Beyond mathematical content knowledge: A mathematician’s knowledge needed for teaching an inquiry-oriented differential equations course

Joseph F. Wagnera,*, Natasha M. Speerb, Bernd Rossa a

a Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH 45207-4441, United States
b Division of Science and Mathematics Education, Michigan State University, East Lansing, MI 48824-1034, United States

Abstract

In this research report we examine knowledge other than content knowledge needed by a mathematician in his first use of an inquiry-oriented curriculum for teaching an undergraduate course in differential equations. Collaboratively, the mathematician and two mathematics education researchers identified the challenges faced by the mathematician as he began to adopt reform-minded teaching practices. Our analysis reveals that responding to those challenges entailed formulating and addressing particular instructional goals, previously unfamiliar to the instructor. From a cognitive analytical perspective, we argue that the instructor’s knowledge — or lack of knowledge — influenced his ability to set and accomplish his instructional goals as he planned for, reflected on, and enacted instruction. By studying the teaching practices of a professional mathematician, we identify forms of knowledge apart from mathematical content knowledge that are essential to reform-oriented teaching, and we highlight how knowledge acquired through more traditional instructional practices may fail to support research-based forms of student-centered teaching.

© 2007 Elsevier Inc. All rights reserved.

Keywords: Undergraduate teaching; Teaching practices; Teacher knowledge; Pedagogical content knowledge

One outgrowth of efforts at instructional reform over the past several decades is a substantial enrichment to the mathematics education community’s understanding of the complexities of teaching and of learning to teach in new ways. The work of educational researchers to document and understand the ways teachers enact instruction of the sort advocated by, for example, the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 2000) has helped to identify factors that shape teachers’ practices, including their implementation of curricular materials. There is compelling evidence that teaching in ways consistent with the vision of the designers of reform can be challenging (Cohen, 1990a; Cuban, 1990; Fennema & Nelson, 1997).

As researchers have made efforts to understand why some teachers readily carry out reform-minded approaches to instruction while others face notable challenges, the community’s understanding of the factors that shape teachers’ practices has been broadened and deepened. One often-implicated influence on teachers’ practices — whether those practices be in the context of reform or not — is teachers’ knowledge (Borko & Putnam, 1996; Calderhead, 1991, 1996; Fennema & Franke, 1992). Although there is ample reason to expect that mathematical content knowledge and forms of general pedagogical knowledge are essential to effective mathematics teaching, there is strong (and growing)
evidence that researchers and teacher educators have the potential to benefit from further research that examines other types of knowledge used by teachers.

Despite existing research efforts concerning the knowledge required for teaching, the literature still suffers from some significant limitations. Most studies of teacher knowledge and curricular reform have focused on K-12 mathematics instruction, with very few examining instruction at the college level. Even fewer — virtually none, in fact — have studied the knowledge required for teaching by university mathematicians. Furthermore, few systematic attempts have been made to implement research-based instructional strategies in upper-level mathematics courses. Moreover, research mathematicians are unlikely to lack mathematical content knowledge, and as a result, there may be much for the mathematics education research community to learn about the other kinds of knowledge required for effective reform-minded teaching. Studies of mathematicians’ by studying their efforts to implement innovative curricula in their classrooms are apt to highlight the role that forms of knowledge other than pure content play in teaching practices.

In this paper we report on our attempts to address some of the existing research limitations described above. We consider a case of an experienced, practicing university mathematician and teacher, Bernd Rossa, a co-author of this paper, in his first experience teaching a course in differential equations using a set of curricular materials developed to support a student-centered, “inquiry-oriented” approach to the instruction of advanced mathematics. Rossa’s reports of the challenges he faced in implementing such a non-traditional instructional approach offer insight into the types of knowledge he himself identified as lacking from his own experience teaching in more traditional ways. His perceptions are corroborated by observations of his teaching and post-class interviews carried out by a mathematics education researcher.

