar length, we overstretched it and destroyed the tight coiling. This did not occur during the filming of the videos because the time scale of the stretching was so short.

In total, we created a set of ten videos of waves propagating on springs. The six that are used in tutorials are:

- *triangle.mov* – a single triangular-shaped pulse travels across the screen (see Figure 6-2),
- *diffside.mov* – two wavepulses on different sides of the spring meet and pass through each other,
- *sameside.mov* – two wavepulses on the same side of a spring meet and pass through each other,
- *diffshape.mov* – two asymmetric wavepulses with mirrored shapes travel side by side down two separate springs with identical mass density and tension (see Figure 6-3, video number 5),
- *diffamp.mov* – two wavepulses of different amplitudes travel side by side down two separate springs with identical mass density and tension (see Figure 6-3, video number 3), and
- *difftens.mov* – two wavepulses travel down two separate spring with different mass density and tension (see Figure 6-3, video number 4).

In the videos which showed a comparison of two different properties of the wave or the system, we had two springs lying side by side. We were able to create this situation by using both of the legs of the tables to which we had attached the springs. These table legs were separated by roughly one meter. We created different waves on each spring and were able to videotape how waves traveled side by side down the spring. Again, the timing issue played a role in creating these videos. We

Figure 6-3



Screen captures of the videos *diffamps.mov* (numbered 003 in the bottom right corner), *diffshape.mov* (005), and *difftens.mov* (004). The springs in *diffamps.mov* and *diffshape.mov* are identical and pulled to the same length and tension. The springs in *difftens.mov* are identical but stretched to unequal tensions (and therefore of different mass densities).

wanted the peaks of the waves to be side by side. This problem was easily solved by having one person with long arms hold each spring and release them at the same time. Examples of the video created can be seen in Figure 6-3.

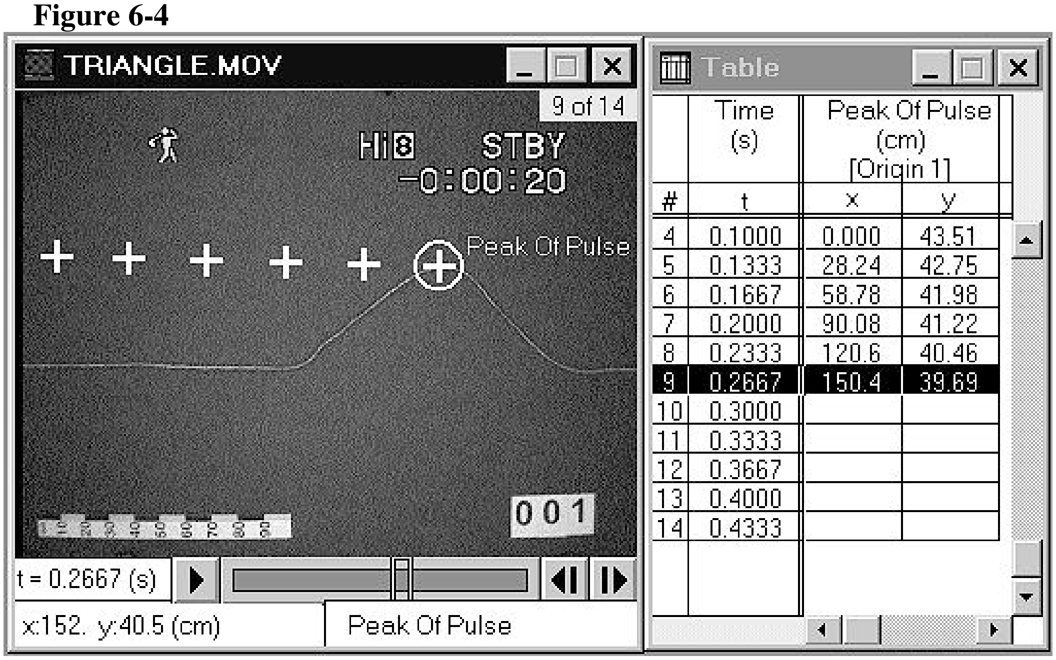
In the video *difftens.mov*, we had two springs of different tension. Note that the spring with the higher tension also had a lower mass density, since the higher tension was created by pulling the spring tighter. In this video, the timing issue was critical. We had calculated that in the given situation, the faster wavepulse would move roughly 1.4 times the speed of the slower wavepulse. The end of the video recorder's range was about 3 m from where the wavepulses were created. Therefore, the slower wavepulse moving at 8 m/s would reach this point in 3/8 s, the faster in 3/11 s (roughly 1/4 s). To have the faster wavepulse catch up to and pass the slower one while on screen, the faster wavepulse had to be released about 1/8 s later than the slower one. As with the video of destructive interference, the first attempt was the most successful. In this video, the faster wavepulse catches the slower wavepulse in the last frame of the video, such that their peaks are almost exactly lined up with each other.

To analyze the speed and the dimensions of the wavepulses on the video screen, we used two different software tools. The first was Apple Computer's *QuickTime*[4] multimedia software. Using this software, we could view the propagating waves and count how many frames it took the wavepulses to traverse a known distance (since we had determined the video's viewing range previously). This gave us a good estimate of the speed of the waves.

To gain more detailed measurements of the wave speed, amplitude, loss of amplitude, width, and other variables, we used software developed at Dickinson College. *VideoPoint*[5] software was in beta testing at the time of our summer workshop, but has been released since then. The videos described in this section form part of the commercially available software package and are available for use by anyone who purchases VideoPoint. To analyze the video, we begin by measuring a known length scale. In our case, we had placed a clearly marked and large meter stick in each of the videos. This meter stick is visible at the bottom left corner of each video screen capture shown in Figure 6-2 and Figure 6-3. By clicking on the each side of the meter stick, the person using the software can define a length scale for the images in the video. VideoPoint scales all distances on the screen according to the given length, and allows the person manipulating the software to use the cursor on screen to describe distances from an origin.

Measurements are made by placing the cursor at a location on the video window and reading its position off the screen (see Figure 6-4). Also, one can click on a given point, leaving behind a marker at that location. The position of this marker is then given in a data table. More than one marker can be placed, and the data table shows the coordinates of each marker at the correct time (where the time scale is

Figure 6-4



Screen capture from VideoPoint. The data point "Peak of Pulse" is shown for this frame, along with the time at which the frame is shown and the *x* and *y* position of the data point.

chosen in 1/30 s increments from the beginning of the video). As an example of how this allows measurements to be made, consider placing markers at the location of the peak of a propagating pulse. By measuring the location of the peak of the pulse in 1/30 s increments, we can show that the speed of the wave is constant as it crosses the screen. But, we can also show that the amplitude of the wave decreases by 10% from its original value. Students use VideoPoint mainly in the tutorial on sound waves, described below.

The video for the sound wave tutorial was filmed by Mel Sabella, also a member of PERG at UMd, during his stay at the 1996 Dickinson Workshop. In this video, a burning candle is placed roughly 5 cm from a large (25 cm diameter) loudspeaker. In this region, the waves from the speaker can be considered planar. By creating a low frequency but high volume wave, one can cause the candle flame to oscillate with an amplitude of roughly 4 mm. The physics of the situation have been discussed previously in chapter 3. In the video, we see the flame oscillating back and forth. The video was created by using a strong telephoto (zoom) to show both the loudspeaker and the flame. An image from the video is shown in appendix C in the tutorial on sound waves. The tutorial, discussed below, asks students to interpret the wave physics based on the oscillation of an element of the system through which the wave is propagating. To do so, they must make use of data gathered from VideoPoint.

The videos on mechanical waves and sound produced by the members of UMd PERG have been published and are commercially available on the VideoPoint CD,[5] where they can be found under the category "UMD movies."

Wave Propagation and Wave Superposition

Description of Tutorial

The Propagation/Superposition tutorial has been designed to address two profound difficulties that students show in pre-instruction investigations of their understanding. The tutorial is found in Appendix A of the dissertation. Through the use of video analysis, students have an opportunity to address their use of the Particle Pulses Pattern of Association (loosely referred to as the Particle Model, or PM, of waves). Our hope is that the tutorial will provide students with the opportunity to overcome their difficulties with wave propagation (i.e. the incorrect description that wave speed depends on the motion of the hand) and superposition (i.e. the incorrect description that waves add only at or with their highest points and nowhere else).

The tutorial we have developed is based partially on work originally done at the University of Washington, Seattle. Although much of our tutorial has been written to include video analysis of propagation and superposition, some parts still contain material from the UW tutorials. Interested readers can compare the UMd tutorial in Appendix A with the UW tutorial.[6]

Students begin with a pretest that investigates their understanding of wave propagation and wave superposition (see Appendix A). In taking the pretest, students are forced to think through the problem on their own, commit to an answer, and articulate that answer in writing. Because of the student difficulties we have found with the questions in the pretest, we believe the problems are both challenging and relevant.

In the tutorial itself, students begin by participating in a class-wide discussion based on demonstrations carried out by a facilitator. (This part of the tutorial is adapted from the one developed at UW.) In response to facilitator questions, students describe their observations of wave motion on a stretched snake spring (like the one used in the video). The facilitators use quick hand motions (a flick of the wrist back and forth) to give students an example of how to create a spring using a hand motion like the one presented in the tutorial. Students are asked to distinguish between transverse and longitudinal waves by comparing the motion of a piece of tape on the spring to the motion of the wavepulse. They are also asked to describe how different hand motions by the facilitator affect the shape of the wavepulse. Discussions led by the facilitator emphasize observations of how the shape and the motion of the wavepulse might be related. The facilitator also changes the tension of the spring and asks students to compare their observations with previous demonstrations of wave propagation. Class discussions use student terminology rather than imposing language from the facilitator. By constantly asking if the whole class agrees with a student's comments, the facilitator allows the students to regulate each other. Students build their understanding through observation and discussion. We find that the demonstrations alone are inadequate to help students observe certain aspects of wave propagation because students often see what they believe occurs rather than observing what actually happens.

Due to the high speeds of wavepulses in the demonstrations and student confusions about their observations during the class-wide discussion, the remaining activities in the tutorial attempt to address lingering difficulties. Students split into groups of three or four to work on the rest of the tutorial. They watch videos of wavepulses to view wave phenomena in slow motion. In these videos, individual wavepulses on two identical springs travel across the computer screen. Students use QuickTime[4] to advance the video frames individually or watch the whole video. In each video, wavepulses on the springs have some fundamental difference. Either their amplitude is noticeably different (*diffamps.mov*), their shapes are noticeably different (asymmetric triangular shaped pulses with mirrored asymmetry, *diffshape.mov*), or the tension in the springs is different. (Students are told this, since tension is not a directly observable difference, *difftens.mov*.) Figure 6-3 shows a typical screen shot of each of the videos.

For each video, students are asked to describe the hand motion that could have caused wavepulses with the shapes on the screen. For example, in *diffamps.mov*, the different amplitude wavepulses are of the same width at the base. Since the waves move at the same speed, equal width implies that the waves were created in the same amount of time. The distance of motion in the same amount of time differs, so the hand speed needed to create the different wavepulses hand differs. Those students who have stated on the pretest that different speed hand motions lead to different speed pulses must reconcile their expectations with observations of same-speed waves in the video. The movie *diffshape.mov* takes this idea further, showing that waves created through two different motions (mixed fast and then slow) would produce waves that travel at one speed. In the *difftens.mov* video, students are told that the tension in the springs is different. They observe different wave speeds, indicating that the tension on the spring affects the speed of wave propagation.

To further address student difficulties with the differences between transverse motion of the medium and longitudinal propagation of the wave, we next ask students to sketch velocity vectors for parts of a wavepulse propagating on a taut spring. Students use the wave motion to describe changes in position of the medium, and then use simple ideas of kinematics to describe the average velocity of the medium at different points. They describe that medium motion and wave motion differ in fundamental ways. These activities extend the previous discussion of differences between the motion of the medium and the motion of a disturbance to the medium by bringing in a more quantitative description of each motion.

After students have observed that the speed of the wave is constant at all times and that the motion of the medium is transverse to the direction of propagation, they predict the behavior of superposing waves. They are asked to sketch the shape of a string with two asymmetric wavepulses on it, much like on the pretest. The shapes given in the tutorial are chosen to match those on a video, "sameside.mov," that students watch on their computers after making their predictions. Students are asked to account for the shape of the string at different times, and guided to an understanding that displacement of the string depends on the displacement due to each individual pulse. The rest of the tutorial develops this idea as students predict the effects of destructive interference and view "diffside.mov."

Student Understanding of Wave Propagation

Student performance on both FR and MCMR wave propagation questions before and after tutorial instruction has been described in chapter 4. The tables showing student performance on the FR and MCMR question before and after instruction (as discussed in chapter 4) are shown in Table 6-1 and Table 6-2.

To use the language developed in chapter 5 to describe student performance after instruction, we found that students still answered many questions using the PM (speed depends on hand motion), though a greater number used the CM exclusively in their responses (speed depends on tension and mass density). Also, many students seemed to be triggered by the additional offered responses in the MCMR question into giving the CM response when they had previously given only a PM response. If we look at only the MCMR responses, we can still discern that many more students give an exclusively CM response and the number of students who give mixed CM/PM responses has gone down greatly.

The MCMR question is an interesting tool to evaluate lingering student difficulties with wave propagation after instruction because students already perform quite well on it before having any instruction on waves (in terms of recognizing the correct answer). The interesting measures in the MCMR question are how many students give completely incorrect (PM) responses or mixed (PM and CM) responses.

In addition to the tutorial classes described above, we have given the MCMR wave propagation question (after instruction) to 116 students who did not have a tutorial that specifically addressed their difficulties with wave propagation. In the S96 semester, students worked through a tutorial that did not include the use of videos in

Table 6-1:

(a)

		Student responses on free response question			
	Speed changes due to change in:	Only tension and density	both the medium and hand motion	the motion of the hand	other
Student responses On MCMR question	only tension and density	7%	1%	2%	1%
	both the medium and hand motion	1%	2%	60%	10%
	the motion of the hand	1%	1%	11%	3%

Comparison of student pre-instruction responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions before all instruction.

Table 6-2:

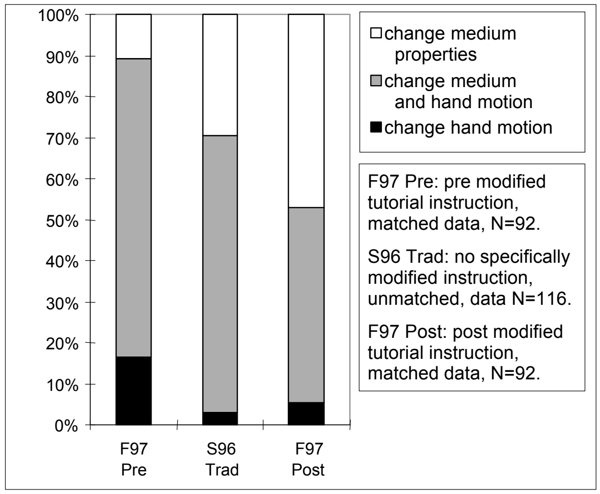
		Student responses on free response question			
	Student Response:	Only tension and density	both the medium and hand motion	the motion of the hand	Other
Student responses on MCMR question	Only tension and density	40%	2%	2%	2%
	Both the medium and hand motion	8%	17%	20%	2%
	the motion of the hand	2%	1%	2%	0%

Comparison of student post-instruction (lecture and tutorial) responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions after all instruction on waves.

the same fashion as in F97. The class did not answer the FR question along with the MCMR question (as has been previously described in the dissertation) due to logistical reasons that prevented us from asking it that semester. We also do not have pre-instruction results from this class, but we suggest that the pre-instruction results from F97can be taken as suggestive of student performance in S96.

Figure 6-5 shows student performance on the MCMR question at three different stages of instruction. Note that some of the columns are matched (F97 data) while the middle column (from S96) is not. Also, we compared the performance of the F97 class as a whole to the performance of the matched students whose data is presented in the figure and found no great difference. The results show that students begin the semester (in F97) already using the correct response very often (more than 80%), but predominantly giving responses which we categorize with the PM (90%). After both traditional and tutorial instruction, nearly all students in both S96 and F97 semesters give the correct response (98%). But, in S96, after instruction that did not specifically address student use of the PM in wave propagation; nearly 70% give responses indicative of the PM. After instruction that addressed student use of the PM (in F97), roughly 50% of the students give answers consistent with the PM. These results illustrate the contrast of student answers among pre-instruction, post traditional instruction, and post modified instruction. (In this case, modified instruction that did

Figure 6-5



Comparison of student responses on the MCMR wave propagation question, F97 (matched pre/post tutorial instruction, N=92) and S96 (unmatched, post traditional instruction, N=116). Students answered the question on diagnostic tests given before and after all instruction on waves.

not specifically address the relevant issue is considered traditional instruction, since students did not receive any instruction on the material outside of the typical lecture setting). The effect of the modified tutorial instruction is evident when considering the differences in mixed CM/PM responses in S96 and F97. The results from F97 indicate that specially designed curriculum can play a role in affecting what is otherwise a very robust incorrect response.

Student Understanding of Wave Superposition

On the topic of wave superposition, we also see improvement in student performance after students participate in modified instruction. Student performance on wave superposition questions before any instruction, after traditional instruction, and after all instruction (including tutorial instruction) shows a definite shift in student performance and understanding of the physics of wave superposition. Table 6-3 shows student responses to the superposition questions asked during the course of the semester. The question shown in Figure 3-13 (described in chapter 3) was asked before and after all instruction. The question shown in Figure 3-9 was asked on a pretest which followed lecture instruction on superposition but preceded tutorial instruction. Only those students (N=131) who answered all three questions are included in the data.

At the beginning of the semester, only a quarter of the students correctly show superposition at all locations, while half the class gives answers which we have characterized as evidence of the PM. For example, they do not add displacement between the peaks of the wavepulses, they add the maximum displacement of each pulse even when the points of maximum displacement do not overlap, or they show

Table 6-3

Time during semester: MM used:	Before all instruction (%)	Post lecture (%)	Post lecture, post tutorial (%)
CM (point-by-point addition)	27	26	59
PM (only one point plays a role)	65	52	27
other	6	13	7
Blank	2.3	9.2	6.9

Student performance on wave superposition questions at different times during F97 (N=131 students, data are matched).

the waves canceling in the area where they overlap. We classify these responses as indicative of at least one aspect of the PM, as described in chapter 5.

After traditional instruction, students did not change their descriptions greatly. Even on a question that differed only slightly from the one asked at the beginning of the semester, one fourth answer the question correctly, and one half show evidence of the PM.

After tutorials, we see that the numbers have shifted dramatically. Nearly 60% of the students answer the question correctly, while slightly more than one fourth still show evidence of the PM. Based on these results, we claim that the tutorial has a strong effect on student understanding of wave superposition.

We must qualify this statement by showing evidence that problems persist. Before and after traditional instruction, 25% and 24% of the students (respectively) do not show addition of displacement between the wavepulse peaks (see Figure 3-13b). This accounts for 40% of the students who gave a PM-like response before any instruction and 50% of the students who gave a PM-like response after traditional instruction. (Other PM-like responses include showing waves as colliding, bouncing, canceling permanently, or adding amplitudes even when the peaks do not overlap.) After tutorial instruction, 17% of the students give the answer that there is no addition of displacement between the wavepulse peaks. This represents 63% of those showing evidence of the PM after instruction. A majority (68%) of the students who state before instruction that there is no addition between the peaks of the wavepulses do not change their responses after instruction. The students who move away from a PM-like response are those that gave other PM-like answers. This suggests that some aspects of PM reasoning when applied to superposition are very hard to overcome and that the present materials are not completely successful in suppressing student use of it.

Mathematical Description of Waves

Description of Tutorial

The tutorial that addresses the student difficulties with the wave-math problem described in chapters 3 and 5 is based directly on the wave-math problem itself (see Figure 3-7). In tutorial, groups of three or four students work through guided worksheets. The worksheet for this tutorial is included in Appendix B. Students begin by considering the mathematical form of a pulse at $t = 0$. In order to minimize the confusion related to the exponential, we begin this tutorial with the equation:

$$y(x) = \frac{50cm}{\left(\frac{x}{b}\right)^2 + 1} \tag{6-1}$$

where $b = 20\ cm$.[7] In order to help students develop a functional understanding of a function, they explicitly graph the shape of the string based on the equation representing its shape. Students are also given a screen capture of a propagating pulse (as shown in Figure 6-2) and asked to estimate the values of the amplitude and b in the equation above. (Though noticeably a very inexact Lorentzian pulseshape, the general shape is sufficient for this exercise and provides an excellent opportunity for discussion of modeling with the more advanced students.) When students are asked to sketch the shape of the spring on which the wavepulse is moving after the pulse has moved a distance of $3b$, we find that many sketch the shape with a lower amplitude. This is consistent with the analysis of the wave-math problem, where the variables x and y are misinterpreted as the location of the peak and peak amplitude of the pulse, respectively. Students are asked to watch a movie of the propagating pulse and those who made incorrect predictions based on the mathematics are confronted with their incorrect predictions and forced to describe the relationship between the mathematics and the physics. Thus, through their observations and their own reasoning, students see the need for modification to the mathematical function so that the wave shape stays the same while the shape propagates through the medium.

Students then sketch the shape the string would have after the pulse traveled some distance without dissipation, and are guided into constructing the mathematical form. In this way, students not only construct the shape of the string from an equation, they construct an equation from the shape of a string. After considering the functional form of the pulse at two different times, the students are given the opportunity to construct a single equation that describes the pulse as a function of both position and time. The key here is that it is the students that are constructing this equation based on their own work and on consideration of a specific physical system.

In the second part of the tutorial, students consider a pulse of a slightly different shape propagating on a string. In particular, they consider a pulse represented at $t = 0$ by the same equation considered in the pretest:

$$y(x) = Ae^{-\left(\frac{x}{b}\right)^2} \tag{6-2}$$

Students again are asked to construct an equation that describes the displacement of the string as a function of position and time. This time, students are not guided to this answer. Instead, they are forced to generalize their results from earlier in the tutorial and, when appropriate, resolve the conflict with their answers on the pretest.

In the final part of the tutorial, students apply and interpret the ideas that they developed by considering the motion of a tagged part of the string. Here they extract useful information about the motion of the tag by interpreting the mathematics of the problem. Because of student difficulties relating a physical situation to the corresponding equation, students use video software to mathematically model the shape of an actual pulse. As part of this, they explore the physical significance of the parameters A and b in equation 2 in more detail than they did with equation 6-1 in the first part of the tutorial.

The homework which accompanies this tutorial is given in Appendix B. Students apply the ideas covered in the tutorial while considering pulses of different shapes moving in different directions. They consider the shape of the pulses at different times physically and mathematically. They also consider the motion of the tagged part of the string in different situations.

Student Understanding of the Mathematics to Describe Waves

We have not investigated student understanding of the mathematical description of waves in as great a detail as other student difficulties for several reasons. To ask students to commit to answers about the mathematical description of waves (including two-variable functions) before instruction would be difficult with students who have no experience with such equations in physics. In addition, because of a shortage of time, we did not investigate this issue on the F97 diagnostic test. Finally, though certain examination questions were asked after student instruction on the mathematics of waves, no clear analysis of student understanding was possible because the questions asked avoided most of the issues which would have elicited student difficulties.

In S97, interviews were carried out with twenty students, fifteen of whom had tutorial instruction and five of whom hadn't. In these interviews, not all students answered the mathematics question because the question was not included in early versions of the diagnostic interview protocol. Of the 10 tutorial students who answered the question described in Figure 3-7 on the pretest before instruction, five sketched the shape of the spring with a smaller amplitude after it had propagated a certain distance, and none were able write a correct equation. Most who sketched a smaller wavepulse indicated that the exponential was the reason. Eight students plugged in x_0 or left x as the variable to describe the equation of the string. We consider that the PM can be used to describe both these responses. Students seem to be using the point primitive in their attempts to make sense of the mathematics, as has been discussed in chapter 5.

After tutorial instruction, student performance improved. Eight sketched the shape of the wavepulse correctly, and the two who did not indicated that they thought of the exponential term to guide their reasoning. Six of the students wrote the correct equation, though four of the students either plugged in x_0 or x to describe the shape of the string. The tutorial seems to have addressed some of the students' difficulties. More research needs to be done to investigate student understanding more deeply.

Sound Waves

Description of Tutorial

The sound tutorial, like the previous two, builds on our observations of student difficulties with fundamental ideas of physics. These difficulties have been illustrated in detail by quotes from interviews with Alex presented in chapter 3 and Kyle presented in chapter 5. In the discussion of student difficulties with sound waves, student difficulties have been described in terms of the description of the motion of a dust particle. The tutorial discusses the motion of the medium through which sound waves move in the context of an oscillating candle flame. Data from a pretest from F97 (see Table 6-4) show that students have generally the same difficulties with describing the motion of a candle flame that they have with describing the motion of a dust particle. Only matched data are presented in the table. Note that the high number of blank responses on the candle flame response are due to the candle flame question coming in the later half of the pretest. Since the pretest was asked in one class (of the two that took it) on the same day as a mid-term examination, many students simply did not attempt the majority of the pretest. Also, their time was much shorter than the students in the other class. Still, the data are very similar, suggesting that the context of the tutorial is relevant to student understanding of sound waves. Detailed results will be shown below.

In the tutorial, students begin by predicting and then viewing a video of the motion of a single candle due to a sound wave coming from a large loudspeaker. Students must describe the motion of the candle due to the sound wave, and must resolve any conflicts between their predictions and observations. In addition, we ask that they explicitly apply their predictions to the context of the dust particle that was part of the pretest.

In the next section of the tutorial, students are given data that shows the position of the left edge of the candle flame at different times. The data points have been taken beforehand using the program VideoPoint[5] (see above for a description of

Table 6-4

Object Whose Motion is Being Described / MM used:	Dust Particle (%)	Candle Flame (%)
CM (longitudinal oscillation)	22	39
Other oscillation	26	3
PM (pushed away linearly or sinusoidally)	38	21
Other	11	17
Blank	3	20

Performance on student pretest, comparing descriptions of dust particle and candle flame motion. Students answered the two questions at the same time (F97, data are matched, N=215). The high number of blank responses on the candle flame question is due to lack of time during the pretest, which was followed by an mid-term examination.

VideoPoint) and are presented to the students in a data table in the tutorial. Due to time limitations during the tutorial, students are not asked to take the data themselves. In the tutorial, students must observe the connection between the data points and the cross-mark on the screen. Students are asked to graph the data points on a provided graph. From the graph, they then find the period of the sound wave. In the activity, students go from a description of a single candle (which represents the motion of the medium) to describing the frequency of the sound wave. Thus, they are given the opportunity to connect observations, mathematical descriptions, and physical properties that they have discussed in class and used in their homework.

Students are then presented with a photograph of *two* candles sitting in front of a loudspeaker. They are asked to describe the motion of both candles and to sketch separate displacement vs. time graphs for each candle. To answer the question, students must generalize from their previous description of a single candle's motion to think about any possible changes between the motion of the first and the second candle. Students must use the idea of a propagating wave with a finite speed to develop the idea of a phase difference between the motion of the two candles. This idea is developed through a Gedankenexperiment where students are asked to think of more and more candles placed at different locations along a path away from the speaker. They are asked to sketch the displacement from equilibrium of each candle at a specific instant in time. From this activity, they able to find the wavelength of the sound wave. Again, students are given the opportunity to connect their mathematical knowledge from class with reasoning based on simple ideas that build on the model of wave propagation from the previous two tutorials.

The ideas of wave propagation and wave-math form an integral part of students' opportunities to build an understanding of the phase difference between parts of the medium which are different distances from the wave source. Thus, at the end of the tutorial, students have had to revisit material and concepts from their first two tutorials. They have built a model of waves as propagating disturbances, and they have described the propagation of these disturbances on a taut spring. In the sound wave tutorial, students use this model of wave propagation to describe a different area of physics. They are able to develop the idea that the concepts discussed in the tutorials are general and applicable to different topics that are more general than the specific areas in which they were first developed.

Student Understanding of Sound Waves

The sound waves tutorial was developed after preliminary results showed that students' difficulties were not changing as a result of traditional lecture instruction. At the end of F95 (after all instruction) and the beginning of S96 (after all instruction), students answered identical questions. They were asked to describe the motion of a dust particle after a loudspeaker has been turned on. Student difficulties with this topic have been discussed in detail in chapter 4.[8] Table 6-5 shows student performance in these two semesters. The data are not matched, since different student populations were involved in the testing. We see that lecture instruction makes no sizable difference in student performance. The comparison between student responses from F95 and F97 is illustrative of the effect of the research-based tutorial instruction.

Table 6-5

Time during semester: MM used:	Before all instruction S96 (%)	Post lecture F95 (%)
CM (longitudinal oscillation)	14	24
Other oscillation	17	22
PM (pushed away linearly or sinusoidally)	45	40
Other and blank	24	14

Comparison of student responses describing the motion of a dust particle due to a loudspeaker. Data are from F95 post-instruction and S96 pre-instruction and are not matched (S96, N = 104; F95, N = 96)

In the beginning of F97, students answered a question (shown in Figure 3-3a) in which they had to describe the motion of a dust particle due to a sound wave. This same question was asked in a pretest in which a question about the motion of a candle flame due to a sound wave was also presented. (Note that Table 6-4 compares student performance on the dust particle question and the candle flame question which is similar to the actual content of the tutorial.) Finally, the dust particle question was asked against the end of the semester. The data comparing student responses to the dust particle question at these three times are shown in Table 6-6.

We see that students show little difference in their performance before and after traditional instruction on sound. They have profound difficulties connecting the physics that is taught in the classroom to any simple physical situations that might help them imagine and understand the situation in detail. After the tutorial, a much larger number of them are able to describe the correct motion of the dust particle. The large increase in the CM response and the large decrease in the PM response indicate that the tutorial is having a strong effect on student understanding.

Though the improvement in student performance is encouraging, we still see lingering difficulties. The total number of students giving CM responses is still less than 2/3 of the class. Also, a large number are still unsure of longitudinal or transverse oscillation of the dust particle, showing that the mathematical and graphical representations we use in class may adversely affect student reasoning about the physics.

Table 6-6

Time during semester: MM used:	Before all instruction (%)	Post lecture (%)	Post lecture, post tutorial (%)
CM (longitudinal oscillation)	9	26	45
Other oscillation	23	22	18
PM (pushed away linearly or sinusoidally)	50	39	11
Other	7	12	6
Blank	11	2	21

Student performance on sound wave questions before, after traditional lecture, and after additional modified tutorial instruction. Data are matched (N=137 students). The large number of blank responses in the post-all instruction category is due to the number of students who did not complete the pretest on which the question was asked.

Conclusion

Tutorials have been designed to replace the smallest possible amount of the common lecture format by replacing the one hour, traditional, TA-led recitation with a set of group activities that provide students with the opportunity to develop their own understanding of the physics while interacting closely with their peers and facilitators. In this chapter, I have shown that research into student difficulties can lead to more effective instruction.

The tutorials described in this chapter serve as an example of the curriculum development that can grow out of research into student difficulties. By knowing student difficulties with wave propagation, we were able to design video-based activities that helped students visualize the manner in which waves propagate. We were also able to provide students with a set of videos that allowed them to see the process of superposition. The relationship between the mathematics and physics of propagation (i.e. the relationship between functions of two variables and the physical situation) was investigated through simple activities that helped students develop the idea of a coordinate transformation without explicitly stating that this is what they were doing. In the sound wave tutorial, the ideas of the previous two tutorials were used to help students move from a description of a piece of the medium to the description of the sound wave making the medium oscillate. Each of these activities is related to an area in which we have found that students have difficulty.

The tutorials have met with measurable success. In some cases, such as sound waves and superposition, students show great improvement in their ability to describe the correct physics. In other cases, such as propagation, we find that the room for improvement is not as large, since many students enter our classes already aware of the correct answer. But after tutorial instruction, a larger fraction of students give only the correct answer when answering FR and MCMR questions, showing that the strength of their understanding has changed. Finally, in the case of wave-math, we have not been able to carry out sufficient investigations to show whether or not the tutorial shows great improvement over traditional instruction. Preliminary results are encouraging, but more work needs to be done.

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[1] For a discussion of different research-based curricula and their effectiveness at the introductory level, see chapter 1, reference 1 (the UMd dissertation in physics by Jeff Saul).

[2] A discussion of the tutorials and the role of research in their development can be found in McDermott, L. C., "Bridging the gap between teaching and learning: The role of research," AIP Conf. Proc. **399**, 139-165 (1997). In addition, a sample class on the tutorials was presented at this conference. See McDermott, L.C., Vokos, S., and Shaffer, P. S., "Sample Class on Tutorials in Introductory Physics," in the same Proceedings. For other examples of tutorials and of the research that underlies their development, see the discussion in chapter 2.

[3] For a description of the development and investigation into the effectiveness of tutorials at UMd, see Redish E. F., J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys.

65 45-54 (1997) and Steinberg, R. N., M. C. Wittmann, and E. F. Redish, "Mathematical Tutorials in Introductory Physics," AIP Conf. Proc. **399** 1075 - 1092 (American Institute of Physics, Woodbury, NY 1997).

[4] QuickTime is a cross-platform multi-media software package and is a registered trademark of Apple Computer (www.apple.com). More information can be found at URL www.apple.com/quicktime.

[5] The VideoPoint™ CD-ROM is a video analysis program developed at Dickinson College by P. Laws and Mark Luetzelschwab. It is commercially available from Lenox Softworks, Lenox MA.

[6] McDermott, L. C., P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice Hall, New York NY, 1998).

[7] We have found through informal observations and one interview that many students interpret the variable x in this equation similarly to how they interpret it in the exponential equation discussed in chapter 4. These students fail to include a time variable in the equation and interpret x to mean the location of the peak of the pulse.

[8] Because the diagram included with the question indicated walls which created a tube around the dust particle, we saw a variety of additional answers which went beyond those discussed in chapter 4. In many, students seemed to use the walls to guide their reasoning; the existence of the wall seemed to trigger responses dealing with harmonics in closed tubes. A non-trivial set of responses involved the sketching of standing wave patterns in the tube. In later semesters, we removed the walls from the question to provide more clear insight into student difficulties with the fundamentals of the physics of sound.

Chapter 7: Investigating the Dynamics of Student Reasoning

Introduction

A goal of our research has been to go beyond simply categorizing student difficulties. In addition, we would like to investigate the dynamics of student responses during instruction. In chapter 6, data were presented to show that individual topics of student understanding were affected by a modified curriculum, but the overall picture of student understanding of wave physics was not discussed. The data were discussed in terms of two reasoning methods that students use when answering a specific set of questions. We can describe students as using either:

- the correct model of waves (Community Consensus Model, or CM), elements of which students learn during the semester, or
- the more problematic pattern of associations to Newtonian (mechanical) particle physics (loosely referred to as the Particle Model, or PM, of waves), elements of which students bring to the classroom.

To investigate the dynamics of student reasoning in more detail, we have developed a diagnostic test that can be administered before and after student instruction. Though the developed diagnostic test included many questions that did not specifically address the distinction between student use of the CM and PM, the discussion in this chapter focuses on questions that elicited student difficulties related to their use of the two models. The questions not discussed in this chapter will be part of future work in investigating student understanding of wave physics.

In this chapter, I discuss the development of the diagnostic test and its usefulness in coming to an understanding of how individual students and an entire class develop an understanding of wave physics. The distinction between the CM and PM gives one example of many different aspects of reasoning that students use when thinking about waves. The questions chosen in the diagnostic represent questions that possibly elicit both CM and PM responses (and possibly both). The preliminary and pre- and post-instruction final diagnostic tests are given Appendix D of the dissertation.

The analysis of the data is similar to the analysis described in chapters 5 and 6. Student responses are categorized as representative of either the CM, the PM, both, or neither. We find that students begin the semester using primarily the PM but move to a mixed state where both the CM and the PM play a role in their thinking. This mixture of reasoning patterns is not due to understanding one topic of wave physics with the CM and another with the PM but seems to exist within a single wave physics topic.

Preliminary Diagnostic Test

A preliminary version of a wave diagnostic test was designed for use in interviews in the S97 semester. The interview setting was chosen to give the opportunity to probe student responses in more detail. By following up on student responses, we were able to compare the reasoning in their responses to the reasoning we believed was occurring. Our beginning assumptions were based on previous research into student difficulties. The diagnostic test was developed using questions we had found were effective in uncovering student use of the CM and PM in reasoning about waves.

The diagnostic was prepared with multiple-choice versions of questions previously used in free response formats in interviews or written tests. The multiple-choice questions were designed with research-based distractors so that students would have the opportunity to make common errors. The distractors were based on responses that students had given during our previous research projects. An example of a question with research-based distractors has been discussed extensively in this dissertation in the context of the free response (FR) and multiple-choice multiple-response (MCMR) question on wave propagation speed (see chapter 3, 5, and 6).

Many of the multiple-choice questions were in an MCMR format. In many of these questions, students were offered a series of questions and a series of possible responses for that set of questions (for example, questions 1 to 4 might have possible responses *a* to *f*). Students were told that they could use one response more than once to answer more than one question.

During the implementation of the interview diagnostic test (see below for more details), two free response versions of MCMR questions were added. This gave us the opportunity to ask nearly identical questions using different formats, which we had found effective in previous research settings.[1] The FR questions were asked at the beginning of the diagnostic, so that students would not use offered responses from the MCMR questions in their FR responses.

The preliminary diagnostic test is presented in Appendix D-1. The entire diagnostic test is presented, but only those parts which directly deal with student use of the CM and PM are discussed in this chapter (as discussed above). The wave physics issues included in the diagnostic are:

- wave propagation (both mechanical waves and sound waves),
- superposition of mechanical waves,
- the mathematics used to describe waves (both mechanical and sound),
- the motion of elements of the system through which the wave propagates,
- reflection of mechanical waves from a boundary.

Most of the questions used in the diagnostic had been used in previous research and common student responses to these questions were well known. For most of these questions, we did both interviews and written tests. In many cases, we explicitly used interviews to check to see if students answered the written questions consistently with the way that they actually thought about the situation. We had found that students answered the questions in the preliminary diagnostic test consistently. (These questions are said to be validated in such a situation.)

As stated above, the design of the preliminary diagnostic test changed during the course of its implementation. Some questions were rephrased due to student comments, some questions were added, and others were dropped. The questions on the last version of the preliminary diagnostic test that were most effective in uncovering student use of the PM are shown in Table 7-1. For each question from the diagnostic test in Appendix D-1, the correct (CM) response is given along with the possible PM responses offered as distractors to the students.

Table 7-1

S97 Wave Diagnostic Question	Possible PM Response(s)	Possible CM Response(s)
Wave Propagation		
FR1 and 5	a, b, c, d, i, j	e, f, g, h
Wave-Math		
FR2a) and 9	a, f	g
FR2b) and 10	a, b, c, d, e	f
Sound Waves		
1	b (if without c)	c
3 and 4	depends on response to question 1…	depends on response to question 1…
Superposition		
12	a or b (if a bouncing explanation given), g	c
17	f, g	e
Reflection		
20	h	e
22	a (bouncing), i (pulse absorbed into wall)	d
23	a, i (see above)	b

Table of S97 wave diagnostic questions that were used to determine if students were answering using the PM or CM. The diagnostic test can be found in Appendix D-1. Not all questions had clear PM and CM responses and are therefore not included in this analysis.

The interview diagnostic test was administered to 20 students. Five of these students had completed traditional instruction in which their recitation sections were led by a department professor. Fifteen students had completed tutorial instruction. Due to time limitations, not all students answered all questions. Also, for reasons stated above, the test itself was changed during the course of the interviews. Although all the interviewed students had completed instruction on waves, many still used problematic reasoning and showed difficulties with the material.

Each student's response was categorized according to the type of reasoning used. The criteria involved have been discussed in chapters 3, 5, and 6. Student responses were first categorized according to the difficulties that they had with the problem. These difficulties were tabulated using a spreadsheet program. Then, the various classes of difficulties were organized. Finally, each student and each question were analyzed according to:

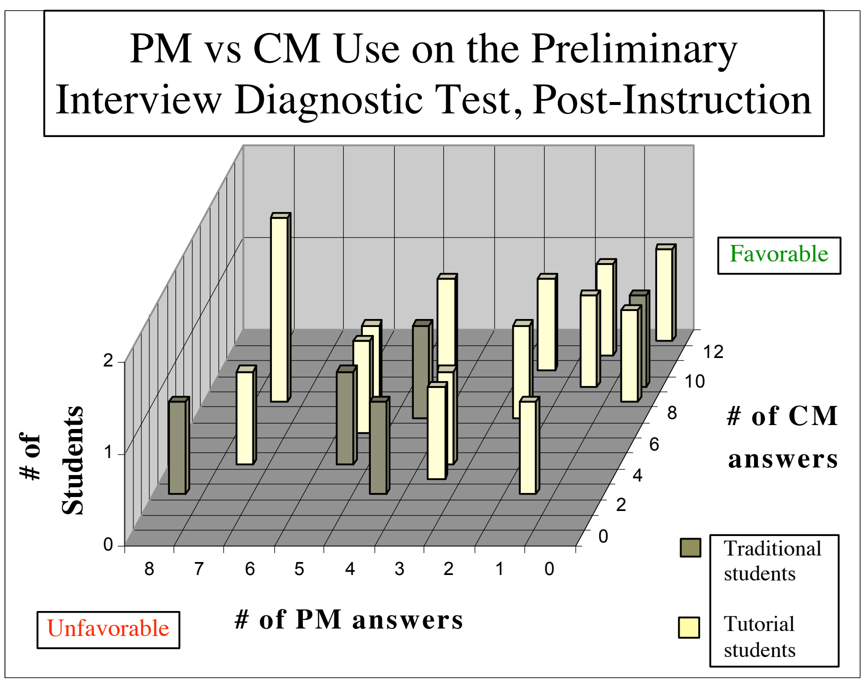
- the number of correct responses (those categorized best by the CM)
- the number of responses best categorized by the PM,
- the number of responses not categorized by either CM or PM, and
- the number of unanswered questions.

The organization of student responses into these 4 categories was used for all the wave diagnostic tests that will be discussed in this chapter.

The summary of how students responded to the 15 questions most likely to show evidence of the PM is shown in a two-dimensional histogram plot in Figure 7-1. The data in the table represent student performance according to how many questions they answered using a specific number of responses that are best classified as either PM or CM responses. For example, we classified the 13 responses of one tutorial student as indicative of the CM and none as indicative of the PM. We consider this a generally favorable result (i.e. we would like all our students to show such performance). We consider a student using primarily PM-like responses as showing unfavorable performance. Note that the sum of student PM and CM responses does not add to 15 in many instances, for reasons stated above and the additional reason that not all responses were classifiable with the CM or PM.

Consistent with our previous findings, we observe that many students consistently misapply otherwise reasonable primitives in their reasoning about wave physics. They are not consistent in their use of the PM, though. If we consider the responses given by Kyle (previously discussed in chapter 5), we see an example of this mixture in student reasoning. Kyle answered six of fifteen questions in a manner best classified by the PM and four in a way best described by the CM. In one question, he used reasoning that was indicative of both the CM and the PM. Thus, he answered a

Figure 7-1



Comparison of post-instruction PM and CM use by 15 tutorial and 5 traditional students on 15 questions from the preliminary wave diagnostic test. PM = Particle Pulses Pattern of Association. CM = Community Consensus (correct) Model. Mixed responses were counted as both PM and CM responses. “Favorable” describes a student who gives only answers best classified by the CM.

total of seven questions using the PM and five using the CM. In addition, four of his responses were not indicative of either the PM or CM. Thus, a student who clearly showed that inappropriately applied primitives guide part of his reasoning in wave physics also showed that he used multiple reasoning methods when thinking about the physics.

If we compare tutorial and traditional instruction students' performances, we note that most tutorial students performed better than the non-tutorial students on the material. This result is consistent with the results presented in chapter 6, which show that the tutorials effectively address student use of the PM. But, with the small number of students participating in the interviews, these data are merely suggestive and not conclusive.[2]

The preliminary diagnostic test was designed as a precursor to a written diagnostic test that would be applied in future semesters. To determine the effectiveness of the questions in uncovering student difficulties with the use of the PM in their reasoning, we counted how many students used the CM, PM, or other explanations when answering each question. The results are shown in Table 7-2. Note that the results show student performance after (both traditional and tutorial) instruction.

Students seemed to have the greatest difficulty with the wave propagation questions. The common "mixed" responses on the MCMR questions stand in contrast to the very polarized responses on the FR question, consistent with results from other investigations. Both wave propagation questions were effective in uncovering student

Table 7-2

# of responses Question	CM	Mixed CM/PM	PM	Other	Total
Propagation					
FR1	8	2	8	0	18
5	9	9	1	1	20
Sound Waves					
1	13	0	7	0	20
3	12	0	6	2	20
4	11	0	2	7	20
Wave-Math					
FR2 a	8	0	2	0	10
FR2 b	5	0	2	3	10
9	7	1	2	0	10
10	6	0	2	2	10
Superposition					
12	17	0	1	0	18
17	7	0	11	0	18
Reflection					
20	9	0	2	6	17
21	7	0	4	6	17
22	14	0	1	2	17
23	10	0	7	0	17

S97 wave diagnostic test responses split by topic. For each question, the number of CM, PM, mixed, and other responses is given, followed by the total number of students who answered that specific question.

use of the PM in their reasoning after instruction.

The sound wave question in which students describe the motion of a dust particle in front of a loudspeaker also elicited many PM responses. Of the five traditional instruction students, four answered this question using the PM (the fifth gave a CM response). Many students had difficulty with the question about the effects of a change in frequency on the dust particle, but did fewer used the PM when describing the effects of a change in volume.