We argue that the knowledge required for experienced mathematicians to implement effective, reform practices in their classrooms includes knowledge that differs from the mathematical content knowledge, pedagogical content knowledge, and pedagogical knowledge that support traditional instruction. These other kinds of essential knowledge include pedagogical content knowledge not accessible through traditional instructional experience and even forms of mathematical knowledge not typically used by professional mathematicians. These findings offer insight into the types of support required for university instructors who desire to adopt non-traditional curricula and instructional practices.

The analytical portion of this paper is organized around four primary challenges identified by Rossa as he implemented a reform-oriented curriculum for the first time. Rossa’s experiences offered research opportunities for insight into the following questions:

1. What kinds of challenges do mathematicians such as Rossa perceive as they implement an innovative, reform-oriented curriculum for the first time?
2. What challenges for such a teacher are apparent to an educational researcher through observations and interviews?
3. What can such challenges reveal about the knowledge needed to teach innovative, reform-oriented curricula?

Rossa’s response to the challenges he faced entailed addressing particular instructional goals in ways that were new to him. We therefore supplement our analysis with a taxonomy of instructional goals related to these challenges that raises questions that may be important to teachers as well as to the educational researchers who are studying and accompanying those teachers in transition to more innovative forms of instruction.

1. Research on teachers’ practices

1.1. Teachers’ changing their practices

In this section we provide an overview of research findings on the relationship between teachers’ knowledge and their teaching practices, as well as the relationship between teachers’ knowledge and their students’ achievement. We discuss how, as this area of research has developed, researchers have found it necessary (and productive) to develop various sub-categories of teachers’ knowledge. The section concludes with a discussion of the specific theoretical framing for the present study.

1.2. Developments in the study of teachers’ knowledge

It has become clear that teachers make use of various types of knowledge when they plan for and enact instruction. While early work focused primarily on teachers’ knowledge of mathematical content, it is now accepted that teachers
make use of knowledge beyond that of content. Broadly speaking, in their work, teachers draw on knowledge they have of mathematics, knowledge about teaching and learning that are connected to particular mathematical ideas, and non-mathematics-specific knowledge.

As obvious as it may seem to say that teachers’ knowledge of mathematics influences their teaching practices (and hence their students’ learning), this claim has been difficult to substantiate. Researchers have found it challenging to establish relationships between measures of teachers’ content knowledge and student achievement (Wilson, Floden, & Ferrini-Mundy, 2001, 2002). In these cases, the “measures” used of content knowledge have typically been the number of mathematics courses taken or degrees attained by the teachers. Some early findings indicated no relationship between these measures and student achievement (Begle, 1979), while findings of positive such relationships (e.g., Darling-Hammond, 2000; Monk, 1994) have been modest and limited. It appears that “the amount” of mathematics teachers learn while in school says little about the quality or nature of the learning opportunities that teachers create for their students.

1.3. Pedagogical content knowledge

A major milestone in the study of teachers’ knowledge was the identification of knowledge that teachers have about teaching and learning that is connected to specific mathematical content. The naming of “pedagogical content knowledge” (Grossman, Wilson, & Shulman, 1989; Shulman, 1986) prompted substantial work aimed at characterizing, identifying, and assessing knowledge that teachers have about, among other things, which mathematical topics typically cause students difficulty; how different mathematical ideas tie together and support later mathematical topics; or how particular examples, explanations, or strategies can be useful in teaching particular mathematical concepts. These examples of pedagogical content knowledge (PCK) all represent knowledge that is unique to mathematics instruction yet not obtained merely through the study of mathematics subject matter itself.

The identification of PCK sparked many lines of research. We point, however, to the Cognitively Guided Instruction (CGI, Carpenter, Fennema, Peterson, & Carey, 1988; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) research program as exemplifying the importance of efforts to study the significance of pedagogical content knowledge. The CGI researchers examined a particular type of pedagogical content knowledge — knowledge of young children’s thinking and strategies for solving addition and subtraction word problems — and found demonstrable relationships between teachers’ knowledge of their students’ thinking, the teachers’ instructional practices, and the subsequent mathematical achievement of their students. Teachers’ knowledge of the different strategies that their students would use to approach problems was positively correlated with student achievement (Carpenter et al., 1988). Even more, Carpenter et al. (1989) demonstrated that teachers who participated in programs designed to increase their pedagogical content knowledge of this aspect of students’ mathematical thinking tended to shift their teaching practices in ways that included listening and attending more closely to students’ mathematical reasoning, and encouraging students to use multiple solution strategies — practices associated with current efforts toward mathematics education reform. The results of CGI research offered clear demonstrations that at least some types of pedagogical content knowledge can be acquired through professional development experiences, and that the teachers’ use of such knowledge is observable and positively correlated with student achievement.

While the move to research on teachers’ PCK and away from studies of quantifications of mathematical content knowledge has been a major development in the field, many questions remain. The accumulated work on teachers builds a strong case for the claim that teaching is a complex activity and that the relationships among various aspects of knowledge are complex (Borko & Putnam, 1996; Calderhead, 1996; Even & Tirosh, 1995, 2002; Schoenfeld, 1998, 1999). Cognitively Guided Instruction provides an example of research designed to identify knowledge used by teachers in the practice of teaching. Many other studies and programs of research have been focused exclusively on accessing and assessing PCK outside of the context of teachers’ actual teaching practices (on written assessments, for example). This issue of the nature of the research designs used in studies of teachers’ knowledge has become a topic for discussion in recent years, as described below.

1.4. Ways of examining teachers’ knowledge

In a call for new directions in the study of teachers’ knowledge, Ball, Lubienski, and Mewborn (2001) argued that two common approaches to such studies have only limited value for advancing the mathematics education community’s
understanding of what knowledge is held by teachers and how that knowledge is used in practice. The first approach, described earlier, is essentially an attempt to quantify teachers’ knowledge, typically by examining their coursework, number of credits taken, or degrees obtained. As noted above, such methods have not generated clear or compelling findings about the role that knowledge obtained through schooling plays in teachers’ practices. The second approach uses teachers’ responses to various instruments (e.g., surveys, interviews) as data. These studies have generated extensive findings about the knowledge teachers do (and do not) possess (see, e.g., Ball, 1990; Brown & Borko, 1992; Even & Tirosh, 2002; Ma, 1999). As Ball et al. (2001) noted, however, although useful in many ways, these investigations have not examined teachers in actual classroom practice, and have generally not had as their central goal the characterization of knowledge (mathematical or otherwise) that is essential to the practices of teaching (e.g., planning, instructing, assessing).

1.5. Mathematics-specific knowledge used in teaching

The heart of Ball et al.’s (2001) critique of earlier approaches to studying teacher knowledge is that by examining courses taken or administering instruments designed to evaluate mathematics content knowledge, researchers were essentially studying knowledge that was already assumed to be important to teaching — knowledge that anyone who studied relevant mathematics would be expected to know. By not examining teachers in action, however, these research methods risked failing to identify other types of knowledge (including mathematical knowledge) of the sort used by teachers that might not arise through classroom or textbook study of mathematics. The implication of this is that some forms of mathematical knowledge used by teachers in the classroom might not be typically acquired and used by other mathematics professionals, including mathematicians.

Consequently, recent attempts have been made to identify forms of what may rightfully be considered mathematical knowledge that is not necessarily used by or readily available even to expert mathematicians — mathematical knowledge that is acquired through the unique opportunities that teaching affords, especially through the type of work done while making sense of and interacting with students’ written and spoken mathematical ideas. Ball and co-workers have referred to such knowledge as pedagogically useful mathematical understanding (Ball & Bass, 2000; Ball et al., 2001; Hill & Ball, 2004), and Ferrini-Mundy, Floden, McCrory, Burrill, and Sandow (2003) have named it mathematical knowledge for teaching.

These researchers and others (e.g., Mewborn, 2003) argued that deep, insightful knowledge of mathematics is required to make sense of students’ reasoning, respond to unexpected questions, analyze students’ errors, develop meaningful assessments of student learning, and provide on-the-spot representations, examples, or explanations for ideas that arise spontaneously in the real-time practice of teaching. Ball and Bass (2000) argued that such knowledge is required for teachers to handle well the unpredictable yet daily classroom occurrences that require quick judgments, decisions, and responses. Moreover, Hill, Rowan, and Ball (2005) provided what they identify as the first empirical evidence that mathematical knowledge for teaching is positively related to student achievement.