The superposition questions were partially successful in uncovering student use of the PM. Consistent with previous results, very few students state after instruction that waves permanently cancel each other. Thus, they answered one question very well. But, many students showed a lack of functional understanding of the point-by-point superposition of displacement from equilibrium when the wavepulses coincided but their peaks did not overlap.

Finally, many students had difficulty with the wave reflection question in which they described the shape of the wavepulse reflected from a free end. Many stated that the wavepulse was absorbed into the. Few had specific problems with the other wave reflection questions, though many "other" responses were given. These questions were only partially successful in uncovering student use of the PM.

The difficulties students had with the questions on the preliminary diagnostic test were consistent with previous results. Therefore, the diagnostic test was not modified very much when it was next used.

Final Diagnostic Test

Based on the results from the S97 preliminary diagnostic test interviews, we developed a final version diagnostic test for F97. We asked a pre-instruction and post-instruction written diagnostic test of two Physics 262 classes at the University of Maryland. In the following sections, I discuss the design and implementation of the pre-instruction and post-instruction diagnostics. The design changes that led to the post-instruction test were partially based on a data analysis of the pre-instruction test, which will be described below. I end the chapter with a comparison of student performance on the two diagnostic tests and draw conclusions about student reasoning and classroom performance based on the data.

Pre-Instruction Diagnostic Test, Final Version

The pre-instruction written diagnostic test is presented in Appendix D-2. The pre-instruction diagnostic consisted of the 15 questions shown in the preliminary interview diagnostic to often elicit responses that could be classified as PM responses.

Two new questions were written for this test. The purpose of these questions was to raise issues from the dust particle sound wave question (discussed in chapters 3 and 6) in a different setting. One question asked students to compare the speed of sound of two people's voices, given that one person's voice was deeper and louder than the other's. A correct response would state that the speed of sound for the two is the same, regardless of volume or frequency of the sound. Students also answered variations on this question (e.g. how does your answer change if the volumes are equal). A second question asked students to consider the motion of a dust particle close to a wall when a sound wave reached the wall and reflected from it. A correct answer would state that the air near the wall is incapable of moving from its equilibrium position due to the wall, and therefore the dust particle will not move. This question also asked students the effect of a louder volume on the speed of sound. For both questions, we expected students to make similar errors to the ones they made when describing the motion of a dust particle in front of a loudspeaker. We expected students to show an incomplete understanding of the relationship between frequency, volume, and speed of sound. We also expected students to have difficulty describing the motion of the medium through which the sound traveled, especially in a setting involving reflection from what is effectively a "fixed end."

One important difference between the preliminary (interview) diagnostic test and the final (written) diagnostic test was that the final version consisted of primarily free response (FR) questions. The predominantly multiple-choice multiple-response (MCMR) format of the preliminary test had been feasible in an interview setting, but students had many difficulties with certain questions. As a result, it was decided to make the F97 test a free response test. One exception was the wave propagation question already discussed in chapters 3 and 6. This question was asked in both FR and MCMR formats

The pre-instruction diagnostic was administered during the first week of the semester during the tutorial period. Tutorials had not originally been scheduled for that week because the first day of the semester was a Tuesday. Those students who had tutorial period on a Monday were asked to come to another section during the course of the week. During the tutorial period, students were asked to answer all the FR questions, turn them in, and were then handed the wave propagation MCMR question separately. In this way, we were assured that they could not change their responses on the FR question due to the offered responses on the MCMR version.

Most of the analysis of the final diagnostic test involves a comparison of pre- and post-instruction data that will be discussed below. The analysis that led to the post-instruction diagnostic must be discussed before introducing the post-instruction diagnostic.

Many of the questions had similar content, though their surface features were different. For example, students answered questions about the speed of sound in the context of two people yelling (at different volumes and frequencies), sound waves created by a loudspeaker, and sound waves created by a clap. Students took much longer to answer the pre-instruction written diagnostic test than had been expected. As a result, we were aware that we had to shorten the test for its use after instruction. Thus, we planned to drop questions in which students gave consistent responses. For example, if students consistently used the same reasoning to describe the effect of a change in frequency on the speed of sound, then we would use only one question addressing that issue.

To see the correlation between student responses on the written diagnostic test, we first classified all student responses according to their use of the PM, CM or other explanation. We then compared how the students answered sets of questions. (In the discussion below, I will refer to the parts of questions as *a*, *b*, and *c*, even when that distinction did not exist in the actual numbering of the questions.) For the three parts of question 1 (on the speed of sound waves), we found that students used the same explanation to answer parts *b* and *c* as they used to answer part *a* (80% and 90%, respectively). Similarly, on question 3 (on the motion of the medium due to sound waves), we found that students answered parts *b* and *c* consistent with part *a* 80% and 90% of the time, respectively. But, they did not necessarily respond to parts *a* of questions 1 and 3 consistently (only 50% of the time). Thus, their responses showed that they were consistent when answering a single problem, but not when thinking about a single physics topic.

Students were also not consistent across physics topics. Students might answer use the PM consistently when describing how to change the speed of a sound wave but use the CM to describe how to change the speed of a mechanical wavepulse. The correlation in student responses between the part *a* of question 1 (on the speed of sound) and the part *a* of question 4 (on the speed of mechanical waves) was only 45%. This could be interpreted as saying that the questions are inconsistent and do not give us insight into student understanding of the material. Such an interpretation would assume that students use only one form of reasoning when thinking about wave physics. Results discussed in previous chapters indicate that students are inconsistent in their reasoning. Therefore, we believe that the questions are accurately uncovering areas in which students think inconsistently about the physics.

Post-Instruction Diagnostic, Final Version

The post-instruction diagnostic was shortened from the pre-instruction diagnostic due to time limitations. Based on the analysis of questions described above, we only used questions that gave unique information about student reasoning. For example, we asked students only part *a* of question 1 from the pre-instruction diagnostic. The wave reflection questions were completely dropped, though their inclusion would have been interesting because it would have given us insight into how student performance changed when there was no tutorial instruction on a given topic. The diagnostic is given in Appendix D-3.

The post-instruction wave diagnostic test was given in two parts. In the week before Thanksgiving (roughly 6 weeks after students had taken a mid-term examination on waves), students answered the FR wave propagation and dust particle (sound wave) questions as part of that week's tutorial pretest. (The material usually covered in that week's pretest was shortened and the extra space used for the wave diagnostic test.) During Thanksgiving week, students took the remainder of the wave diagnostic test during the commonly scheduled pretest time. There was no pretest because tutorials are not held during a holiday week. On this part of the wave diagnostic test, students answered the MCMR wave propagation question, the two "real world" sound wave questions, the superposition question with asymmetric waves, and an MCMR version of the dust particle question. This MCMR question had not been asked on the pre-instruction diagnostic.

Comparison of Student Pre- and Post-Instruction Performance

Two different types of analysis were done on the data. First, one specific topic of the wave diagnostic test was investigated according to the modes of reasoning (CM or PM) students used to answer four very similar questions. The evidence suggests that students use multiple reasoning methods within individual topics of wave physics rather than separate consistent reasoning methods for different topics. Our result is consistent with the analysis of FR and MCMR questions described in chapter 3 and it is more robust than the results from preliminary diagnostic test, described above. Second, a statistical analysis of the data was developed. Using a mathematical description allowed us to parameterize the results and compare parameter variables from before and after tutorial instruction.

Because a subset of pre-instruction questions was used on the post-instruction diagnostic, only those questions used in both pre- and post-instruction diagnostics are compared. In addition, only those students who answered a majority of these eight questions before and after instruction are compared.[3] This lets us restrict the discussion of the data to only those students who answered identical questions before and after instruction.

Inconsistent reasoning to describe a single wave physics topic

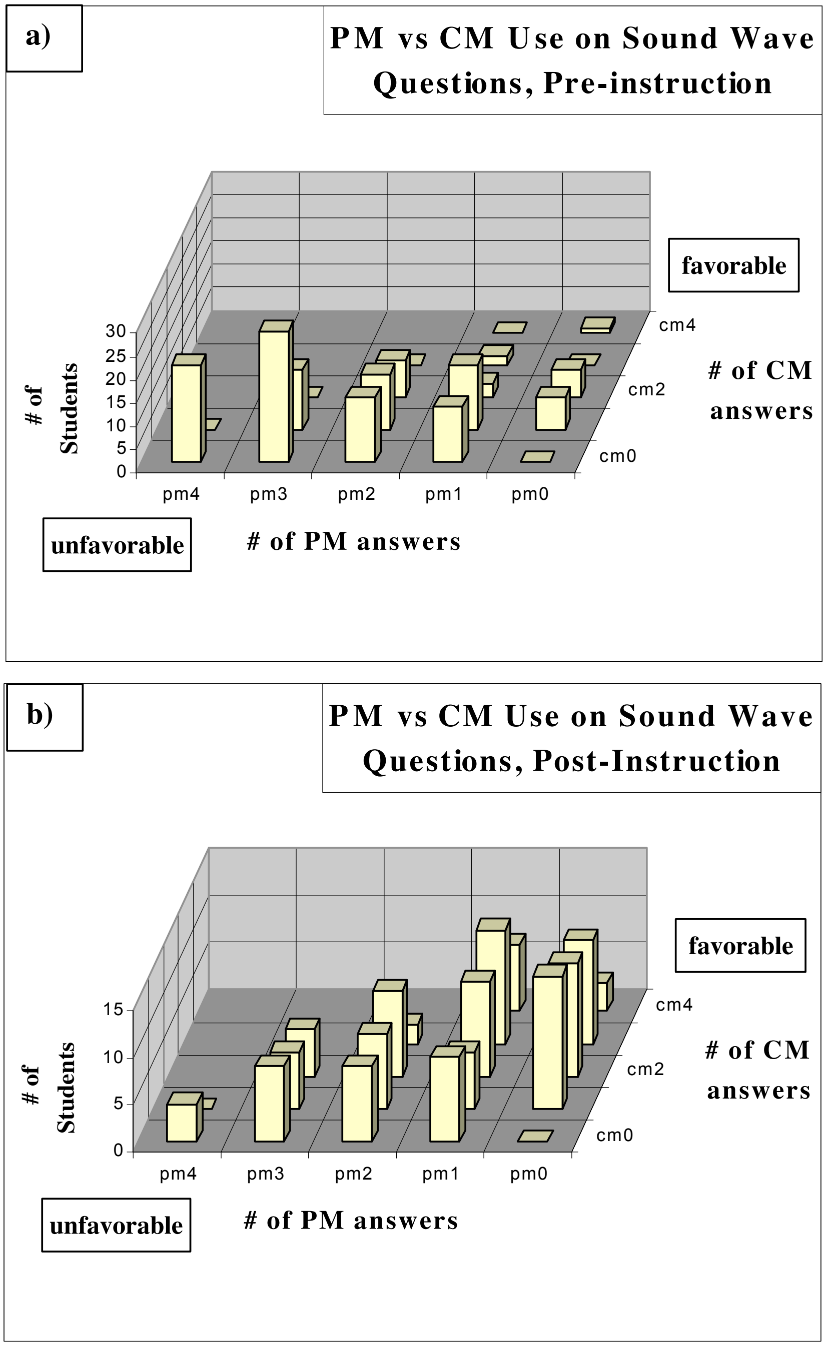
By focusing on student responses on a single topic (such as sound waves), we can see how students use multiple reasoning methods to describe a single physics topic. We find that most students begin the semester using the PM in their reasoning, but students at the end of the semester are more mixed in their responses.

On the pre-instruction diagnostic, questions 1a, 3a, and 6a have the same physics (how a change in the creation of a wave affects fast the wave propagates). Of the 182 students who answered each question, 99, 94, and 108 students (respectively) gave a PM response. Of the other students, 68, 16, and 14 (respectively) gave CM responses. The second most common category (after PM reasoning) for the last two questions was "other." This implies that students are reasoning in different ways about the same physics situation. Students seem to be reasoning about a single physical topic in many different ways (though most consistently use the PM).

Further evidence comes from looking at a plot which shows how many students gave a specific number of PM and CM responses for all four sound wave questions asked before and after instruction. We find that students are neither consistent nor coherent in their understanding of individual topics of wave physics. Figure 7-2 shows data from pre- and post-instruction wave diagnostic test questions that deal with sound waves. Only matched data are included (i.e. 136 students answered a majority of the questions both before and after instruction). The histogram is like the one discussed in relation to the preliminary diagnostic test. Each histogram bar shows how many students gave a specific number of PM and a specific number of CM responses.

Note that most students begin the semester answering predominately with PM reasoning, but many use mixed reasoning. After instruction, we find that students still answer the four sound wave questions using both reasoning methods. They have moved toward CM reasoning in their responses but have not stopped using PM reasoning.

Figure 7-2



Histograms of student PM and CM responses on sound wave questions in the final version wave diagnostic test. Figures show data from a) before instruction, b) after tutorial instruction. Data are matched, N=141 students. “Favorable” describes a student who gives only answers best classified by the CM.

Students use inconsistent reasoning when thinking about a single wave physics topic such as sound waves both before and after tutorial instruction. It seems that the effect of tutorial instruction was to move students to a hybrid form of reasoning that includes both the CM reasoning that we would like them to have and the PM reasoning with which many enter our courses.

Multiple reasoning methods to describe wave physics

Students also use inconsistent reasoning when describing the investigated wave physics topics both before and after tutorial instruction. Figure 7-3 shows separate histograms of student pre- and post-instruction responses. Again, each column represents the number of students who gave a certain number of CM and PM responses. For example, before instruction, two students answered the eight analyzed questions using one PM response and seven CM responses. We consider this favorable student performance.

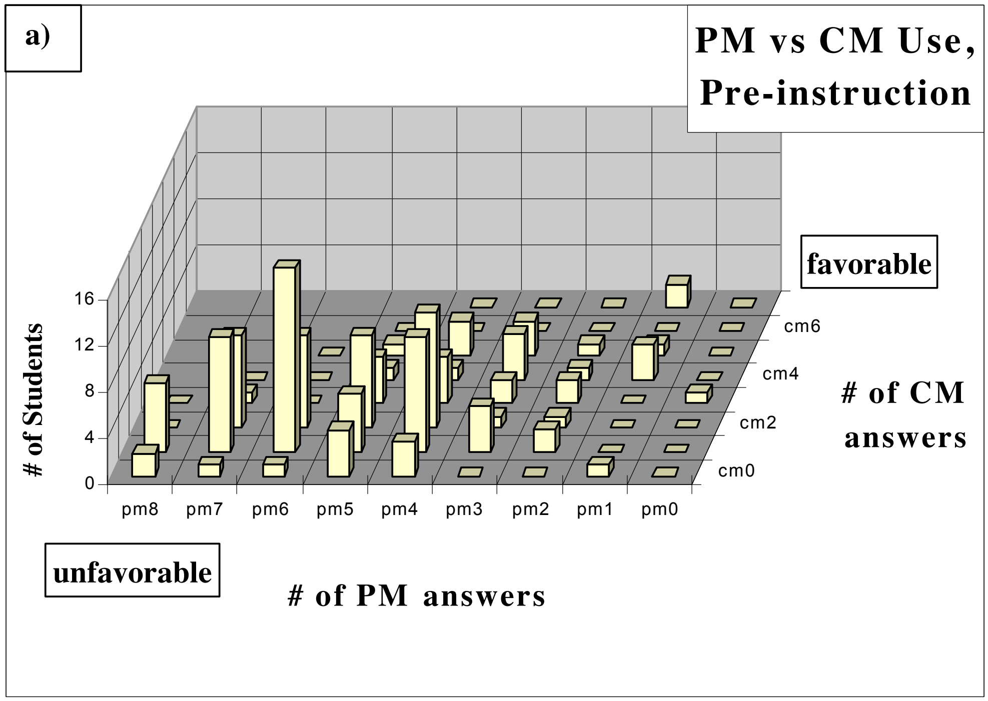
At the beginning of the semester, most students use primarily PM reasoning. They use CM reasoning for only one or two questions. Based on these results, we conclude that students are inexperienced with wave physics and are using the previously learned mechanics to help guide their reasoning for most topics. As stated in a previous chapter, student attempts to use their previous knowledge to guide their reasoning on unfamiliar topics is a quality that we would like them to develop in the classroom. The difficulty in this setting occurs from the incorrect application of otherwise useful primitives to waves.

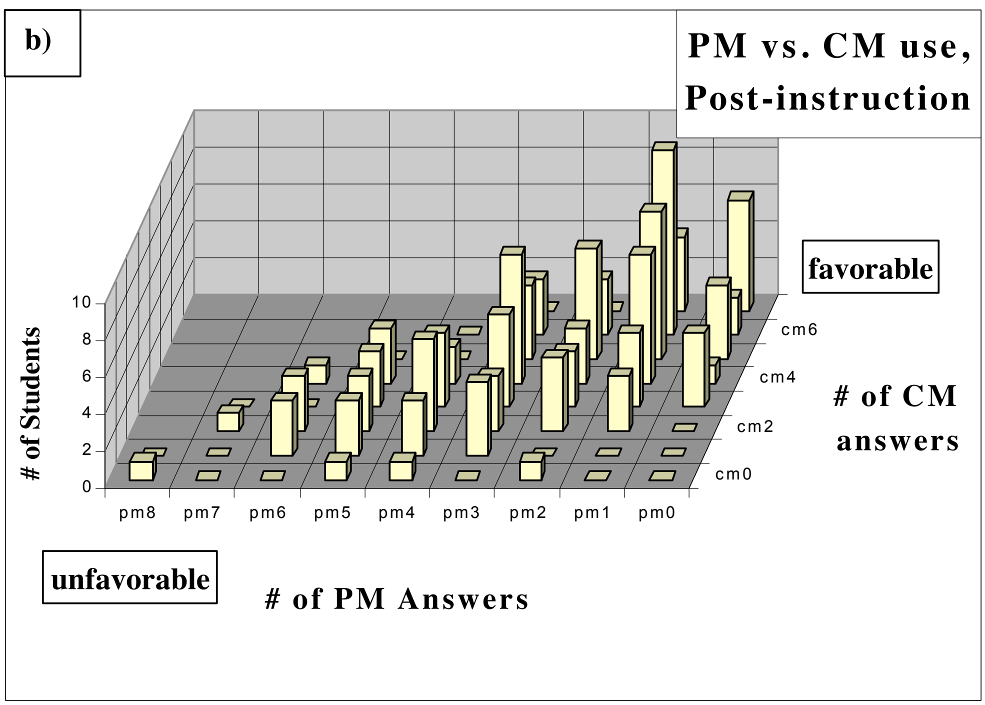
At the beginning of the semester, some students are located in the middle region of the graph, answering between 3 and 5 questions using both the PM and the CM. This indicates that the students are in a mixed state of knowledge about the physics when they enter our course. As was suggested in chapter 3 in the context of the FR and MCMR wave propagation questions, students have difficulty being consistent in their descriptions of physics topics. In the discussion of FR and MCMR responses, we found that students often recognized the correct responses but were unable to call them up on their own. It may be that pre-instruction student performance shows evidence that students are aware of a few correct ideas in wave physics, but predominantly use the PM to guide most of their thinking.

At the end of the semester, students have begun to use more CM reasoning, but still use PM reasoning heavily. Where they began the semester predominantly in the high-PM, low-CM region of the plots, they end the semester spread out in the middle- to high-CM region of the plots. The data as presented do not show that the number of responses categorized as "other" has stayed roughly the same as at the beginning of the semester. Most of the movement in student responses during the semester seems to occur between PM and CM reasoning.

Our results suggest that students have difficulties when learning to describe new phenomena in physics. Students bring to the discussion an ability to make analogies to the knowledge they do have. These analogies are guided by their experience (limited, in the case of waves, since most wave phenomena that we deal with on a daily basis are not visible), and often the analogies are incorrectly applied

Figure 7-3





Histograms of student PM and CM responses on the final version wave diagnostic test. Data are from a) before instruction, b) after tutorial instruction. Data are matched, N=136 students. “Favorable” describes predominantly CM responses.

without a functional understanding of the physics. As students go through our courses, they learn aspects of the correct model of physics, but do not let go of their previous knowledge in all cases.

Describing class use of different reasoning methods

The previous two analyses have focused on an overview of changes in student performance on the wave diagnostic test, but an analysis and description of an entire class's performance is also possible. We have carried out this analysis by considering the use of the PM and CM separately, rather than in a two-dimensional histogram. We can look at the average use of the PM or CM within a class and use these criteria to categorize classroom performance in more detail. By providing a statistical language, this method summarizes the data and allows a discussion of classroom use of multiple reasoning methods that goes beyond a description of student movement from favorable to unfavorable responses.

Each data set consists of a count of how many students answered a specific number of questions using a specific reasoning method (either CM or PM). This is essentially the sum of each row of data in Figure 7-3. These data were plotted on a graph where the number of student responses was compared to the number of responses using the given reasoning method. By fitting equations to the data sets, we are able to parameterize the results in a way that lets us quantify any changes in student reasoning that occur due to instruction. This analysis is data-driven, in the sense that the data fits are chosen based on reasonable descriptions of the population and of the situation. Figure 7-4 shows the data fits for the number of questions to which students responded with the PM before and after instruction. In Figure 7-5, the data for the number of CM responses on the wave diagnostic tests from both before and after instruction are presented. A variety of methods was used to determine the best fits of the data. For example, the pre-instruction PM data were plotted on a lin-log plot to help determine the best data fit. The plot is shown in Figure 7-6. The best fit for the data was parabolic with negative curvature, indicating that the best fit for the actual data would be a Gaussian or normal distribution. For the lin-log plot, a parabolic fit gives an equation with the form

$$\ln(y) = -N(x - x_0)^2 + y_0 \qquad (7\text{-}1)$$

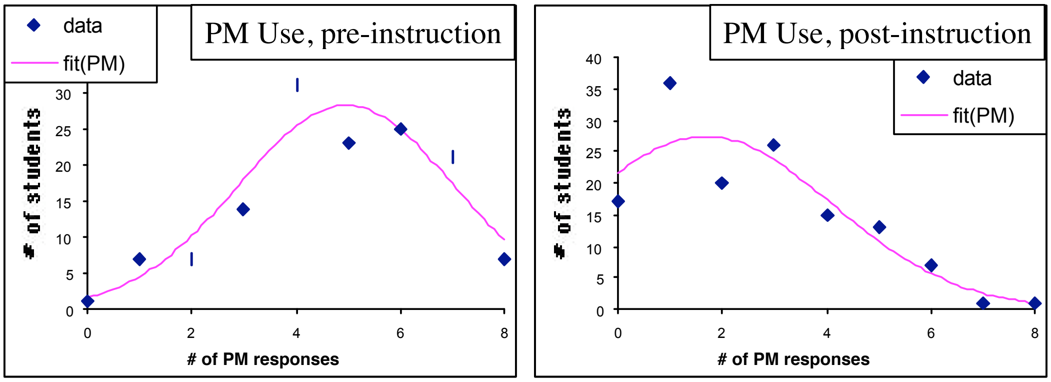
where the parabola has a width determined by N and has its maximum (or minimum) is located at x_0, y_0. The negative sign determines the downward shape of the parabola.