While such forms of knowledge are certainly mathematical in nature, they do not necessarily represent knowledge that professional mathematicians are accustomed to using. Mathematicians value the precision, compactness, and elegance of mathematical explanations, but mathematical knowledge for teaching requires that compact mathematical ideas be “decompressed” (Ball & Bass, 2000; Ferrini-Mundy et al., 2003) into components that are more accessible to students. The issue is not that mathematicians experts cannot have this kind of mathematical knowledge, but that it is not readily acquired by them because they do not necessarily have the opportunities afforded by teaching experience that leads to the development of the particular mathematical insights that are useful to teachers.

Progress in the study of teachers’ knowledge has led to a growing consensus among researchers that traditional forms of teacher preparation that include general courses in mathematics and additional courses in pedagogy are not sufficient for preparing quality mathematics teachers, particularly teachers prepared to accept the challenge of teaching according to contemporary principles of reform. This judgment is growing beyond research into policy and practice as the major professional mathematical societies have recognized that today’s teachers need access to more than just high quality knowledge of mathematics (Conference Board of the Mathematical Sciences, 2001). Continued research is needed, however, to determine more precisely the nature of those additional forms of knowledge needed, and the means by which both pre-service and in-service teachers at all levels of instruction can be supported in developing it (Even & Tirosh, 2002; Hill et al., 2005; Hill, Schilling, & Ball, 2004).
In response to this research need, Ball et al. (2001) called for increased study of the use of teachers’ knowledge in context — in the actual practice of teaching. They asserted that such research is necessary if the field is to move beyond merely observing what knowledge teachers do or do not possess, toward identifying the mathematical knowledge they actually use, as well as how that knowledge contributes to effective instruction.

1.6. Rationale and focus of our research: teacher knowledge at the post-secondary level

If quality mathematics teaching depends on more than quality knowledge of mathematics, then the research cited above may have implications for post-secondary mathematics instruction. Parallel with the developments in the K-12 community, there have been calls for mathematics educators to examine and redesign the content taught as well as the instructional approaches used in undergraduate courses (Douglas, 1986; Seymour & Hewitt, 1997; Steen, 1987, 1989). While some have made efforts to examine and understand the effects of reform efforts at this level (Ganter, 1997, 2001), very little is known about the actual teaching practices of college mathematics instructors (Smith, Speer, & Horvath, 2007). Despite the extensive research on teacher knowledge highlighted above, virtually no related research on teacher knowledge has been carried out at the post-secondary level. The work presented here represents a small, initial step toward identifying the kinds of knowledge — pedagogical content knowledge as well as mathematical knowledge for teaching — required of university mathematicians as they enact reform-minded instructional practices.

As discussed above, researchers have revealed limitations in the conceptual understanding of the mathematics taught by a significant proportion of K-12 teachers in the United States. Because of this, limitations in the mathematical content knowledge of some K-12 teachers may function as a sort of confounding variable to the study of other knowledge necessary for instruction. If, for example, other kinds of knowledge (e.g., PCK, pedagogically useful mathematical knowledge) are essential to effective instruction, the role of such knowledge may be difficult or impossible to detect among teachers whose teaching practices are also affected by limited mathematical content knowledge. It is for this reason that the study of university mathematicians engaged in efforts to reform their own teaching practices can be of significant research interest. Since their content knowledge can rarely be called into question, they offer particularly useful opportunities for researchers to observe what other kinds of knowledge may be essential to effective instruction in research-based, reform-minded ways. We believe that existing research on teachers’ knowledge at the K-12 level and additional research on the knowledge used by university mathematicians in their practice of teaching can mutually inform each other as researchers strive to identify teachers’ knowledge essential to reform-oriented mathematics instruction.

For mathematicians (who typically do not attend teacher preparation programs or take part in teaching-focused professional development), such knowledge is learned primarily through experience as they come face-to-face with the questions and instructional challenges posed by their students. Because of this, however, the pedagogical content knowledge teachers acquire through experience is closely tied to the nature of the instructional experiences they have. What we learn about students through instruction is shaped by the nature of that instruction, so the pedagogical content knowledge supporting traditional instructional methods may not be the same as that which best supports reform-minded instructional practices (Sherin, 2002). Thus, observing experienced teachers in transition to more innovative instructional methods holds the potential to reveal — perhaps by its absence — the nature of the knowledge necessary to support the kinds of learning that are aligned with efforts toward mathematics education reform.

We take a cognitive approach to our analysis — an approach shared by other researchers who examine teachers’ knowledge and the roles that knowledge plays in shaping teaching practices (Borko & Putnam, 1996; Schoenfeld, 1999; Sherin, 2002; Shulman, 1986). In such an approach, knowledge is seen as one of several factors influencing teachers’ goals and the approaches they take to accomplish those goals as they plan for, reflect on, and enact instruction. We consider the acts of planning, in-class instruction, and reflection all to be elements of a teacher’s practice. In the next section, we describe our research design and the particular methods we used to gain insight into the roles that pedagogical content knowledge and other mathematics-specific forms of teacher knowledge played in a mathematician’s efforts to teach in ways that were substantively different from how he had taught in the past.

2. Research setting and design

This research took place in the spring of 2004 as Bernd Rossa taught an undergraduate course in differential equations at Xavier University, a Jesuit liberal arts university of approximately 6,700 students (3,900 undergraduates).
There were 19 students in the class, most pursuing majors or minors in mathematics and/or biology, chemistry or physics. Rossa, a mathematician with 16 years of university teaching experience, had taught differential equations in the past using a traditional text and traditional instructional methods (described in the next section). His dissatisfaction with the learning outcomes he observed of students under traditional lecture instruction led him to seek instructional alternatives. He decided to adopt a new curriculum for teaching differential equations developed by Rasmussen (2002) because it seemed to him to be promising. (Rossa learned of Rasmussen’s work at a professional conference.) Joe Wagner, a mathematics education researcher and teacher in the mathematics department at Xavier, was already familiar with Rasmussen’s work, and he saw in the circumstances an opportunity for research. The introduction of the Inquiry-Oriented Differential Equations (IO-DE) materials to Xavier, and to Rossa’s classroom in particular, provided an opportunity for a mathematician and a mathematics education researcher to collaborate on research to examine the challenges faced by veteran university mathematics instructors in adopting reform-oriented teaching practices in advanced mathematics courses.

Rasmussen’s (2002) IO-DE curriculum distinguishes itself as a rare example of a set of student and instructor materials for a course in an advanced mathematical domain using reform-minded, research-based principles of instruction. The materials include several series of problems, activities, and accompanying Java applets designed to guide students through the discovery of the core concepts of a dynamical systems approach to differential equations. Through group efforts at solving problems and engaging in carefully designed activities, as well as participating in a classroom context in which mathematical argumentation, justification and proof are the norm, students are to learn graphical, numerical and analytic techniques for analyzing, interpreting and solving differential equations. Problems and activities were selected to challenge and encourage ways of thinking about the mathematics, to lead students to discover important ideas on their own, and to build upon ideas in order to propel students along a learning trajectory informed both by the traditional mathematical content of the course and research about student learning. As a result, while the top-level content areas of a traditional course in differential equations were readily visible in the course materials, the “path” through which students were introduced to them differed from most traditional texts.

Preliminary research (Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006) has shown that in classrooms where the IO-DE curriculum materials are implemented with fidelity to the designers’ vision, students’ achievement increases with respect to conceptual understanding. This occurs without cost to the acquisition of analytic techniques and skills in comparison to students studying differential equations under more traditional instructional approaches.