Solving for y gives the normal distribution,

$$y = Ae^{-\frac{1}{2}\left((x-x_0)/\sigma\right)^2} \qquad \mathbf{(7\text{-}}2)$$

where $A = e^{y_0}$ and $\sigma = (2N)^{-1/2}$. In this situation, σ^2 gives the standard deviation of the data around the mean, x_0. Three of the four data plots (pre-instruction and post-instruction PM use, post-instruction CM use) were best described with normal distributions, as determined by the method described above.

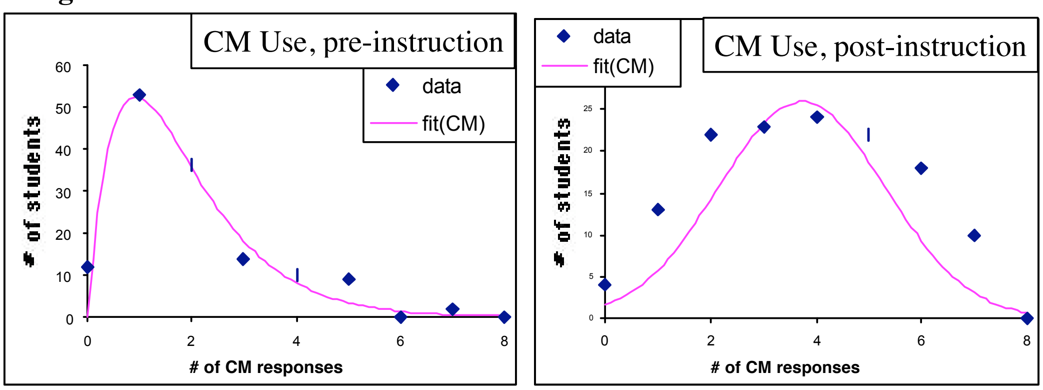
Figure 7-4



Pre- and post-instruction PM use on the wave diagnostic test by N=136 students, F97.

Fits for both sets of data are given by a Gaussian distribution, $y = Ae^{-\frac{1}{2}((x-x_0)/\sigma)^2}$. For the pre-instruction data, A = 28.0, x_0 = 5.03, σ = 2.02. For the post-instruction data, A = 27.4, x_0 = 1.68, σ = 2.43.

Figure 7-5



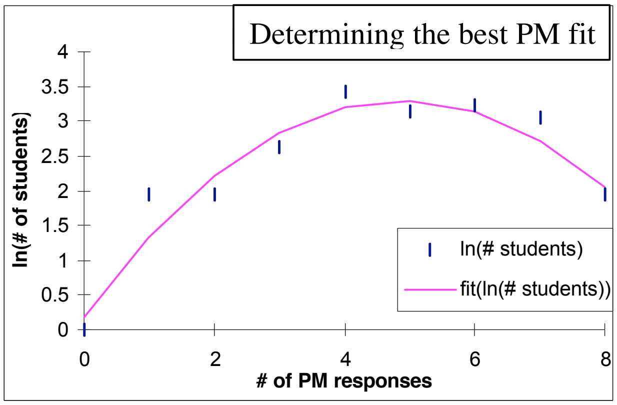
Pre- and post-instruction CM use on the wave diagnostic test by N=136 students, F97. The fit of the pre-instruction data is the integrand of a Gamma function distribution, $y=Ax^{b-1}e^{-x/c}$, where A = 154, b = 2, c = .926 (mean: bc = 1.84, variance: bc^2 = 1.71). The fit of the post-instruction data is given by a Gaussian distribution,

$y = Ae^{-\frac{1}{2}((x-x_0)/\sigma)^2}$, where A = 25.9, x_0 = 3.73, σ = 2.23.

For the fourth data plot, a different function had to be found. The pre-instruction data indicating CM use is heavily skewed to the left. Most of the students did not use any CM responses at the beginning of the semester (which is not necessarily surprising, since the investigation preceded any study of waves in the classroom). But, very few students used *no* CM responses (recall that on the MCMR wave propagation questions, 85% of the students entering the course included the CM in their responses). Thus, a function had to be found that went to zero at the origin but also decayed very quickly to zero as the number of questions answered using the CM increased. A function of the type

$$y = Axe^{-nx} \tag{7-3}$$

Figure 7-6



F97 pre-instruction PM data plotted on a lin-log plot. A parabolic fit was the best fit for the data, indicating that the appropriate fit for the lin-lin plot of pre-instruction PM use vs. # of PM responses would be Gaussian (normal distribution).

would provide this structure. Equation 7-3 is closely related to the integrand of the Gamma function distribution,

$$y = Ax^{b-1}e^{-x/c} \tag{7-4}$$

with values of $b = 2$ and $c = 1/n$. For such a function, the mean value is given by $x_0 = bc$ and the variance (the equivalent of σ^2 for a Gaussian distribution) by bc^2. As opposed to the normal distribution, the mean of the Gamma function distribution is not located at its maximum value (which is $x_{max} = bc-c$, as determined by setting the first derivative of equation 4 equal to zero).

Once the functions had been chosen for the fits, the various parameters had to be fit correctly. Using a spreadsheet program, the sum of the squares of the differences between the actual data value and the fitted value was computed. Using macros in the program, the lowest value of the sum of the squares was computed by varying the mean, standard deviation, and normalization values. Since the normalization values were primarily determined by the size of the class (N=136 students), the normalization is not a measure of student performance on the wave diagnostic tests while the mean and standard deviations are.

Table 7-3 shows a comparison of mean and standard deviation values for PM and CM use before and after instruction. These data describe the class performance rather than an overview of individual students' performances. Students start the semester using the PM to answer most of the questions (5.03 ± 2.02 questions), and only answer a few questions using the CM (1.84 ± 1.71 questions). The sum of PM

Table 7-3

Data fit Value:	Pre PM use	Post PM use	Pre CM use	Post CM use
Mean	5.03	1.68	1.84	3.73
Standard Deviation	2.02	2.43	1.71	2.23

Summary of data presented in Figure 7-4 and Figure 7-5. Class averages are from matched data of 136 students who answered identical wave diagnostic test questions before and after instruction in F97.

and CM means before instruction is nearly 7 out of 8 questions (though many students answered one with mixed MM use, which was then counted for both PM and CM use), showing that for the class average, most students are using these two models. Also, the standard deviations of the pre-instruction data overlap only slightly. This shows that the data are separated far enough to show that there is little overlap in student model use.

At the end of the semester, the mean value of PM use is nearly as low as the pre-instruction mean of CM use, but the standard deviation is larger (1.68 ± 2.43 questions). Thus, the spread in PM use for the class is larger, showing that students have moved away from their pre-instruction reasoning but not in a consistent fashion. The class is still using the PM more after instruction than they used the CM before instruction.

The post-instruction CM data has also not moved as far from the pre-instruction as we would like as instructors (3.73 ± 2.23 questions). The mean value of the data shows that the class as a whole uses the CM for slightly less than half of the questions. Again, the spread of the data is large, implying that students are spread out over a many different levels of CM use in their responses. Class use of the CM after instruction is less than class use of the PM before instruction, implying that the model of waves which they learn in our classes is used less than the inappropriately applied reasoning with which students enter our classes.

Finally, when looking at the sum of the means of post-instruction mental model use, we see that 5.4 questions are answered using one of the two mental models. Recall that on the post-instruction wave diagnostic test there were 9 questions and that mixed reasoning was common to the MCMR wave propagation question. Thus, many students are using other explanations after instruction, and many are leaving questions blank. The latter occurred often on the FR sound wave question, for example, since it was at the end of a lengthy pretest and many students did not complete it. Still, 5.4 out of 8 questions is still very low. In general, we see that students have a strongly mixed understanding of wave physics, with the PM and CM being their predominant models but other explanations also playing a role. In F97, they did not leave our classes with a coherent understanding of wave physics.

Summary

In this chapter, I have described how the development of a diagnostic test to investigate the dynamics of student reasoning about wave physics. The diagnostic was developed to elicit the most common difficulties we saw students having in our previous investigations. These difficulties have been organized in terms of the Particle Pulses Pattern of Association (PM). We have also characterized the model we would like students to learn in our introductory physics classes as the Community Consensus Model (CM).

A preliminary version of a diagnostic test showed that students used both the PM and the CM to describe wave physics after instruction. Based on these results, we developed a written wave diagnostic test that could be administered both before and after instruction. This would let us evaluate the effectiveness of instruction with respect to the broad picture of student understanding of waves.

Results on the F97 wave diagnostic test were interpreted in terms of both student and class performance. Student performance was described by considering two-dimensional histograms which showed both PM and CM performance. The histograms indicated that most students were moving from predominantly PM to mixed PM and CM reasoning. Students seem to leave the classroom with a less coherent (though more correct) model of waves than the model with which they entered our course.

Class performance was described by analyzing use of a single reasoning method (PM or CM) and fitting functions to the data. Again, we saw that the class as a whole began the semester using predominantly the PM with only weak use of the CM. Out of eight questions, an average of 5.03 (±2.02) were answered using the PM and less than 1.68 (±2.43) were answered using the CM before instruction (numbers in parentheses are the standard deviations of the distribution functions used to fit the data). After instruction, the PM is still used to answer some questions (1.84 ± 1.71), and the CM is used more often (3.73 ± 2.23). Still, the CM is not used as often as one would hope, and class performance indicates that most students are finishing instruction on waves with a mixed understanding of wave physics.

Both analyses indicate that students go through a transition in their understanding of wave physics. Students bring previous understanding to the classroom. We can say that they apply their previous understanding to new settings in an attempt to make sense of the material. We have found that students enter our classes using mechanics-based reasoning inappropriately applied to wave physics. They leave our classes using the correct model of physics but still holding on to their original analogies and reasoning patterns.

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[1] See chapters 3 and 6 for a discussion of the use of FR and MCMR questions to gain insight into student understanding wave propagation.

[2] We have not yet had the opportunity to investigate the difference in student performance in the two instructional settings in more detail.

[3] Some students were not present during one of the weeks the post-instruction diagnostic was asked. Others did not complete large parts of the diagnostic.

Chapter 8: Summary

Introduction

Our investigations of student understanding of wave physics show that students do not bring so much a body of pre-existing knowledge but a way of applying their pre-existing knowledge to a new and unfamiliar situation. Much of their knowledge is appropriate in some settings but inappropriately applied in others. During our courses, they learn the correct material that we want them to learn, but they still hold on to their previous way of thinking about the physics. In this chapter, I briefly review the results discussed in previous chapters and summarize our findings.

As part of ongoing research by the Physics Education Research Group (PERG) at the University of Maryland (UMd), I have investigated student understanding of some of the introductory physics concepts of waves taught in the engineering physics sequence at UMd. Investigations used the common research methods of physics education research. In-depth understanding of student reasoning was gained through the use of individual demonstration interviews. In this setting, students are probed as thoroughly as possible about an individual topic because the researcher has the ability to follow up on student comments. Interviews form a sort of "state space" of possible responses that can be used to analyze other probes that gives less insight into student reasoning. For example, written tests allow for more students to answer a single question, but the researcher is usually unable to follow up on student responses. In both written tests and interviews, it is possible to ask questions in a variety of formats. In this dissertation, I describe free response and multiple-choice, multiple-response questions in some detail.

The model of waves which students learn in this course consists of small amplitude waves traveling through ideal media such that there was no loss and no dispersion in the system. This model was investigated in the context of mechanical waves (on a taut string or spring) and sound waves (in air). Topics include wave propagation, superposition, and reflection of mechanical waves (on a string) and the propagation of sound waves. For both mechanical and sound waves, I have also investigated the mathematical descriptions students use to describe the medium through which the waves travel. Students have fundamental difficulties with each of these topics, and their reasoning shows that they often are unable to apply fundamental ideas of physics appropriately.

In our physics classrooms, we expect our students to understand and apply well-defined, coherent models of physical systems. The results presented in this dissertation indicate that many students have a fragmented picture of physics. They seem to access their knowledge depending on criteria triggered by the question and situation at hand. Thus, they may simultaneously have both correct and incorrect ideas about specific physical situations. Both as instructors and as physics education researchers, we benefit from an understanding of the elements of students' reasoning and the criteria by which students organize their understanding.

Specific Examples of Student Reasoning About Waves

The brief examples given below of specific student difficulties with wave physics are described in more detail in chapter 3.

We asked students to describe how they could change the speed of a wavepulse created by a quick flick of a hand holding a long, taut string. Before any instruction, 13% of the students who answered a free response version of this question gave the correct answer that only changes to the medium (its mass density or tension) would have the desired effect. Of the other students, 77% stated that the demonstrator creating the wave would have to move her hand more quickly (or slowly) to create a faster (or slower) wave. Thus, students are unable to separate the propagation of a wave from the initial condition that describes its creation. Instead, students describe the motion of the wave as if it were directly influenced by the manner of the wave's creation.

Student description of the motion of a dust particle floating in air due to a sound wave propagating through the air showed that students are unable to separate the sound wave from the medium through which it travels. Both before and after traditional instruction, more than 40% of the students state that a dust particle in such a situation will be pushed away from the loudspeaker in the direction of wave propagation. (96 students answered the question before instruction and 104 answered after instruction. Data are not matched, in the sense that these are not the same students, but other research results are consistent with these data.) Student explanations indicate that they are thinking of sound as moving air exerting a force (in only the direction of propagation) on the medium through which it travels. After instruction, less than half the students (46% of 104 students) describe the dust particle oscillating due to the sound wave, and only 24% correctly indicate that the motion is longitudinal. Many students are unable to distinguish between a propagating disturbance to a medium and the medium itself. Instead, many students describe the wave as the motion of the medium itself.

When discussing superposition, many students do not always think of a mechanical wave as an extended region that is displaced, but instead describe the wave by a few specific and significant points. For example, when two wavepulses (finite length waves, as opposed to infinitely long, e.g. sinusoidal, wavetrains) coincide but their peaks do not overlap, many students do not show superposition in the appropriate regions. Instead, they state that the wave only superposes when the amplitudes overlap. By "the amplitude," these students mean only the peak amplitude. Before instruction, 65% of 131 students give answers similar to this one, while only 27% show point-by-point addition of displacement at all appropriate locations. Even after traditional instruction, 53% of the students describe superposition in terms of only the amplitude point, and 26% give the correct response. Students giving the amplitude response are not recognizing a wave as a disturbance to the system that covers an extended region. Instead, they use a single point to describe the entire wave and neglect all the other displaced points in their descriptions.

In a fourth area of wave physics, we have investigated student interpretations of the mathematics used to describe waves. Students were given the shape of a Gaussian wavepulse propagating along an ideal, taut string and the equation to describe the shape of the string at time t = 0 s, $y(x) = Ae^{-\left(x/b\right)^2}$. They were then asked to sketch the shape of the spring and write an equation to describe the shape they had sketched after the peak of the wavepulse had moved a distance x_0 from the origin. Of the 57 students, 35% sketched a shape with a smaller amplitude. Though a physically appropriate description (if students were taking into account the loss in the system, which they were told was ideal), the explanations students give indicate that they are instead being guided in their reasoning through a misinterpretation of the mathematics. Many students interpret the variable x to mean the position of the peak of the pulse. The variable y describes the amplitude of the pulse for these students. Thus, when the x value increases (to x_0, for example), the amplitude of the wave decreases. This description is similar to the one given by many students describing superposition. Students do not use the mathematics to describe the entire string. Instead, they focus on the peak of the wave as the important point described by the mathematics.

Organizing Student Responses

The brief description below of how we organize student reasoning is described in more detail in chapter 4, and the interpretation of student results in terms of this approach is described in more detail in chapter 5.

To systematize student reasoning, we have described their reasoning in terms of primitives applied inappropriately to a given setting. A primitive describes a fundamental element of reasoning, in the sense that it is general to many different areas of experience. For example, to push a stationary box over a floor and to motivate an inherently lazy person both require an actuating agency. Or, when describing the motion of a box being pushed or the amount of work someone will do, more effort may be required to attain the same result, depending on the resistance to motion or work in the system. This primitive is referred to as the Ohm's primitive, based on Ohm's law, which describes the relationship between (output) current and (exerted) voltage, depending on the resistance of a circuit.

Many of the primitives that describe student reasoning come from investigations of student reasoning within Newtonian particle physics. These primitives include a set of primitives related to force and motion and a set related to collisions of objects. For example, students learning mechanics often use the actuating agency or Ohm's primitives to describe the effects of forces on the motion of an object. Though appropriate when describing phenomena in a world containing friction, the use of these primitives often indicates that students are not reasoning in terms of physical laws such as Newton's second law or are unable to interpret the many different elements of these laws in order to reach a complete and accurate description of the physics.

In addition to the primitives describing force and motion or collisions, students describing wave physics often make use of a previously undocumented primitive. I have documented student use of the *object as point* (or simply *point*) primitive in wave physics, but it is also commonly used in other areas. In the context of mechanics, it describes the useful manner in which objects are simplified to a single point when appropriate. For example, in free body diagrams or trajectory problems, an object is often described by a single point (the center of mass). Thus, the point primitive is useful and appropriate in many settings, but not necessarily in wave physics.

In the context of wave physics, students often use the point primitive inappropriately. In the context of superposition or the mathematical description of waves, many students seem to make use of it when they describe a wave by a single point, its peak amplitude. In the context of wave propagation, students might make use of it when describing how larger forces might lead to faster wave speeds. In this sense, the point primitive leads to the idea that a larger force can create a faster wave in the same fashion that a larger throwing force leads to a faster baseball.

Many students seem to inappropriately apply more than one primitive at the same time when describing wave physics. We can describe their reasoning in terms of a pattern of association, where these linked primitives seem connected in their reasoning. When asking students a series of wave physics questions on a specially designed diagnostic test, we see that they consistently make use of many of the primitives that are more appropriate in a mechanics than a wave physics setting. We describe student responses in terms of the *Particle Pulses Pattern of Association*, loosely referred to as the Particle Model, or PM. In contrast, we refer to the appropriate responses in a given situation as being indicative of the *Community Consensus Model* (or Correct Model, CM).

Curriculum Development to Develop Appropriate Student Reasoning

To help students move from a primarily PM based reasoning to a more appropriate CM based reasoning, we have developed a set of instructional materials called tutorials. The general design of tutorials as developed by the University of Washington, Seattle, is described in chapter 2. The tutorials designed at UMd as part of this dissertation and the description of their effectiveness in helping students develop more appropriate reasoning are given in chapter 6.

In tutorials, students work in groups of three or four on worksheets designed to change student reasoning about a specific topic. The three wave tutorials use the physics contexts of propagation and superposition, the mathematical description of waves, and sound waves to address many of the issues summarized above. In each tutorial, students view computerized videos of propagating waves to give the students the opportunity to see the otherwise very fast phenomena at a more interpretable speed. These videos were filmed by me and other PERG members and are commercially available as part of a video analysis software package, VideoPoint.

In the videos that students view while answering questions that deal directly with wave propagation issues, two wavepulses travel on two separate springs. Students must interpret the differences between the wave shapes and compare these differences to the possible differences in wave speed. When viewing the videos showing wave superposition, students are able to see that superposition occurs at all points in the medium where wavepulses coincide. They are then guided through activities that help them develop this idea more formally. In the wave-mathematics tutorial, students model the shape of a single wavepulse using both Lorentzian and Gaussian waveshapes. In the sound tutorial, students view a candle flame oscillating due to a sound wave. They graph the position of the candle as a function of time and use this information to develop ideas of period and frequency of sound. Further activities build on the video they have viewed and help students build an understanding of wavelength and the relationship between wavelength and frequency of sound.

To investigate the effectiveness of tutorials, we have compared student responses on a common set of questions before and after instruction. For each tutorial, we find that student performance improves more due to the tutorial than due to the traditional instruction that preceded it. For example, before instruction, only 9% of 137 students correctly state that a sound wave will make a dust particle oscillate longitudinally and another 23% state that the particle will oscillate but do not specify how. Some of the latter students describe transverse motion, possibly indicating that they are misrepresenting a displacement graph as a picture of the motion. The most common response is given by 50% of the students who state that the sound wave will push the dust particle away. Based on interviews we have done with students, we believe that this response is indicative of student use of the Particle Model, described above. These students seem to be applying inappropriate reasoning to their description of sound waves. After lecture instruction, 26% of these same students correctly describe the dust particle's motion (22% describe oscillation but not the direction), and 39% still describe the sound wave pushing the dust particle away. After tutorial instruction, 45% describe the motion correctly (18% more describe oscillatory motion without being clear about its direction), and only 11% describe the dust particle being pushed away by the sound wave.

Similar results are found in student responses toward wave propagation and superposition. Student performance both before and after lecture instruction indicate that many students use inappropriate reasoning when describing the physics of waves. The tutorials provide students with the opportunity to develop a more appropriate way of describing the physics, as can be seen from data indicating that far fewer students use the Particle Model after tutorial instruction than before. As a result, we believe that the tutorials are successful in helping students overcome the most common difficulties that they have with the material.

Investigating the Dynamics of Student Reasoning

As part of the investigation of the effectiveness of the tutorial materials, a diagnostic test was developed. This diagnostic test probed student understanding of wave physics in terms of student use of the PM and CM. In the final version of the diagnostic test, 137 students were asked eight identical questions dealing with propagation, superposition, and sound waves both before and after all instruction on waves. The diagnostic test contained both free response and multiple-choice, multiple-response questions. When a question was asked using both question formats, the free response question was asked first to prevent students from getting reasoning hints from the offered multiple-choice responses. Student responses were categorized according to whether or not their responses were indicative of either the PM or CM. Only students who answered a majority of the questions both before and after instruction were included in the analysis. Many students left some questions blank because they did not have time to complete either the pre- or post-instruction diagnostic, and many student responses were not classifiable as either PM or CM responses.

Before instruction, a majority of students use the PM in their reasoning. The average number of PM responses per student is 5.03 ± 2.02 (the standard deviation) while the average number of CM responses is 1.84 ± 1.71. Thus, we see that most students use the PM to guide their reasoning, and very few students use the CM consistently. After all instruction on waves (including tutorial instruction), students perform better. The average number of PM responses is now 1.68 ± 2.43, while the average number of CM responses is 3.73 ± 2.23. Students are using the PM much less often to guide their reasoning, but are also not using the CM as often as we would like.

In another analysis, we found that student use of multiple ways of describing the physics was not dependent on using appropriate reasoning in some topics of wave physics and inappropriate reasoning in others. Four of the questions on the diagnostic test addressed the physics of sound waves and the motion of the medium through which they travel. Student responses on these four questions before and after instruction are similar to their responses on the diagnostic test as a whole. Students begin the semester giving primarily PM descriptions of the physics, and end the semester using a hybrid of PM and CM reasoning to describe the physics. Thus, even in a specific area of wave physics, students give conflicting descriptions and show inconsistent reasoning.