Wagner attended, observed, and videotaped nearly all of Rossa’s differential equations classes during the entire semester, taking notes on the class content and keeping a list of topics he wished to discuss with Rossa. These discussions took place during audiotaped “debriefing” sessions that occurred after most classes. During these discussions, Rossa reflected on his perceptions of how the class went as well as the challenges he faced. Wagner took an active role in offering feedback and advice, and the two often collaborated in planning for the next class. As a “participant observer,” it is likely that Wagner influenced some of Rossa’s decisions about his teaching. Nonetheless, their discussions surrounding the assistance Rossa sought, and the suggestions Wagner made were mechanisms through which examples of the kinds of knowledge Rossa was lacking and developing during their collaboration became apparent. The nature of their cooperative efforts, as seen by each, focused on the issues most essential to Rossa.

At the end of the semester, Wagner asked Rossa to reflect in writing on what Rossa considered to be the biggest challenges he faced in implementing the IO-DE materials. In response, Rossa produced four “stories” about his experiences with the course. Rossa wrote these stories independently and without knowledge of the analytical work that Wagner was simultaneously carrying out on the data gathered about Rossa’s experiences. In addition, Rossa provided a written self-report of what his teaching of this course had been like during previous semesters.

In reviewing his field notes and reflecting on his classroom observations and debriefing sessions with Rossa, Wagner detected themes in his interactions with Rossa throughout the semester. In particular, analysis of his notes and observations revealed that Wagner’s responses to many of the issues raised by Rossa throughout the semester were to encourage Rossa to adopt particular practices in preparation for his classes. These practices often included setting and attending to particular goals for instruction that were new to Rossa or different from those he had grown accustomed to in his prior teaching experience. The goals were further analyzed to determine whether they could be characterized in particular ways. This analysis led to the development of four main categories of goals: classroom orchestration, cognitive/learning, assessment, and content. This categorization of goals...
was the basis for the “taxonomy” Wagner developed to capture the kinds of challenges Rossa faced during the semester.

To examine Rossa’s stories for evidence suggestive of knowledge-related issues, without access to Wagner’s notes, audio-data, or earlier analyses of instructional goals, co-author Speer analyzed the text of Rossa’s stories, identifying statements where issues of knowledge might be present. These conjectures were presented to Wagner who provided supporting or refuting evidence from his data. This was the first step in their collaborative work to identify specific types of knowledge Rossa lacked. Finally, Rossa reviewed Wagner and Speer’s analysis. He corroborated the analyses of the elements of knowledge relevant to the stories, providing only minor edits to the document. This member check (Rossman & Rallis, 1998) was used for triangulation on the analyses and claims.

In this paper, we take the entire semester of the study as one “point in time” and we do not examine changes in Rossa’s practices. We make only small claims about what he learned (as identified by Rossa himself); rather, we interpret the challenges Rossa faced in implementing the new curriculum as indicative of the absence of the knowledge necessary to resolve them. In doing so, we point to specific aspects of knowledge beyond mathematical content knowledge that Rossa needed to learn to enact reform-oriented teaching practices.

We believe that it is of more than passing interest that Rossa’s four stories and Wagner’s four categories of instructional goals demonstrate significant parallels in their identification of the challenges Rossa faced in adopting the IO-DE curriculum and related teaching practices. The parallels between Rossa’s stories and Wagner’s taxonomy represent a form of investigator–participant triangulation, strengthening warrants for our claims. The two perspectives represent not merely two presentations of the same story: they demonstrate independent agreement on the issues of central significance to Rossa’s teaching experience that frame our further analysis of Rossa’s knowledge for teaching. These two, independently produced products of a semester’s collaborative efforts serve as the basis for the analytical section of this paper, and they offer insight into the particular kinds of knowledge that Rossa found himself to be lacking — and slowly developing — as he struggled to effect new teaching practices.

3. Analysis of Rossa’s stories and struggles

In the sections that follow, we briefly examine Rossa’s description of his prior instructional practices when teaching differential equations, and we then present each of Rossa’s four stories in turn, describing the implications each story presents for the types of knowledge with which Rossa struggled. The need for these particular kinds of knowledge is evident in the difficulties Rossa experienced as he tried to formulate and implement particular goals for instruction that are not a typical part of a teacher’s repertoire in traditional, lecture-based classrooms. After each of Rossa’s four stories, we identify particular formulations of goals to which reform-oriented teachers may need to attend but that Rossa was just learning to consider.