Summary

In this dissertation, I have shown that it is possible to organize student reasoning in terms that give us deeper insight into their thinking about wave physics. I have defined the appropriate reasoning primitives, including a previously undocumented primitive called the *object as point* primitive. By organizing sets of commonly but inappropriately used primitives that students apply to the physics of mechanical and sound waves, we are able to discuss student difficulties with the material, the consistency of their reasoning, and how students develop their reasoning over time.

In much the same way that the use of certain primitives may be helpful in some settings but inappropriate in others, student use of the Particle Pulses pattern of association before students have received instruction on wave physics is understandable and not necessarily problematic. Students are applying the physics that they have previously learned and are trying to make sense of material with which they are usually not familiar. They are not always using correct physics in their reasoning, but we observe that students are trying to use their previous understanding to guide them in the new situation.

Student use of the inappropriate pattern of association after instruction is more problematic. Though we cannot compare tutorial students' performance on the diagnostic test to students who have not participated in tutorials, results from other investigations (such as student responses after lecture but before tutorial instruction) indicate that student are better able to reason effectively and accurately after they have participated in tutorial instruction. Further investigation would be required to determine what the differences are in student performance in a non-tutorial class, and to see what aspects of tutorial instruction are most effective in helping students develop more appropriate reasoning in our classes.

But even in a tutorial setting, students leave our classrooms using a mixture of appropriate, helpful ideas and inappropriate, problematic ideas. The research described in this dissertation shows that detailed descriptions of student difficulties with physics present a rich area of investigation relevant to both instructors and physics education researchers. For instructors, a more detailed understanding of possible student difficulties with the material can lead to more appropriate examinations and lecture materials that match more closely to students' actual needs. For researchers, the use of primitives, patterns of association, and mental models to describe student reasoning may provide a more appropriate language with which to describe the richness of student understanding of the physics.

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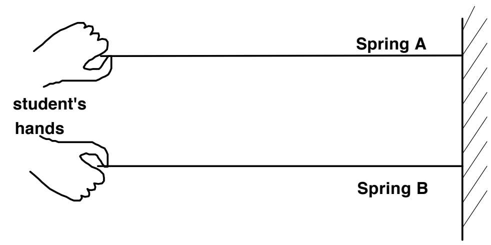
Appendix A: Propagation and Superposition of Wavepulses Tutorial

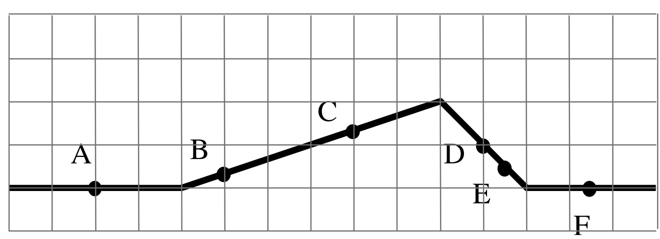
Name

Pretest

1. A demonstrator holds two long, taut springs attached to a distant wall (see figure). The demonstrator starts to move both hands at the same time and in the same direction. The wavepulses created by the demonstrator both move toward the wall, but one reaches the wall sooner than the other. How can you account for the difference in speed of the two pulses? List all possible ways.

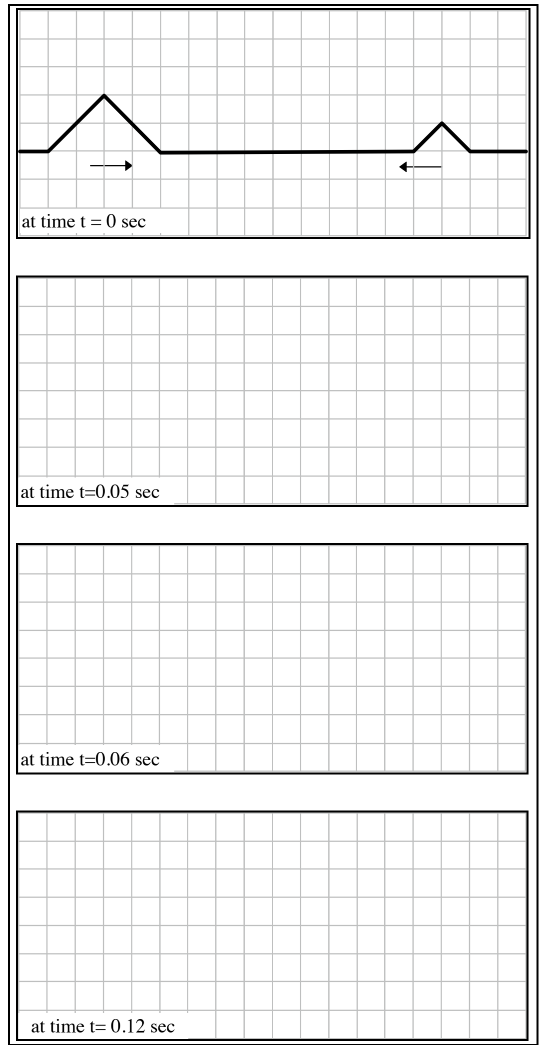


2. The solid line shown at right indicates the position of a wavepulse traveling to the right on a spring. Each dot on the line indicates the location of a piece of tape on the spring. Rank the speed of each piece of tape from highest to lowest. Explain how you arrived at your answer.



(over)

3. Consider the following situation. Two wavepulses are moving toward each other on a spring (see figure at right). The wavepulses are shown at time t = 0 sec. Each wavepulse moves at a speed of 1 m/sec (=100 cm/sec). Each block represents
1 cm (=0.01 m).



A. Sketch the shape of the string at time t = 0.05 sec in the space at the right. Explain how you arrived at your answer.

B. Sketch the shape of the string at time t = 0.06 sec in the space at the right. Explain how you arrived at your answer.

C. Sketch the shape of the string at time t = 0.12 sec in the space at the right. Explain how you arrived at your answer.

Propagation and Superposition of Wavepulses

I. Demonstrations and Videos of Wavepulses on a Spring

A. A facilitator will create wavepulses on a stretched spring by quickly moving his or her hand back and forth exactly once. The facilitator will use different hand motions to create wavepulses with different amplitudes and shapes. Observe the motion of the wavepulse and of the spring in each case. A piece of tape has been attached to the spring near its middle.

How did the motion of the tape compare to the motion of the wavepulse for each type of wavepulse that you observed?

Did the speed of the hand motion affect the speed of the wave? Explain.

Did the amplitude of the wave affect the speed of the wave? Explain.

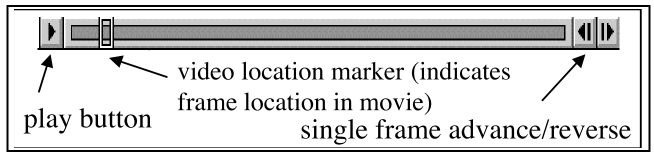
Did the tension in the spring affect the speed of the wave? Explain.

The demonstrator will create a pulse by pulling a piece of the stretched spring toward him or herself and releasing it. Compare the motion of the tape in this situation to the previous motion of the tape.

You have described the difference between transverse and longitudinal waves. In this tutorial, we focus only on transverse waves.

 These tutorials are developed for Activity Based Physics, NSF Grant DUE-9455561. Some of the materials were based on materials in *Tutorials in Introductory Physics*, L. C. McDermott, P. S. Shaffer, and the Physics Education Research Group at the University of Washington, Seattle (Prentice Hall, Upper Saddle River, NJ, 1998).

B. We have made videos of wavepulses traveling on identical springs. On your computer, open ***Shapes.mov***. Use the controls shown below to play the video on your computer. If the video plays too quickly, use the single advance buttons to go frame-by-frame. To return to the beginning of a movie, drag the "video location marker" to the left on its slider or click on the left edge of the play strip.



Imagine you are holding each of the springs in the video.

1. Describe what you see in the video. What hand motion could you use to create wavepulses having these shapes?

Open and play the movie ***Amplitud.mov***.

2. Describe what you see in the video. What hand motion could you use to create wavepulses having these shapes?

Consider that one of the springs used in questions 1 and 2 is stretched out to a greater length.

3. What physical properties of the spring have been changed by doing this? Explain.

Open and play the movie ***Stretched.mov***. In this video, one of the springs has been changed as described in question 3, the other is unchanged.

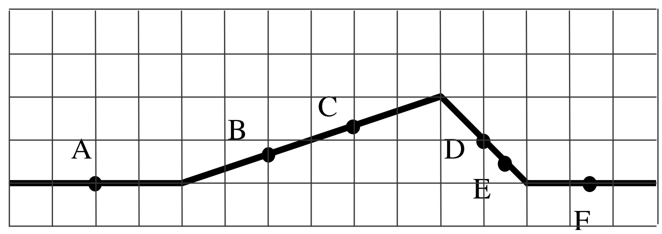
4. Describe what you see in the video.

5. Based on your observations, how can you affect the speed of a wave traveling along a spring? What changes can you make that *do not* affect the speed of the wave? Explain.

6. Are your observations consistent with the equation that describes the speed of a wave on a string, which you learned in class? Resolve any discrepancies.

II. Analysis of a single wavepulse

The solid line shown at right indicates the position of a wavepulse traveling along a spring at a time t_0. Each block represents 1 cm. The wavepulse is moving to the right with a speed of 100 cm/s.



1. In the graph located above, sketch the position of the wavepulse after 0.01 sec has elapsed.
2. How can you use your diagram to find the velocity of a piece of the spring at time t_0 (e.g., the part of the spring labeled D)?

   Determine the velocity of the piece of spring located at position D at time t_0. Explain.

   Determine the velocity of the piece of spring located at position C at time t_0. Explain.

   Draw vectors on the diagram to represent the instantaneous velocity of the pieces of spring labeled A – F at time t_0. Draw your vectors to scale.
3. Compare the direction of motion of the wavepulse and of the spring.

III. Superposition of Wavepulses:

A. Open the movie ***SameSide.mov*** on your computer. Play the video.

1. Describe what happens as the two wavepulses meet. Discuss each pulse and the spring.

Use the single advance buttons to find the frame showing the moment when the two wavepulses overlap as completely as possible.

2. How could you determine the maximum displacement of the spring at the instant of perfect overlap? Explain.

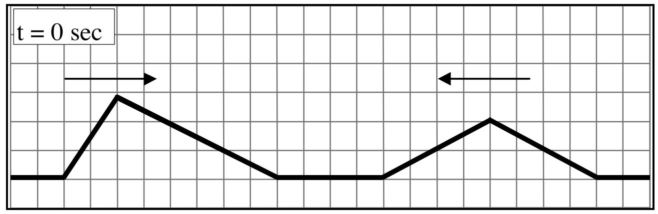
   Explain how you can determine the displacement of the spring at locations *other than* the point of maximum displacement at this instant.

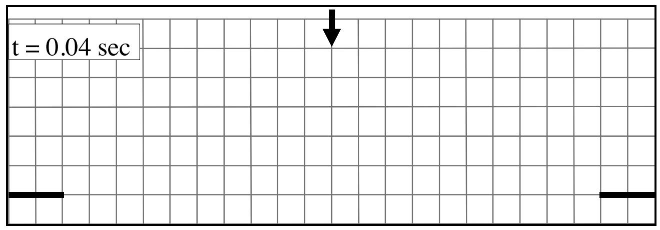
Find the frame *just before* the moment analyzed above.

3. Describe and sketch the shape of the spring.

   Account for the shape of the spring between the two peaks. Is your explanation consistent with the explanation you gave in question 2? Resolve any discrepancies.

B. Consider the following situation. Two wavepulses are moving toward each other on a spring (see upper figure at right). The wavepulses are shown at t = 0 sec. The left pulse moves at a speed of 1 m/sec (=100 cm/sec). Each block represents 1 cm (= 0.01 m).



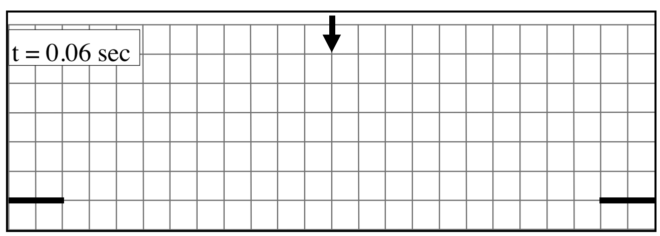


1. What is the speed of the *right* wavepulse? Explain.

2. Sketch the shape of the spring at time t = 0.04 sec in the figure above. Explain how you arrived at your answer.

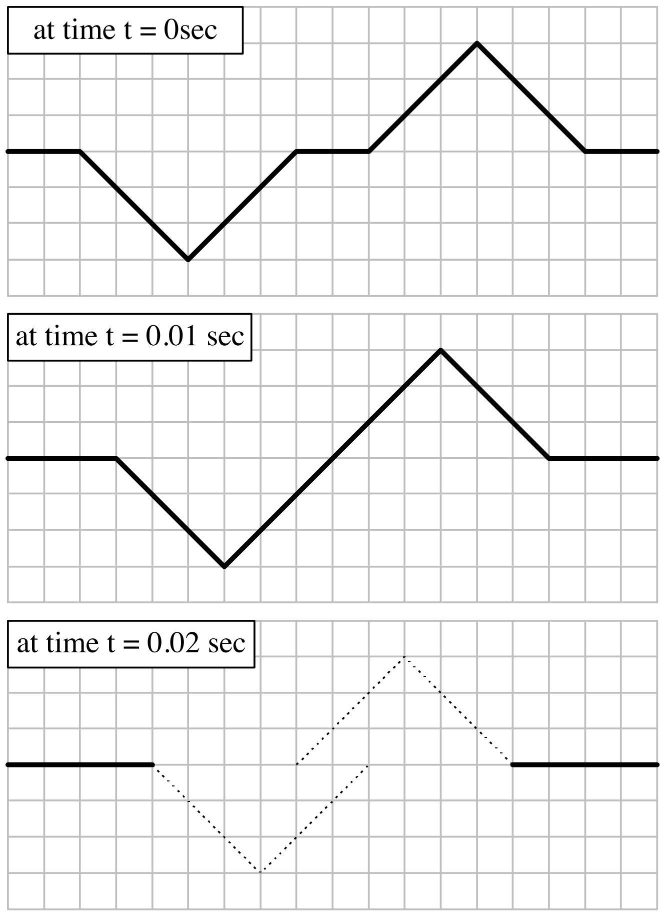
   How did you determine the displacement of the spring at the location of the gridline indicated by the arrow? Explain.

3. Sketch the shape of the spring at time t = 0.06 sec in the space at right. Draw your sketch to the same scale as above. Explain how you arrived at your answer.



   How can you determine the displacement of the spring at *any location* and *any time* when two wavepulses overlap. Explain you reasoning.

C. Two wavepulses on opposite sides of a spring move toward each other at 100 cm/s. The diagrams below show the wavepulse locations at three successive instants, 0.01 s apart. (In the last diagram, the individual wavepulses are shown dashed.) Each square represents 1 cm.



1. Use the principle of superposition to find the shape of the spring at t = 0.02 sec. Draw it in the graph above. Make sure all of your group agrees on how you arrived at your answer before continuing.

The diagrams above and blanks for three further time steps are reproduced on the last two pages of this tutorial. Each person in your group should draw what the spring will look like for ONE of the times shown from 0.03 sec to 0.05 sec. Draw the shape of the **spring** for each of the instants shown. After constructing your diagram, discuss your results with the rest of your group until you all agree what the spring should look like at each instant.

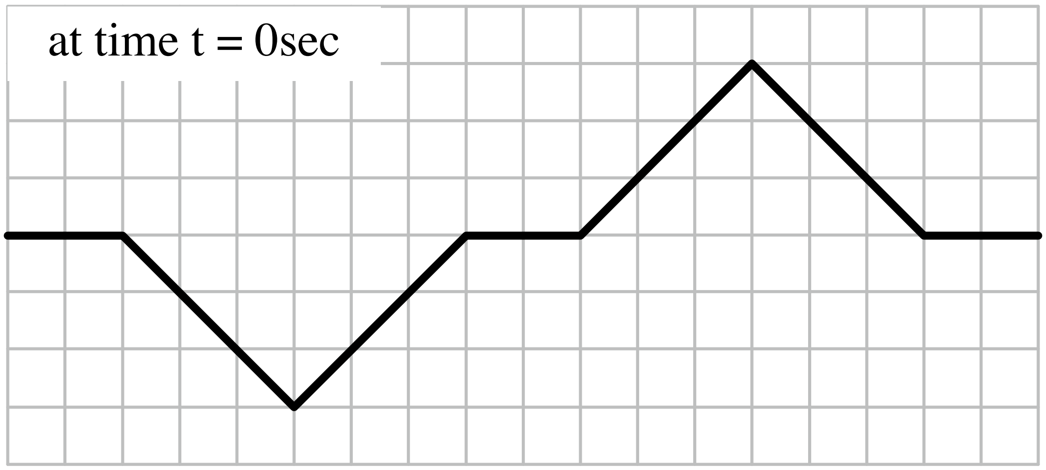
2. Are your graphs consistent with your explanation on the bottom of page 4? Resolve any discrepancies.

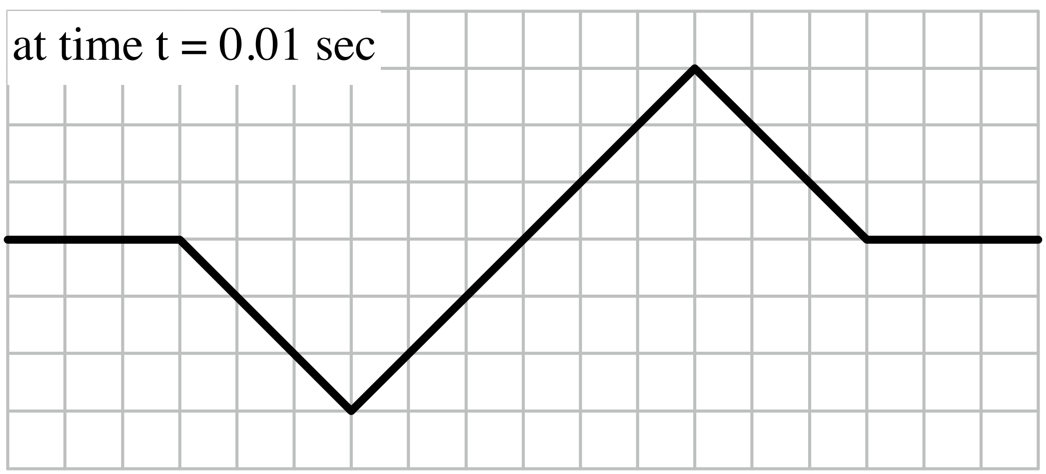
3. How can you account for the continued propagation of the wavepulses after the time
   $t = 0.4$ sec?

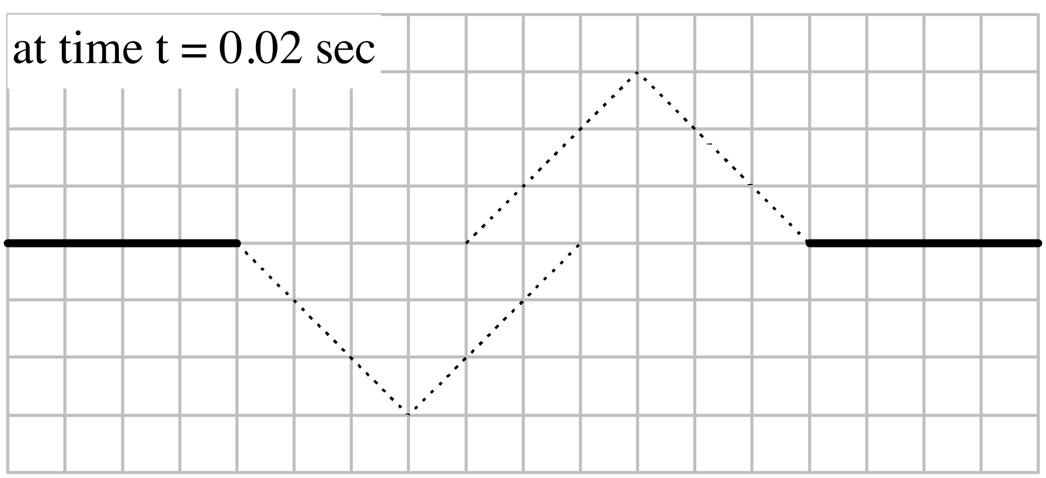
4. Sketch a graph of the velocity as a function of position for the spring at time $t = 0.4$ sec in the space below. Explain how you arrived at your answer.

On your computer, open and play the movie ***Opposite.mov***.

5. Is the movie consistent with the sketches you made on the large diagrams? Resolve any discrepancies.


at time t = 0sec


at time t = 0.01 sec


at time t = 0.02 sec

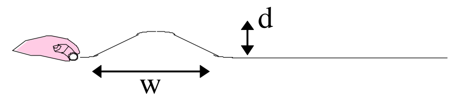
at time t = 0.03 sec

at time t = 0.04 sec

at time t = 0.05 sec

Name **Tutorial Homework**

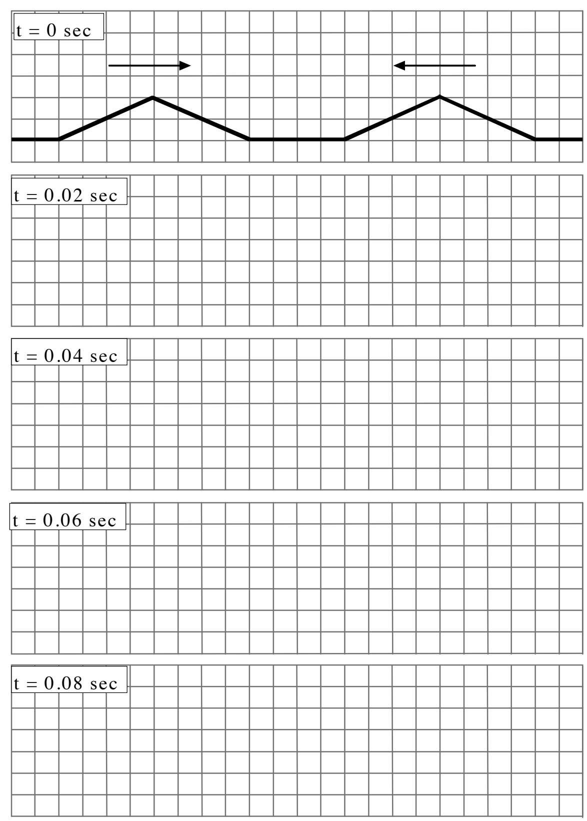
A. A method for generating a wavepulse is to move one end of a spring quickly up a distance *d* and then back down (see figure). The hand takes the same amount of time to move up as to move down. Consider a second wavepulse generated with the same amplitude, *d*, on a different spring (spring 2). It is observed that the wave speed on spring 2 is half that in the original spring (spring 1).



1. How can you account for the difference in speed of the wavepulse on the two springs? Explain.

2. What could you change about the creation of the second wavepulse or spring 2 so that the wavepulse on spring 2 traveled at the same speed as the wavepulse on spring 1? Explain.

B. The diagram at right shows two wavepulses at time t = 0 sec moving toward each other on the same side of a spring. Each pulse is moving at a speed 100 cm/sec. Each block represents 1 cm.



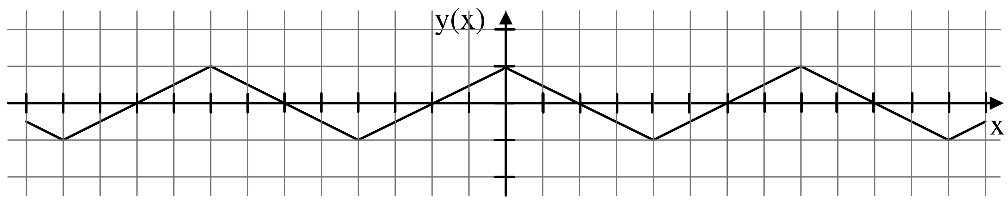
1. In the grids provided to the right, sketch a sequence of diagrams that show both the positions of the individual pulses (with dashed lines) and the shape of the spring (with a solid line) at 0.02 sec intervals.

2. Draw velocity vectors to indicate the *velocity* of the piece of spring located at the horizontal midpoint of each square at time t = 0.04 sec. Explain how you arrived at your answer.

Propagation and Superposition - Tutorial Homework

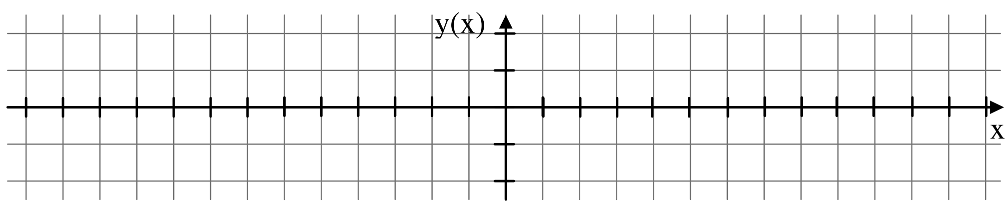
Bridging Problem

Two infinite (continuing in both directions) waves are traveling along a taut spring of uniform mass density. At time t = 0 seconds, the waves have the same shape and are in the same location. One is traveling to the *right*, the other is traveling to the *left*. *One* of the waves is shown in the space below. (At time t = 0 sec, the other wave's peaks perfectly overlap the first wave's peaks.) In the diagram, each block represents 10 cm. After t_0 seconds, the wave traveling to the *right* has traveled 20 cm.

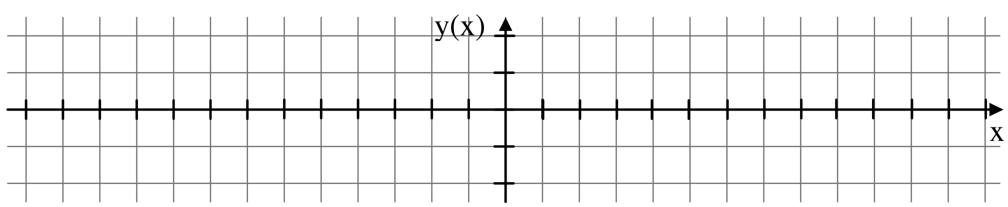


a) Compare the speed of the two waves. Explain how you arrived at your answer.

b) Using two different colors of pen or pencil, sketch each individual wave at time t_0 in the graph below. (Do not sketch the shape of the spring, just each wave.) Indicate the direction that each wave is traveling on your sketch.



c) In the graph below, sketch the shape of the spring (with both waves traveling on it) at time t_0. Explain how you arrived at your answer.



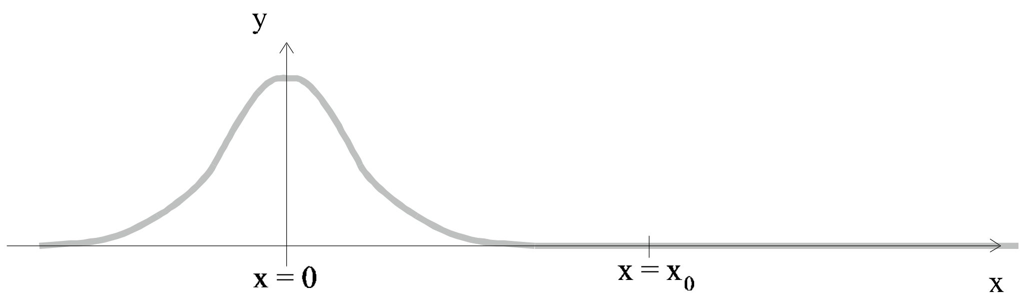
d) If t_0 = 0.2 sec, find the velocity of the piece of spring located at x = 0 at the instant you have drawn in part c. Explain how you arrived at your answer.

Appendix B: Mathematical Description of Wavepulses Tutorial

Name

Pretest

1. Consider a pulse propagating along a long, taut spring in the +x-direction. The diagram below shows the shape of the pulse at t = 0 sec. Suppose the displacement of the spring at this time at various values of x is given by $y(x) = Ae^{-\left(x/b\right)^2}$.



A. On the diagram above, sketch the shape of the spring after the pulse has traveled a distance x_0, where x_0 is shown in the figure. Explain why you sketched the shape as you did.

B. For the instant of time that you have sketched, find the displacement of the spring as a function of x. Explain how you determined your answer.

(over)

2. The experiment described in question 1 is repeated, except that at t = 0 sec, $y(x) = Ae^{-\left(x/c\right)^2}$, where c is twice as big as b (c = 2b).

A. Compare the shape of the *pulse* in this experiment to the shape of the *pulse* in the previous experiment. Explain.

B. Compare the motion of the *pulse* in this experiment with the motion of the *pulse* in question 1. Explain.

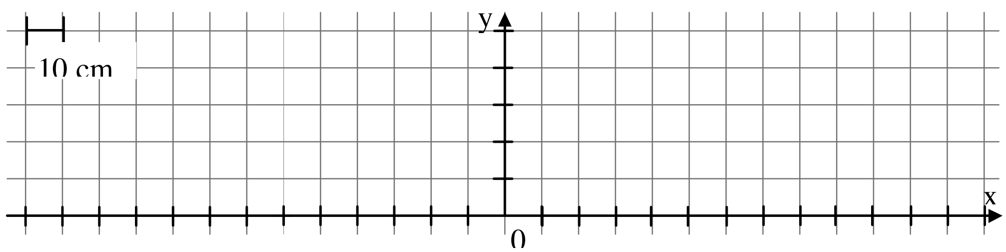
C. Consider a small piece of tape attached to the spring at x_0. Compare the motion of the *piece of tape* in this experiment with the motion of the *tape* in question 1. Explain your reasoning.

Mathematical Description of Wavepulses

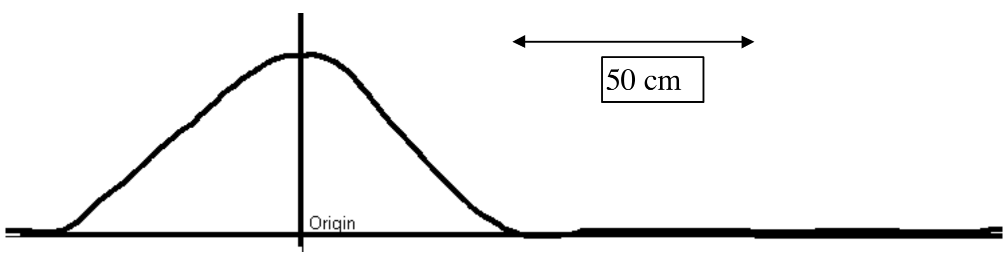
I. Describing the Movement of a Wavepulse

A. At t = 0 sec, the displacement of the spring from its equilibrium position can be written as a function of x, $y(x) = \dfrac{50cm}{\left(x/b\right)^2 + 1}$, where b = 20 cm.

1. Sketch the shape of the spring at t = 0 sec in the graph below. Use a scale where each block represents 10 cm on a side.



2. The digitized photograph below shows a wavepulse propagating to the right. The wavepulse can be roughly described by an equation of the form $y(x) = \dfrac{A}{\left(x/b\right)^2 + 1}$. Using the indicated scale, find approximate values for A and b. Explain how you arrived at your answer.



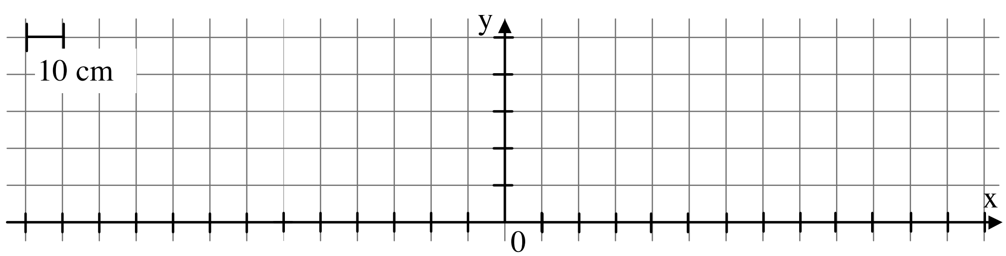
3. Predict the maximum amplitude of the wavepulse after its peak has moved a distance 3b to the right. Explain how you arrived at your answer.

4. On your computer, open the movie ***pulse.mov***. Play the video. Compare what you see in the video to your answer to question 3. Resolve any discrepancies.

5. Describe what the symbol "x" represents in this problem. Explain.

B. Consider a wavepulse propagating to the right along an ideal spring. The shape of the spring at time t = 0 seconds is given by $y(x) = \dfrac{50cm}{\left(x/b\right)^2 + 1}$, b = 20 cm.

1. In the space below, sketch the shape of the spring after the wavepulse has moved so that its peak is at x = 70 cm. Compare this graph to the graph you sketched in question 1 on the previous page.



2. On the graph above, draw a coordinate system with its vertical axis at 70 cm. Label its horizontal axis with the variable *s*.

3. Write a formula y(s) that describes the displacement of any piece of the spring (when the peak of the wavepulse is located at 70 cm) *as a function of s*. Explain.

4. Write an equation for *s* as a function of *x*.

5. Write a formula that describes the shape of the wavepulse at the time it is centered at 70 cm *as a function of x*. Explain.

6. Consider that the wavepulse had moved an arbitrary distance x_0. How would your formula change? Explain.

7. Suppose the wavepulse moved a distance x_0 at a speed v in a time t_0. Write an equation for x_0 in terms of v and t_0.

8. How could you use this information to find the displacement of any piece of the spring at time t_0? Write an equation that would let you do this. Explain.

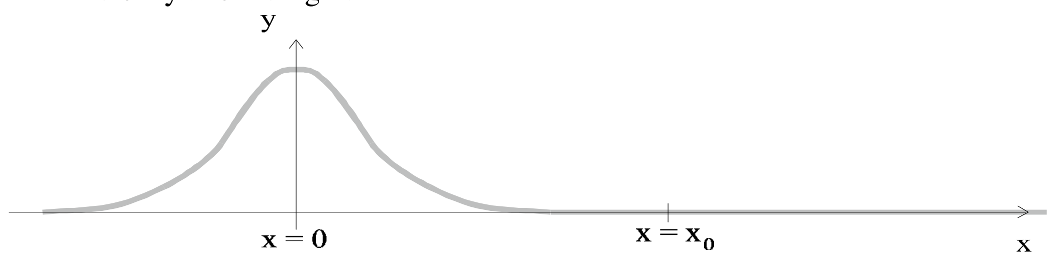
9. Write an equation that describes the displacement of any piece of the spring at *any* time. Explain.

10. Describe how you would find the displacement of any piece of the spring at any time.

11. Would your equation in question 8 be correct if the spring were *not* ideal, or if there were friction between the spring and the ground? Explain.

II. Measuring the Shape of the Wavepulse

A. Consider a different wavepulse propagating along a long, taut spring. The diagram below shows the shape of the wavepulse at time t = 0 sec. Suppose the displacement of the spring at various values of x is given by $y(x) = Ae^{-(x/b)^2}$. The wavepulse moves with a velocity *v* to the right.



1. On the diagram above, sketch the shape of the spring after it has traveled a distance x_0, where x_0 is shown in the figure. Explain why you sketched the shape you did.

2. Write an equation that describes the displacement of the spring as a function of x and t for the instant of time that you have sketched. Explain how you determined your answer.

A piece of tape is attached to the spring at position x = x_0.

3. In the space below, *qualitatively* sketch the velocity of the piece of tape as a function of time. Explain how you arrived at your answer.

4. In the space below, *qualitatively* sketch the acceleration of the piece of tape as a function of time. Explain how you arrived at your answer.

B. On your computer, play ***pulse.vpt*** to show a video of a single wavepulse traveling along a spring. Suppose the wavepulse in the video can be described by an equation like the one you wrote in question 2 on the previous page.

1. What effect would changing the parameters "A," "b," and "v" in the equation (from page 4) have on the wavepulse?

2. Find numerical values for each parameter in the equation. Show all work.

3. Use your equation to find the displacement of the spring at position x = 125 cm at time t = 0.1 sec. Show all work.

4. Advance the video 0.1 sec beyond the instant of time where its peak is located at the origin. Find the displacement of the spring at position x = 125 cm and time t = 0.1 sec.

5. Compare your answer to question 4 with your answer to question 3. Resolve any discrepancies.

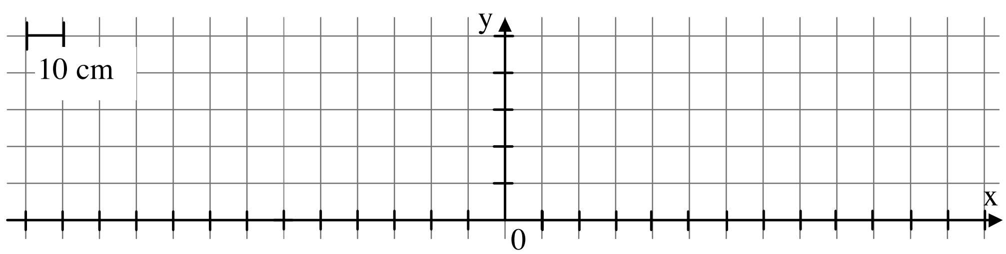
Name **Tutorial Homework**

Suppose a pulse is propagating along a spring with a velocity 600 cm/sec *to the left*. At time t = 0 sec, the displacement of the spring from its equilibrium position can be written as a function of x, $y(x) = \frac{50cm}{\left(x/b\right)^2 + 1}$, where b = 20 cm.

1. Write an equation that describes the displacement of the spring from its equilibrium position at any position, x, and at any time, t. Explain how you arrived at this answer.

2. Compare your equation to the equation which you derived in section I.B, question 9 of the tutorial. What has changed? Explain the effect of this change.

3. In the space below, graph the displacement of the spring after 0.1 seconds have passed.



4. Compare the shape of the graph you have sketched to the equation you wrote in question 1. Find the displacement of a point located 20 cm to the left of the peak of the pulse after 0.1 seconds have passed.

5. What point had this same displacement at time t = 0 seconds? Explain how you arrived at your answer.

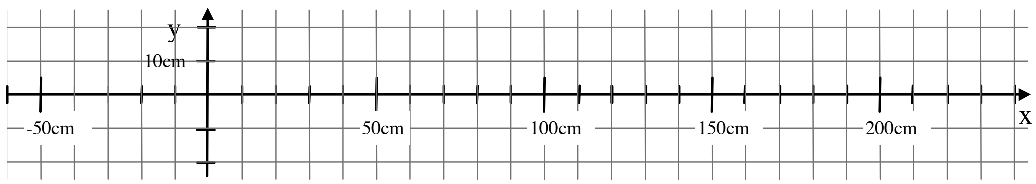
Mathematical Description of Waves - Tutorial Homework

Bridging Problem

Consider a pulse traveling *to the right* with a speed of 600 cm/sec. The equation describing the displacement of the spring from equilibrium at time t = 0 sec is

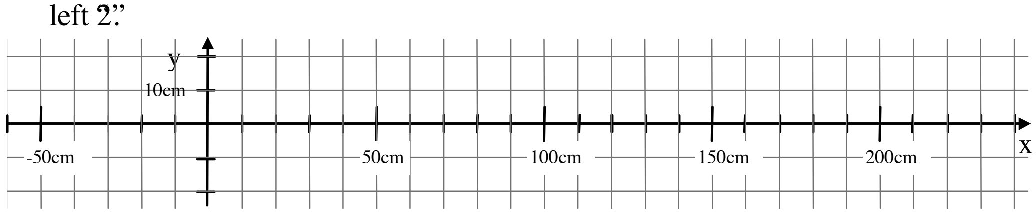
$y_1(x) = A_1 e^{-\left(\frac{x}{b_1}\right)^2}$, where A_1 = 20 cm and b_1 = 20 cm.

a) Sketch the shape of the spring at time t = 0 sec in the graph below. Use the indicated scale.



b) Write an equation that describes the displacement of any piece of the spring at any time for this pulse. Explain how you arrived at this answer.

c) Now suppose a second pulse moving *to the left* is also present on the spring. At t = 0.1 sec the equation describing this 2nd pulse is $y_2(x) = A_2 e^{-\left(\frac{x-200cm}{b_2}\right)^2}$ where A_2 = 10 cm and b_2 = 10 cm. Sketch the shape of the spring at time t = 0.1 sec in the graph below, labeling the pulse going to the right "1" and the pulse going to the left "2."



d) At what time and position do the maxima of the two pulses meet? Show your work.

e) Write an equation that describes the shape of the spring when the maxima meet. Show your work.

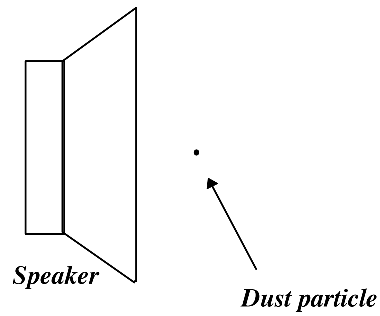
Appendix C: Sound Waves Tutorial

Name

Pretest

1. Consider a small piece of dust placed in front of a large, silent loudspeaker (see figure).

A. The speaker is turned on and plays a note with a constant frequency, f. How, if at all, does this affect the motion of the dust particle. Explain.

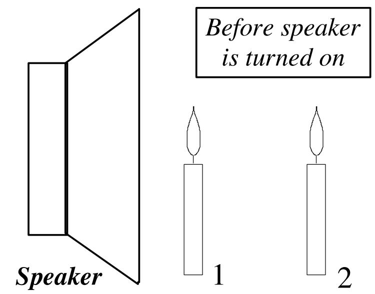


B. Consider an identical dust particle placed in front of an identical loudspeaker. The speaker is turned on and plays a note with frequency $2f$. How, if at all, does this affect the motion of the dust particle. Explain. Compare your answer to your answer to question A.

C. Consider an identical dust particle placed in front of an identical loudspeaker. The speaker is turned on and plays a note at the original frequency, f, but with a higher volume. How, if at all, does this affect the motion of the dust particle. Explain. Compare your answer to your answer to question A.

(over)

2. Consider two lit candles placed in front of a silent loudspeaker (see figure). The candle flames, flame 1 and flame 2, are perfectly upright and still.



A. The speaker is turned on and plays a note at a constant frequency, f. How, if at all, does this affect the motion of *flame 1*? If it does not move, state so explicitly.

B. Compare the motion, if any, of *flame 1* and *flame 2* after the loudspeaker is turned on. Explain how you arrived at your answer. If neither moves, state so explicitly.

C. Consider a situation where the speaker plays a note with the frequency $2f$. Compare the motion, if any, of *flame 1* and *flame 2*. Explain how you arrived at your answer. If neither moves, state so explicitly.

Sound Waves

I. Introduction

Last week we investigated propagating waves on springs. This week, we will consider sound waves. Experiments show that sound waves travel at about 340 m/s through air at room temperature.

Consider a flame placed in front of a speaker as shown in the figure at right. No wind is blowing.

A. The speaker plays a note at a constant pitch. Explain how, if at all, the sound produced by the speaker affects the flame. If the sound does not affect the flame, state that explicitly.

B. Open the movie ***Sound Movie.mov*** on your desktop. Play the movie.

1. Describe your observations. Do your observations agree with your earlier predictions?

2. How can you account for the flame's motion? Explain your reasoning.

Consider a dust particle in front of a silent loudspeaker. The dust particle is in the same location as the original location of the flame.

3. The speaker is turned on and plays a note with a constant frequency, f. How, if at all, does this affect the motion of the dust particle? Explain.

II. Motion of a Single Flame

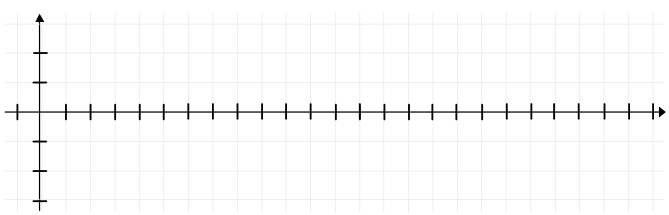
A. The program VideoPoint lets you analyze the motion of the flame by giving information about the positions of points on the screen at different times. On the desktop, open the file ***Sound Data File.vpt***. This lets you analyze the position of points on the video screen for the movie. Play the video using the single advance buttons.

1. What is the red cross on the video screen measuring? Explain.

2. The ‘Table’ window of ***Sound Data File.vpt*** includes data already obtained. These data are also shown to the right. What does this data represent?

Time (s)	x position (mm)	y position (mm)
0	0	0
0.03333	2.8	0
0.06667	2.1	0
0.1	0	0
0.1333	-3.5	0
0.1667	-1.4	0
0.2	1.4	0
0.2333	2.8	0
0.2667	2.1	0
0.3	-2.1	0
0.3333	-3.5	0
0.3667	-1.4	0
0.4	2.1	0
0.4333	2.8	0
0.4667	-2.1	0
0.5	-3.5	0
0.5333	-1.4	0
0.5667	2.1	0
0.6	3.5	0
0.6333	2.1	0
0.6667	-2.1	0
0.7	-2.1	0

3. In the space below, plot the position of the flame.



What does each axis on your graph represent? Explain.

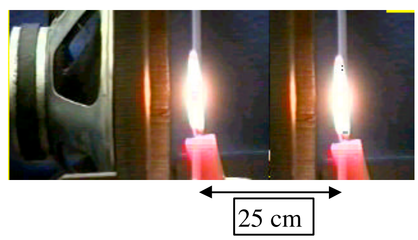
4. Describe the shape of your graph. (It may help to sketch a continuous curve on the basis of your data points.)