We present these goals in the form of a taxonomy consisting of four categories that correspond to overarching themes of Wagner’s conversations with Rossa: *classroom orchestration goals*, *cognitive/learning goals*, *assessment goals*, and *content goals*. Each category represents goals that may be equally important to traditional and reform-oriented instructional practices. Different instructional practices, however, place different demands and constraints on how such goals can be accomplished. In turn, the ways a teacher goes about accomplishing those goals are shaped by the knowledge (of various sorts) the teacher brings to teaching-related decisions. We occasionally offer descriptions of Rossa’s classroom practices to illustrate the ways that particular knowledge-related issues were apparent in his instruction.

Wagner found that in response to Rossa’s questions and perceived challenges, he often encouraged Rossa to reflect on and develop new sub-goals for his classes that would help him accomplish his overall goals for instruction. These sub-goals (or objectives) were distinct from those that Rossa already considered during his usual preparation processes. As we discuss in the analysis that follows, Rossa’s unfamiliarity with such sub-goals can be linked to the lack of certain kinds of knowledge that would not likely have developed through his prior experiences of teaching in more traditional ways.

Instructional goals of different kinds are always at play in classrooms, and we recognize that each story may introduce several such goals. Nevertheless, we believe that each story illustrates best one of the types of goals identified in the taxonomy. The order in which the stories are analyzed reflects the order in which Rossa presented them in his written reflection. In the service of presentation and discussion, we break the stories into parts, but they are otherwise presented in their entirety and without editing Rossa’s own words.
3.1. Prelude: Rossa’s prior and new teaching practices

The changes Rossa set out to make to his instructional practices were substantial. In previous incarnations of the course, Rossa had implemented a mostly lecture-based way of supporting students’ learning. In his own words, the course transpired like this:

I followed the text rather closely, introducing new ideas, definitions, results, and techniques in a lecture style format, followed by examples. Of course, I allowed for questions during class, but the number of questions raised was usually minimal.

Students did work on problems, but as part of their homework assignments or on quizzes:

At the end of each class I gave a homework assignment containing exercises from the text, which were similar to the material discussed in class. Often, the assignment contained reading of portions of the next section in preparation for class. Sometimes I started class with a little “quiz,” which the students worked on in pairs, to see if they could do some of the basic steps discussed in the previous class (e.g. draw a few arrows of a given vector field by hand . . .). Rossa supplemented his lectures with computer demonstrations using Maple. He said he did this “in class to visualize the differential equations as slope fields, to calculate and graph numerical solutions with (varying) initial conditions, to calculate solutions analytically, graph them and compare them to numerical solutions, etc.” Students were expected to learn how to use Maple and to use it to complete two 2-week-long projects (from the textbook) in small groups. Rossa was quite disappointed with students’ experiences in the course. On course evaluations, students complained about “the lack of seeing ‘the big picture’” and the expectation that they become proficient with Maple. These reactions to the course were of particular concern to Rossa because he “had been charged with spearheading a differential equations course that leads away from the very mechanical, technique-focused course [the department] had taught up to then to a more investigative and qualitative introduction.”

In contrast to the lectures and presentation of examples that dominated Rossa’s prior teaching practices in this course, he set out in the spring of 2004 to implement the inquiry-oriented differential equations curriculum materials. Working closely with Wagner and in communication with the principal developers of the curriculum, Rossa structured the course around series of problems and activities with which students engaged in groups during class time. Each activity or set of activities was followed by opportunities for full-class discussions during which students were encouraged and expected to present the results of their work (verbally, at the board, or using technology), justify their arguments, and work toward arriving at consensus on the mathematical ideas at hand. Rossa orchestrated the discussions, posed questions, added contributions of his own, and at times summarized and reviewed arguments for the entire class. He also made sure that as new ideas or practices emerged, they were eventually aligned with conventional mathematical language, practice, and notation. The nature of Rossa’s role in the classroom developed throughout the semester, and this development is discussed further below. As the semester progressed, Rossa continually made deliberate efforts to consult Wagner, Rasmussen, and others involved in the IO-DE project to be sure that he was implementing the curriculum according to the intent of its developers.