B. Find the period with which the flame and speaker oscillate. Explain how you obtained your answer.

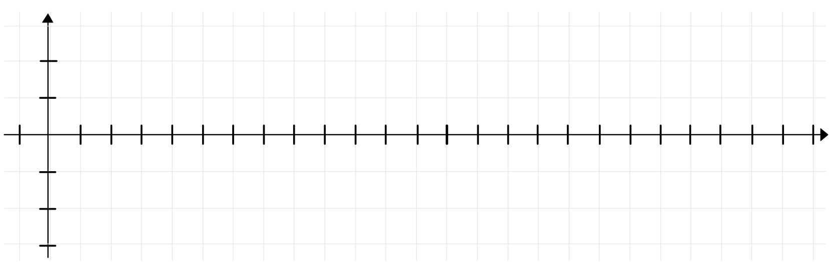
C. How many oscillations are there in a 1 second time interval. Explain how you obtained your answer.

III. Motion of Many Flames

A. Consider two candles sitting 25 cm apart in front of a loudspeaker oscillating at 680 Hz. A clock is started at an arbitrary time. At time t = 0 seconds, the first flame is perfectly vertical and moving away from the speaker. Its maximum displacement from equilibrium is 5 mm.



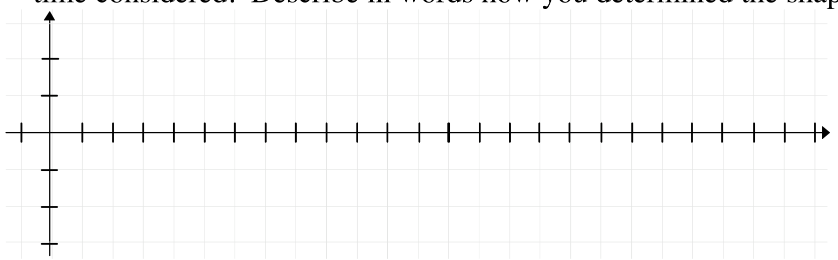
1. In the graph below, sketch the displacement of the first flame from equilibrium at different times. Label axes clearly. Explain how you arrived at your answer.



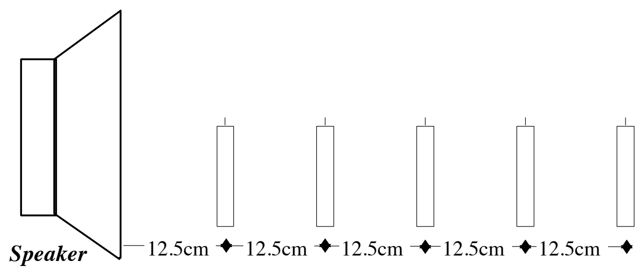
2. Compare the graph above to the graph you plotted on page 2. Explain.

3. Compare the motion of the second flame to the motion of the first flame. Explain.

4. In the graph below, sketch the displacement of the second flame for the entire time considered. Describe in words how you determined the shape of the sketch.

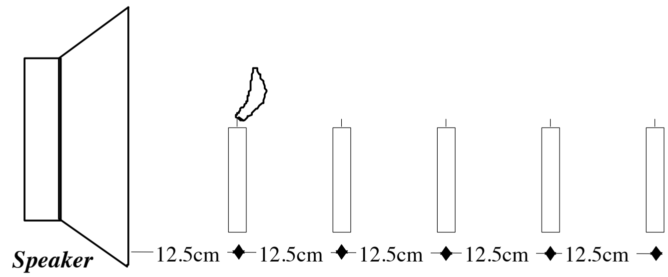


B. Consider an arrangement of candles where each candle is placed 12.5 cm from its nearest neighbor, and the first is located 12.5 cm from the speaker. The speaker plays a note at 680 Hz. At an arbitrary time after all the flames are in motion, a clock is started. At time t = 0 seconds, the first flame is at its maximum displacement of 5 mm from its upright position, away from the loudspeaker.

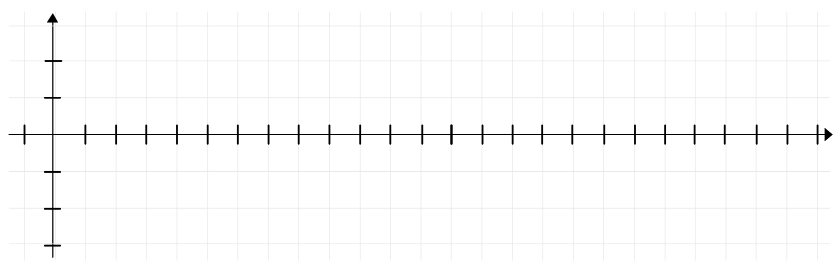


1. How would the graph of displacement vs. time for the second flame compare to the same graph for the first flame? Explain your reasoning.

2. The displacement of flame 1 at time t = 0 seconds is at a maximum and is shown at right. (The displacement is exaggerated for easy viewing). Sketch the displacement of each of the other flames at time t = 0 seconds in the diagram. Explain how you arrived at your answer.



3. In the graph below, sketch the displacement of each flame vs. the distance of the flame with respect to the speaker at time t = 0 seconds. Let each block on the horizontal axis represent 2.5 cm. Label axes clearly.



4. Consider a very large number of flames placed from 5 cm to 60 cm away from the speaker. On the graph on the previous page, plot the displacement of each of these flames at time t = 0 seconds as a function of distance from the speaker. Explain how you arrived at your answer.

5. Describe the shape of the graph you have sketched.

6. How can you use the graph you have sketched to find the wavelength of the sound produced by the loudspeaker? Explain.

7. Find the wavelength of the sound produced by the speaker in the movie ***sound.mov***.

VI. Comparing Graphs

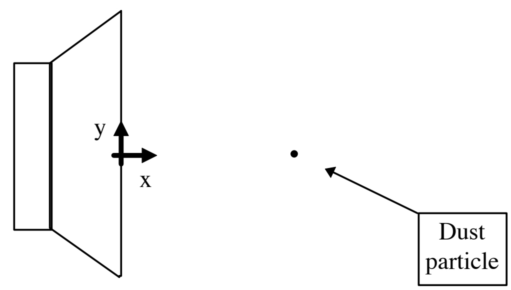
1. How could you use the graph on page 2 (instead of the graph on page 4) to find the wavelength of the sound wave?

2. How could you use the graph on page 4 (instead of the graph on page 2) to find the frequency of the sound wave? the period of the sound wave?

3. How could you use the graphs on pages 2 and 4 to find the amplitude of the sound wave?

Name **Tutorial Homework**

Consider a single dust particle floating a distance x_0 from a loudspeaker (see figure). The loudspeaker is turned on and produces a sound with a constant frequency f. The speed of sound is v. In the indicated coordinate system, the origin is located at the center of the speaker.



1. How long does it take for the sound wave to reach the dust particle? Explain.

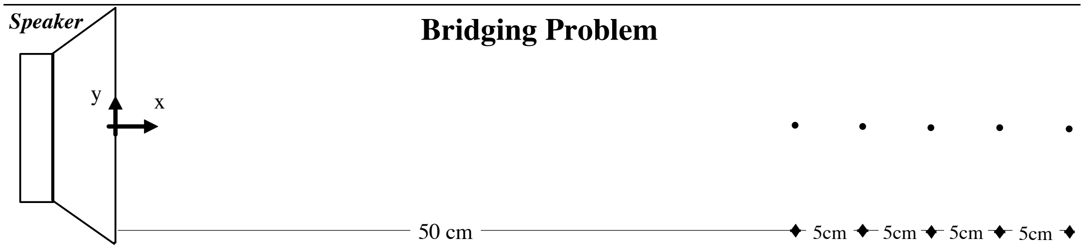
2. At time t = 0 sec, the dust particle begins to move away from the loudspeaker. Write an equation that describes the displacement of the dust particle from equilibrium for all times after t = 0 sec. Explain how you arrived at your answer. Explicitly define any variables you introduce in your equation.

3. Consider an identical dust particle a distance x_0 from an identical loudspeaker. The loudspeaker is turned on and produces a sound with a frequency of $2f$. Does the dust particle begin to move earlier than in question 1? Explain.

4. Write an equation that describes the displacement of the dust particle in question 3 from equilibrium for all times after t = 0 sec. Explain how you arrived at your answer.

Sound - Tutorial Homework

Bridging Problem

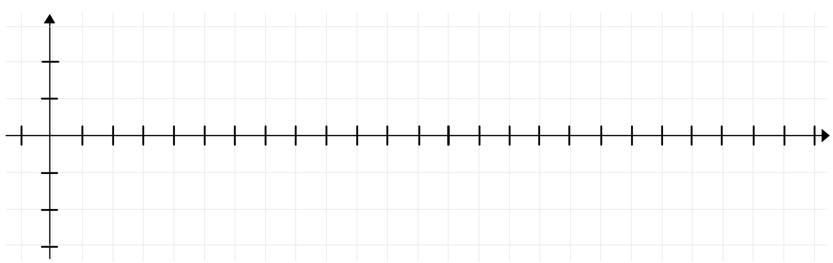


Five dust particles are placed in a row 5 cm apart beginning 50 cm from a loudspeaker (see figure). The speaker plays a note with a frequency of 1700 Hz. The speed of sound is 340 m/s. The maximum displacement of the first dust particle is $s_{max} = 3$ mm. Assume that the intensity of the sound wave is the same for all dust particles. In the indicated coordinate system, the origin is at the center of the loudspeaker. A clock is started at an arbitrary time.

a) At time t = 0 sec, the first dust particle is at equilibrium and moving away from the loudspeaker. Find t_0, the amount of time that elapses until the second dust particle is at its equilibrium position. Explain.

b) What is the displacement from equilibrium of the first dust particle at time t_0? Explain how you arrived at your answer.

c) In the graph below, sketch a graph of s vs. x at time t_0. Define each axis clearly.



d) Find s(x,t) for $x = 65$ cm and $t = 2.941176 \times 10^{-4}$ sec. Show all work.

Appendix D-1: Wave Diagnostic Test, Preliminary Version

University of Maryland
Department of Physics

Spring 1997 Post Wave Test v.3

Name Class Section

Introduction: The Physics Education Research Group is studying how student learn physics in introductory courses. We are developing new methods and new materials like the tutorials for teaching physics.

Request: As you answer these questions, we want to focus on how you respond and how you approach the questions. In order for us to evaluate your responses in more detail, we would like to videotape you answering these questions. The tapes will be transcribed for the group to study.

Confidentiality: These tapes will be edited and transcribed with code names. Your name will be kept confidential.

Grades: Your grade in this course will not be affected in any way by whether you choose to participate or by what you say on tape.

Value: The better we understand what is happening in class and know how you are thinking about physics, the more effectively we can teach you. It also helps us to develop better ways of teaching physics.

If you are willing to allow us to tape you, please write your name, student number, and signature in the space below.

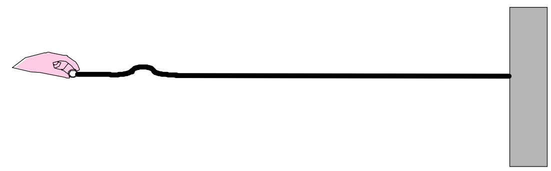
Name____________________________________

Student Number____________________

Signature_________________________________

UMd Wave Diagnostic Test

1. A person holds a long, taut string and quickly moves her hand up and down, creating a pulse which moves toward the wall to which the string is attached and reaches it in a time t_0 (see figure).



How could the person *increase* the amount of time it takes for the pulse to reach the wall? Explain.

2. Consider a pulse propagating along a long, taut string in the +x-direction. The diagram below shows the shape of the pulse at t = 0 sec. Suppose the displacement of the string at t = 0 sec for different values of x is given by $y(x) = Ae^{-\left(x/b\right)^2}$.

a) On the diagram, sketch the shape of the string after the pulse has traveled a distance x_0, where x_0 is shown in the figure. Explain why you sketched the shape as you did.

b) For the instant of time that you have sketched, write an equation for the displacement of the string as a function of x. Explain how you determined your answer.

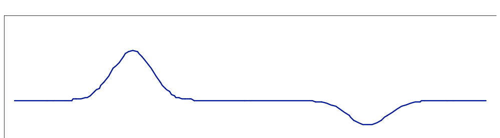
Please note:

For the remaining multiple-choice questions, please answer in the indicated spaces.

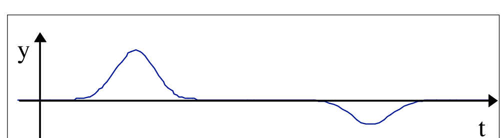
On some questions, more than one answer may be correct (i.e. *b*, *c* and *d* from a list of possible responses *a*–*k*). If so, **give them all**.

In some sections, you may use the same response **more than once** to answer different questions (i.e. use *d* to answer questions 14, 15, and 18).

For some questions, answers may describe both pictures and graphs. For example:



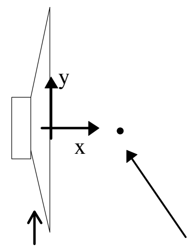
A picture of two wavepulses on a string



A graph of displacement as a function of time

In each case, the same diagram can be used to represent different quantities.

For **questions 1 to 4** consider the following situation. A dust particle is located a distance x_0 from the front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant pitch. At time t = 0, the particle begins to move.



1. Which of the actions *a–f* to the right describes the motion of the dust particle after time t = 0 sec. More than one answer may be correct. If so, **give them all.** _______ Explain your reasoning.

Possible Responses for question 1:

a) The dust particle will move up and down.
b) The dust particle will be pushed away from the speaker.
c) The dust particle will move side to side.
d) The dust particle will not move at all.
e) The dust particle will move in a circular path.
f) None of these answers is correct.

2. Consider the coordinate system shown in the figure. Which equation *a-i* best describes the position of the dust particle for all times t > 0? _______
Explain how the equation you chose relates to the motion you described (be sure to include a description or definition of **all** variables, such as A, ω, k, or s).
If you chose "*i*," explain.

Possible Responses for question 2:

a) $x = A\sin(\omega t)$ b) $y = A\sin(\omega t)$
c) $s = x_0 + v/t$ d) $s = A\sin(kx)$
e) $y = A\sin(kx - \omega t)$ f) $x = x_0 - v/t$
g) $x = A\sin(kx)$ h) $s = A\sin(\omega t)$
i) none of the above

Use possible responses *a–e* to the right to answer the following two questions. You may use the same response more than once. More than one answer may be correct. If so, **give them all.**

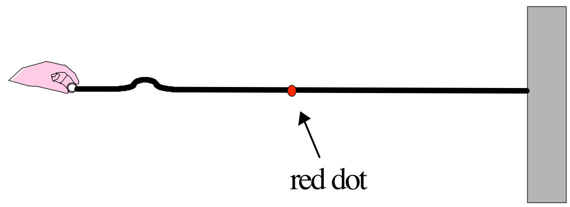
3. How, if at all, would the answer to question 1 change if the speaker played a note at a higher pitch? ________ Explain.

4. How, if at all, would your answer to question 1 change if the speaker played a note at a greater volume (but the original pitch)? ________ Explain.

Possible changes to your answer to question 1:

a) The particle would move exactly as before.
b) The particle would move slower.
c) The particle would move faster.
d) The particle would move with a greater amplitude.
e) The particle would still not move at all

For **questions 5 to 8**, consider the following situation. A long, taut string is attached to a distant wall (see figure). A demonstrator moves her hand and creates a very small amplitude pulse which reaches the wall in a time t_0. A small red dot is painted on the string halfway between the demonstrator's hand and the wall. For each question, state which of the actions *a–k* (listed to the right) **taken by itself** will produce the desired result. For each question, more than one answer may be correct. If so, **give them all.**



How, if at all, can the demonstrator repeat the original experiment to produce:

5. A pulse that takes a longer time to reach the wall. More than one answer may be correct. If so, **give them all.**__________ Explain.

6. A pulse that is wider than the original pulse. More than one answer may be correct. If so, **give them all.**__________ Explain.

7. A pulse that makes the red dot stay in motion for less time than in the original experiment. More than one answer may be correct. If so, **give them all.**__________ Explain.

8. A pulse that makes the red dot travel a further distance than in the original experiment. More than one answer may be correct. If so, **give them all.**__________ Explain.

Possible Responses for questions 5 to 8:

a) Move her hand more quickly (but still only up and down once and still by the same amount).

b) Move her hand more slowly (but still only

c) up and down once and still by the same amount).

d) Move her hand a larger distance but up and down in the same amount of time.

e) Move her hand a smaller distance but up and down in the same amount of time.

f) Use a heavier string of the same length, under the same tension

g) Use a lighter string of the same length, under the same tension

h) Use a string of the same density, but decrease the tension.

i) Use a string of the same density, but increase the tension.

j) Put more force into the wave.

k) Put less force into the wave.

l) none of the above.

For **questions 9 to 10**, consider the following situation. A pulse on a string described at time t = 0 s by the equation $y(x) = Ae^{-\left(x/b\right)^2}$ propagates along a long, taut string in the +x-direction. The diagram above shows the string at t = 0 s.

9) On the diagram, **sketch the shape of the string** after the pulse has traveled a distance x_0, where x_0 is shown in the figure. Which of statements *a-h* to the right describes the shape you have drawn. More than one response may be correct. If so, give them all. ___________ Explain.

Possible Responses for question 9:

a) The pulse will have a smaller amplitude.
b) The pulse will have a larger amplitude
c) The pulse will be narrower.
d) The pulse will be wider.
e) The pulse will have a bigger area.
f) The pulse will have a smaller area.
g) The pulse will have the same shape as before.
h) None of these answers is correct.

10) Which of the equations *a-h* to the right gives an equation that gives the displacement of the string as a function of x at the instant in time that you have sketched. ___________. Explain how you determined your answer.

Possible Responses for question 10:

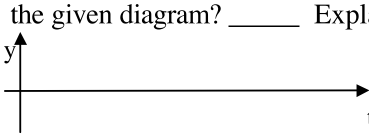
a) $y(x) = Ae^{-\left(x_0/b\right)^2}$ b) $y(x) = Ae^{-\left(x/b\right)^2}$
c) $x = b\sqrt{\ln(y)}$ d) $x_0 = vt$
e) $x = -b\sqrt{\ln y}$ f) $y(x) = Ae^{-\left(x-x_0/b\right)^2}$
g) none of the above

For **questions 11 to 15**, consider the following situation. Two wavepulses with different amplitudes on a string are moving at speed v = 1 m/s toward each other. **At time t = 0.5 sec**, the shape of the string is shown in the diagram to the right, and the wavepulses are separated by a distance of 1 m. Three specific pieces of string are labeled "p," "q," and "r." In answering these questions, you may use the same answer more than once. In each diagram, up is positive.

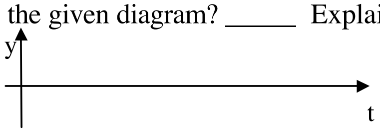
11. Which diagram represents a picture of the string at time t = 1.0 s (i.e. 0.5 s after the time in the given diagram)? ______ Explain.

12. Which diagram represents a picture of the string at time t = 1.5 s (i.e. 1.0 s after the time in the given diagram)? ______ Explain.

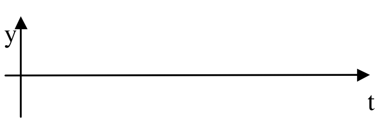
13. Which diagram represents a plot of the displacement (as a function of time) of the piece of string indicated by a "p" in the given diagram? ______ Explain.



14. Which diagram represents a graph of the displacement (as a function of time) of the piece of string indicated by a "q" in the given diagram? ______ Explain.



15. Which diagram represents a graph of the displacement (as a function of time) of the piece of string indicated by an "r" in the given diagram? ______ Explain.



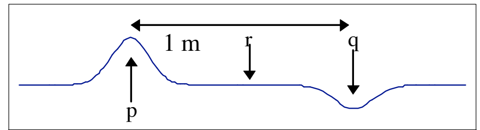
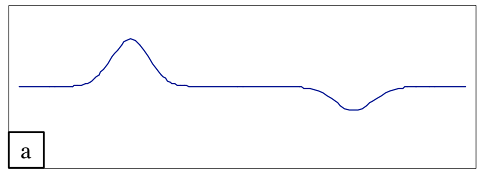
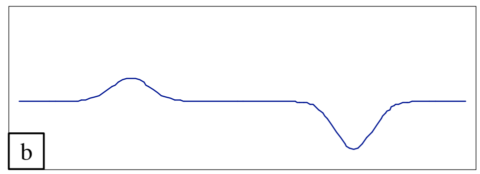


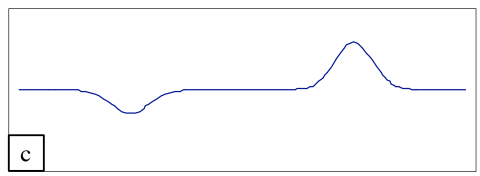
Diagram of string at time t = 0.5 sec



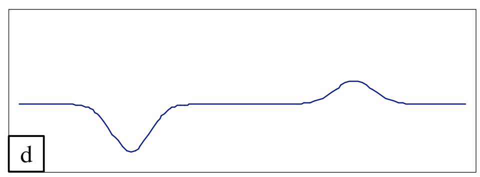
a



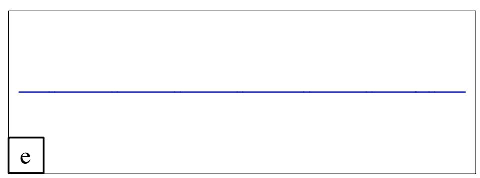
b



c



d



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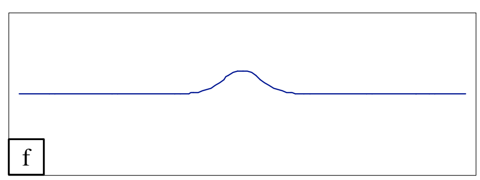
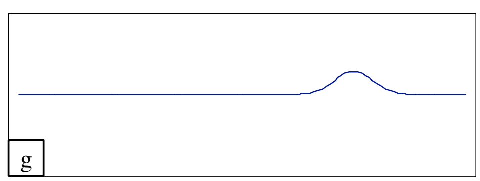


f



g

h) none of the above are correct.

For **questions 16 to 19**, consider the following situation. Two asymmetric wavepulses on a string are moving at speed v = 1 m/s toward each other. **At time t = 0.4 s**, the shape of the string is shown in the diagram to the right, and the peaks of the pulses are separated by a distance 1.2 m. One piece of string is labeled "p." In answering these questions, you may use the same answer more than once. If you choose "h," explain.

16. Which of the diagrams represents a picture of the string at time 1.0 s (0.6 s after the time in the given diagram)? _____ Explain.

17. Which of the diagrams represents a picture of the string at time *a little bit before* time t = 1.0 s (e.g. t = 0.9 s, 0.5 s after the time in the given diagram)? _____ Explain.

18. Which of the diagrams represents a picture of the string at time t = 1.6 s (1.2 s after the time in the given diagram)? _____ Explain.

19. Which of the diagrams represents a graph of the displacement (as a function of time) of the piece of string indicated by a "p" in the given diagram? _____ Explain.

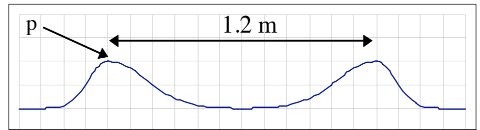
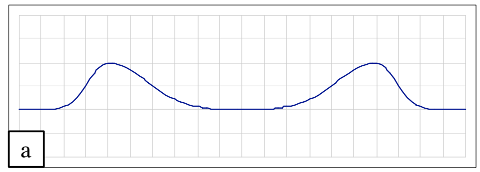
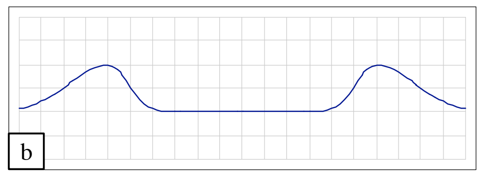


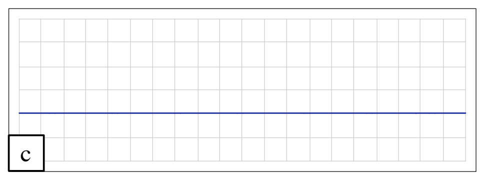
Diagram of string at time t = 0.4 sec



a



b



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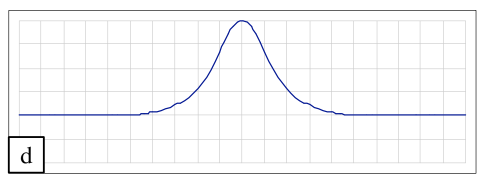
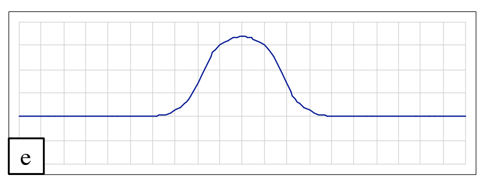
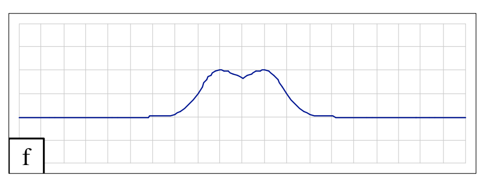


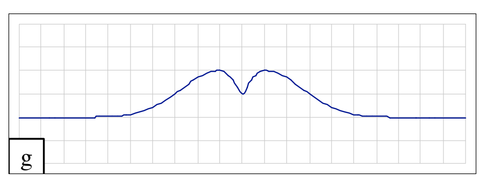
d



e



f



g

h) none of the above are correct.

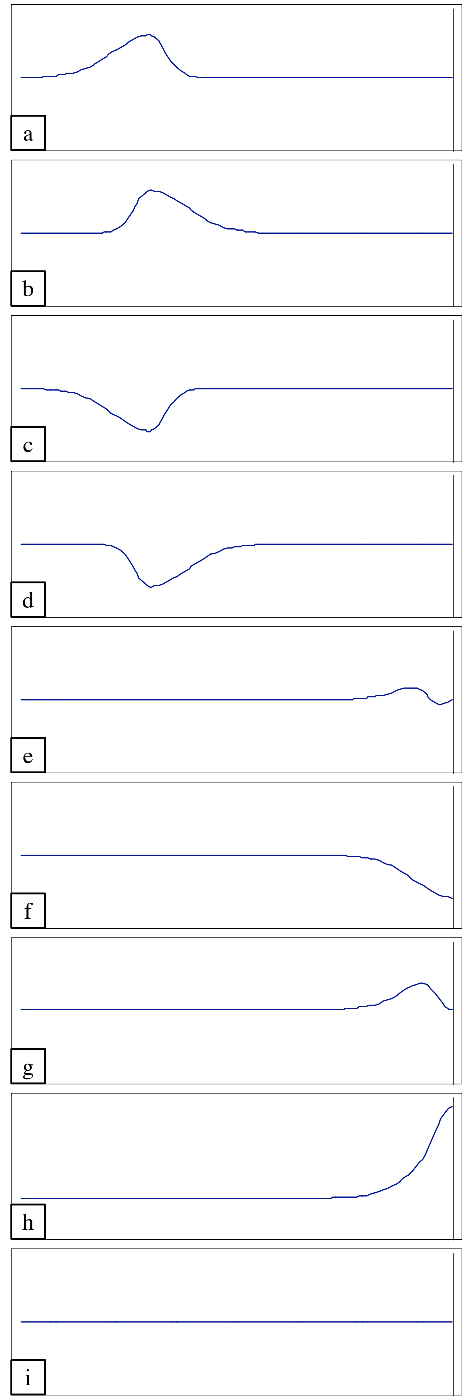
For **questions 20 to 23**, consider the following situation. An asymmetric wavepulse moves on a string at speed 1 m/s toward a pole. At time t = 0 s, the shape of the string is shown in diagram “a,” and the peak of the wavepulse is a distance 1 m from the pole. In answering these questions, you may use the same answer more than once.

20. If the string is firmly attached to the pole, which diagram represents a picture of the string at time t = 1 s? _____
    Explain.

21. If the string is free to move along the pole, which diagram represents a picture of the string at time t = 1 s? _____
    Explain.

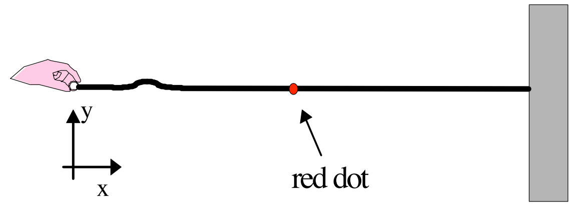
22. If the string is firmly attached to the pole, which diagram represents a picture of the string at time t = 2 s? _____
    Explain.

23. If the string is free to move along the pole, which diagram represents a picture of the string at time t = 2 s? _____
    Explain.



j) none of the above

For **questions 24 to 28**, consider the following situation. A pulse with a shape as shown in diagram "a" to the right is traveling to the right along a string on which a red dot of paint is located (see figure). Consider only the time until the pulse reaches the wall. For each question, identify which figure below would look most like the described quantity. For each graph, consider positive to be up. If none of the figures look like you expect the graph to look, answer "i." In responding to these questions, you may use the same answer more than once.



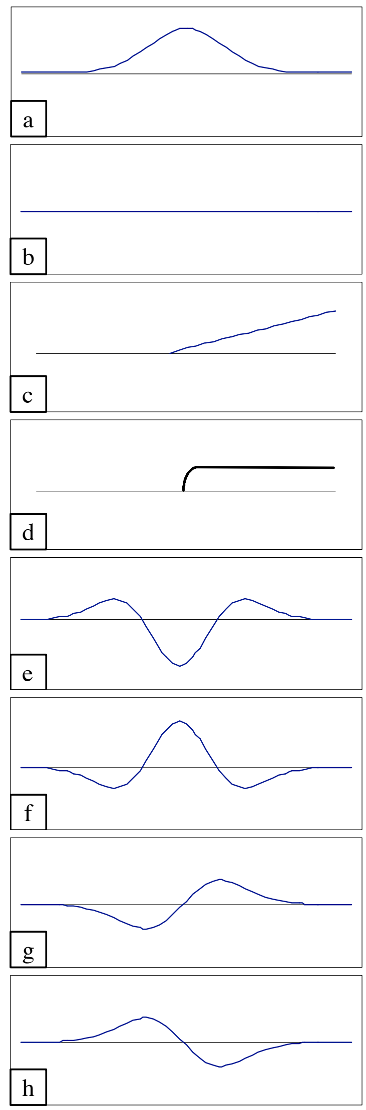
24. The graph of the y displacement of the red dot as a function of time. _________ Explain.

25. The graph of the x displacement of the red dot as a function of time. _________ Explain.

26. The graph of the y velocity of the red dot as a function of time. _________ Explain.

27. The graph of the x velocity of the red dot as a function of time. _________ Explain.

28. The graph of the y component of the force on the red dot as a function of time. ________ Explain.



i) None of these figures is correct.

Appendix D-2: Wave Diagnostic Test, Final Version, Pre-Instruction

Name **UMd Wave Diagnostic Test**

1. Michael and Laura are standing 100 m apart and yell “Yo!” at each other at exactly the same instant. Michael yells louder than Laura, and the pitch (frequency) of his voice is lower.

Will Laura hear Michael first, Michael hear Laura first, or will they hear each other at the same time? Explain how you arrived at your answer.

How, if at all, would your answer change if Laura yelled at the same volume as Michael? Explain your reasoning.

How, if at all, would your answer to the original question change if Michael and Laura yelled at the same pitch but Michael yelled louder? Explain your reasoning.

2. Consider two wavepulses with different amplitudes moving on a string at speed of 10 m/s toward each other. **At time t = 0 sec**, the shape of the string is shown in the diagram to the right, and the wavepulses are separated by a distance of 1 m.

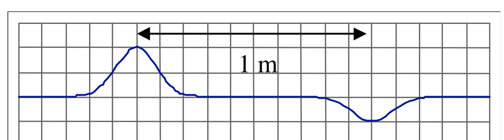
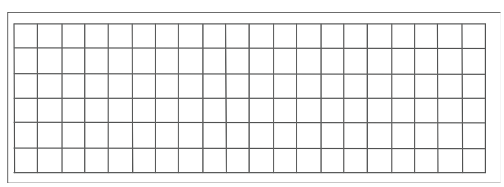
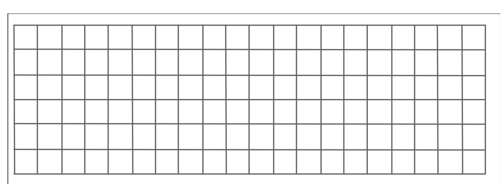


Diagram of string at time t = 0 sec

Sketch the shape of the string at time t = 0.05 sec in the diagram to the right. Explain how you arrived at your answer.

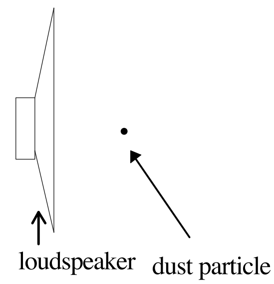


Sketch the shape of the string at time t = 0.1 sec in the diagram to the right. Explain how you arrived at your answer.



3. A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant pitch.

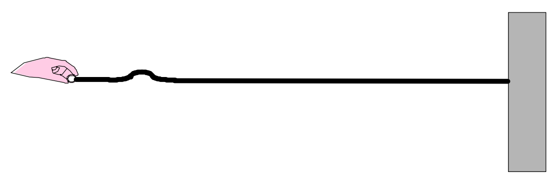
Describe the motion of the dust particle. Explain your reasoning.



How, if at all, would your answer change if the speaker played a note at a higher pitch (frequency)? Explain your reasoning.

How, if at all, would your answer to the original question change if the speaker played a note at a greater volume (but the original pitch)? Explain.

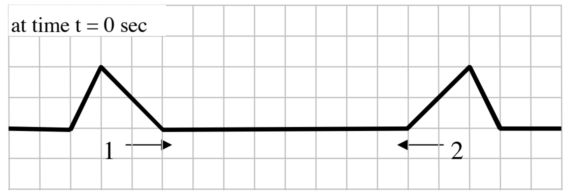
4. A person holds a long, taut string and quickly moves her hand up and down, creating a pulse which moves toward the wall to which the string is attached. The pulse reaches the wall in a time t_0 (see figure).



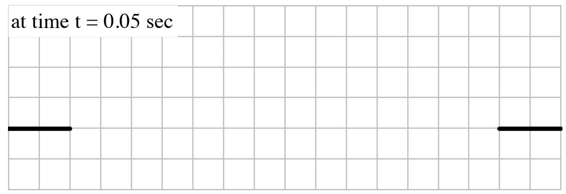
How could the person decrease the amount of time it takes for the pulse to reach the wall? Explain.

How, if at all, would the *speed* of the pulse change if the pulse were wider? Explain your reasoning.

5. Consider two pulses on a spring, as shown in the figure to the right. They are moving toward each other at 100 cm/sec. Each block in the picture represents 1 cm.



In the figure to the right, sketch the shape of the spring after 0.05 sec have elapsed. Explain how you arrived at your answer.



In the figure to the right, sketch the shape of the spring after 0.06 sec have elapsed. Explain how you arrived at your answer.

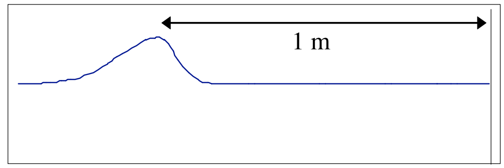
In the figure to the right, sketch the shape of the spring after 0.1 sec have elapsed. Explain how you arrived at your answer.

6. Margaret stands 20 m from a large wall and claps her hands together once. A short moment later, she hears an echo.

How, if at all, would the time it takes for her to hear the echo change if she clapped her hands harder? Explain.

Consider a dust particle floating in the air very close to the wall (within 0.1 mm). Describe the motion, if any, of this dust particle between the moment that Margaret claps and the moment she hears the echo. Explain how you arrived at your answer.

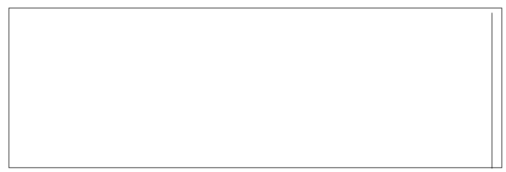
7. An asymmetric wavepulse moves on a string toward a pole at speed 10 m/s. At time t = 0 sec, the shape of the string is shown in the diagram to the right and the peak of the wavepulse is a distance 1 m from the pole.



Consider that the string is **firmly attached** to the pole.

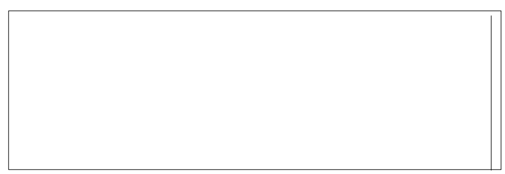
In the figure to the right, sketch the shape of the string at time t = 0.1 sec. Explain how you arrived at your answer.

In the figure to the right, sketch the shape of the string at time t = 0.2 sec. Explain how you arrived at your answer.

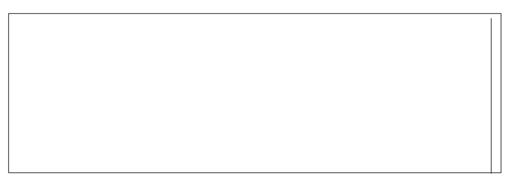


8. Consider that the experiment above (in question 7) is repeated, but the string shown in the figure is **free to move** along the pole to which it is attached.

In the figure to the right, sketch the shape of the string at time t = 0.1 sec. Explain how you arrived at your answer.

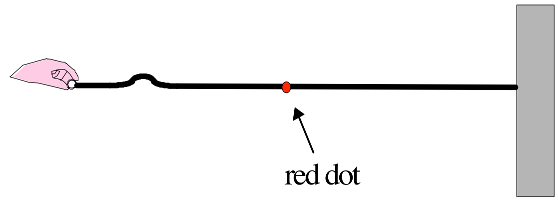


In the figure to the right, sketch the shape of the string at time t = 0.2 sec. Explain how you arrived at your answer.



After completing these questions, please turn in this part of the questionnaire. Then, get the last page of the questionnaire…

9. A long, taut string is attached to a distant wall (see figure). A demonstrator moves her hand and creates a very small amplitude pulse which reaches the wall in a time t_0. A small red dot is painted on the string halfway between the demonstrator's hand and the wall. For each question, state which of the actions
a–k (listed to the right) **taken by itself** will produce the desired result. For each question, more than one answer may be correct. If so, **give them all.**



How, if at all, can the demonstrator repeat the original experiment to produce:

A pulse that takes a longer time to reach the wall. More than one answer may be correct. If so, **give them all.**__________
Explain.

A pulse that is wider than the original pulse. More than one answer may be correct. If so, **give them all.**__________
Explain.

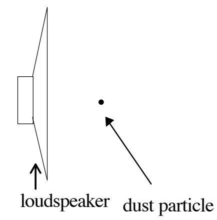
A pulse that makes the red dot stay in motion for less time than in the original experiment. More than one answer may be correct. If so, **give them all.**__________
Explain.

A pulse that makes the red dot travel a further distance than in the original experiment. More than one answer may be correct. If so, **give them all.**__________
Explain.

Possible responses for all parts of question 9:

a) Move her hand more quickly (but still only up and down once and still by the same amount).
b) Move her hand more slowly (but still only up and down once and still by the same amount).
c) Move her hand a larger distance but up and down in the same amount of time.
d) Move her hand a smaller distance but up and down in the same amount of time.
e) Use a heavier string of the same length, under the same tension
f) Use a lighter string of the same length, under the same tension
g) Use a string of the same density, but decrease the tension.
h) Use a string of the same density, but increase the tension.
i) Put more force into the wave.
j) Put less force into the wave.
k) none of the above.

10. A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant (low) pitch. **Which choice or combination of the choices** *a–f* (listed below) can describe the motion of the dust particle after the loudspeaker is turned on? **Circle the correct letter or letters.** Explain.

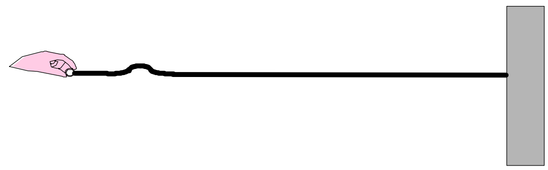


Possible responses for question 10:
a) The dust particle will move up and down.
b) The dust particle will be pushed away from the speaker.
c) The dust particle will move side to side.
d) The dust particle will not move at all.
e) The dust particle will move in a circular path.
f) None of these answers is correct.

Appendix D-3: Wave Diagnostic Test, Final Version, Post-Instruction

Name **UMd Wave Diagnostic Test**

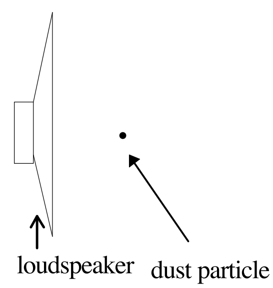
1. A person holds a long, taut string and quickly moves her hand up and down, creating a pulse which moves toward the wall to which the string is attached. The pulse reaches the wall in a time t_0 (see figure).



How could the person decrease the amount of time it takes for the pulse to reach the wall? Explain.

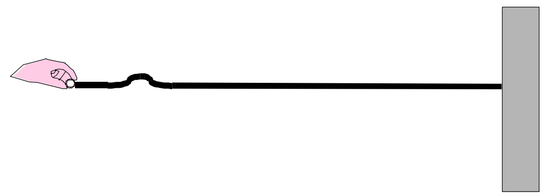
2. A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant pitch.

Describe the motion of the dust particle. Explain your reasoning.



After completing these questions, please turn in this part of the questionnaire. Then, get the last page of the questionnaire…

3. A long, taut string is attached to a distant wall (see figure). A demonstrator moves her hand up and down exactly once and creates a very small amplitude pulse which reaches the wall in a time t_0. For the question below, state which of the actions *a–k* (listed to the right) **taken by itself** will produce the desired result. Note that more than one answer may be correct. If so, **give them all.**

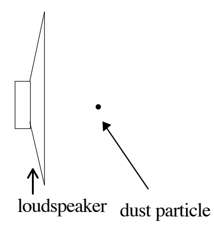


How, if at all, can the demonstrator repeat the original experiment to produce a pulse that takes a longer time to reach the wall. More than one answer may be correct. If so, **give them all. Circle the correct letter or letters.** Explain.

Possible responses for question 1:

a) Move her hand more quickly (but still only up and down once and still by the same amount).
b) Move her hand more slowly (but still only up and down once and still by the same amount).
c) Move her hand a larger distance but up and down in the same amount of time.
d) Move her hand a smaller distance but up and down in the same amount of time.
e) Use a heavier string of the same length, under the same tension
f) Use a lighter string of the same length, under the same tension
g) Use a string of the same density, but decrease the tension.
h) Use a string of the same density, but increase the tension.
i) Put more force into the wave.
j) Put less force into the wave.
k) none of the above.

4. A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant (low) pitch. **Which choice or combination of the choices** *a–f* (listed below) can describe the motion of the dust particle after the loudspeaker is turned on? **Circle the correct letter or letters.** Explain.



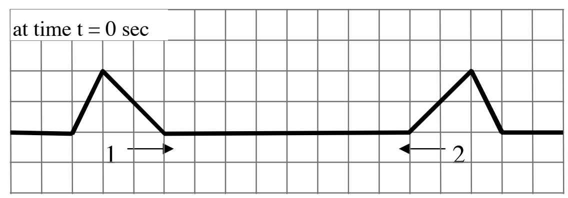
Possible responses for question 2:

g) The dust particle will move up and down.
h) The dust particle will be pushed away from the speaker.
i) The dust particle will move side to side.
j) The dust particle will not move at all.
k) The dust particle will move in a circular path.
l) None of these answers is correct.

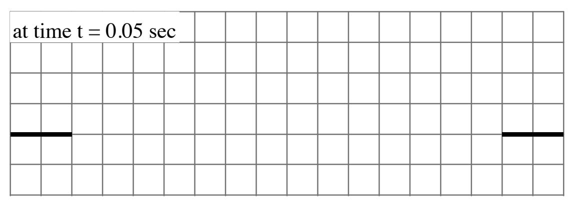
5. Michael and Laura are standing 100 m apart and yell "Yo!" at each other at exactly the same instant. Michael yells louder than Laura, and the pitch (frequency) of his voice is lower. No wind is blowing.

Will Laura hear Michael first, Michael hear Laura first, or will they hear each other at the same time? Explain how you arrived at your answer.

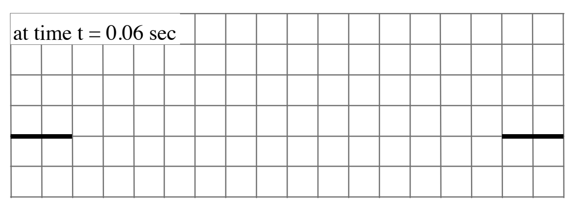
6. Consider two pulses on a spring, as shown in the figure to the right. They are moving toward each other at 100 cm/sec. Each block in the picture represents 1 cm.



In the figure to the right, sketch the shape of the spring after 0.05 sec have elapsed. Explain how you arrived at your answer.



In the figure to the right, sketch the shape of the spring after 0.06 sec have elapsed. Explain how you arrived at your answer.



7. Margaret stands 30 m from a large wall and claps her hands together once. A short moment later, she hears an echo.

How, if at all, would the time it takes for her to hear the echo change if she clapped her hands harder? Explain.

Consider a dust particle floating in the air very close to the wall (within 0.1 mm). Describe the motion, if any, of this dust particle between the moment that Margaret claps and the moment she hears the echo. Explain how you arrived at your answer.