3.2. Story 1: Insecurity in class discussions

Of the concerns that came up in discussions with Wagner during the semester, most related to Rossa’s adjusting to the demands of a “student-centered” classroom. As described earlier, the IO-DE curriculum required extensive group work and class discussion. For an instructor accustomed to a more teacher-centered, lecture-intensive style of instruction, learning how to lead such discussions and learning how to be a part of such discussions without presenting oneself as the principal source of mathematical authority can pose substantial and novel challenges. The first section of Rossa’s story about class discussions captured these challenges:

Especially in the beginning of the course, I did not feel secure about how to guide the class discussion. To illustrate, let’s say students worked on a problem and I’d sometimes stop them after a few minutes to ask for some initial ideas. Some initial ideas are shared. What do I do with these ideas? I don’t want to say: “This one is good, this one is not so good, and that one is really off the wall . . .” If I want them to “discover” the mathematics
for themselves, I had the feeling that I should take whatever their ideas were at the time and help them move these ideas into a productive direction, rather than telling them to go back and re-start, re-direct, or to tell them what was wrong.

At a top level, Rossa faced a dilemma identified by Chazan and Ball (1999) as common to many teachers learning to lead class discussions of mathematical ideas. Rossa understood that an important element of the IO-DE curriculum was that students needed to “discover” or “construct” their own knowledge without the teacher merely “telling” them as entailed by traditional lecture forms of instruction. As Chazan and Ball pointed out, however, observing a rule of “not telling” does not offer prescriptive help for what a teacher’s role becomes in leading a classroom discussion. Rossa was well-acquainted with his role in a traditional classroom in which “telling” was the norm, but what he learned through such a role in traditional instruction offered him little assistance in enacting his role in a student-centered environment.

To orchestrate productive discussions, Rossa needed to elicit, use, and coordinate students’ contributions to lead to a desired mathematical end. This also meant that Rossa had to help students learn what is considered a legitimate mathematical contribution to the discussion. Wagner’s observations, especially during early classes, revealed that Rossa regularly elicited input from students in the class, but without guidance or framing, so that very often students responded not by commenting on or critiquing ideas already in play, but by adding more and more ideas to the discussion. The following excerpt from Rossa’s very first day of class exemplifies this issue. Rossa had posed a problem for students to discuss in groups and a student had just offered his group’s initial thoughts. Without reacting to the student’s response, Rossa turned again to the class:

R: Any responses? Any other thoughts? Does that make sense? […] [looks up student’s name] Nodding? Mike, right?
S1: Yeah.
R: Do you want to add to that somehow?
S1: Uh, we just looked at it and our observation, well, his [points to group mate] observation was that system 1, if you say, well, take away all the y’s, then x will start shrinking. And take all the x away, then y will shrink. So they obviously need each other. Because if you take away all the y’s, then it’s, they’ve both got negative derivatives. For system 2, if you take away one species, the other one does fine on its own. So system 2, they obviously are competing; in system 1 they’re cooperating.
R: OK, so, so uh . . . . [to Beverly] Is it clear to you, Beverly, what was just said?
S1: If you plug in zero for one of the variables, as a population, then the other one does fine in system 2, but in [inaudible] in system 1.
S2: [Beverly nods to Rossa to indicate understanding.]
R: Ryan?
S3: I thought it was, [inaudible] system 1 was cooperating, because, like if you put positives in for x and y, like positive rate of change or something, I’m not sure, you would be like, adding a positive number, which would increase the numbers. Um . . . .
R: OK, it’s very interesting, I think you mentioned x and y, you mentioned the rate of change, and you kind of said, well, you’re not quite sure there. I think that maybe we should, you know, there’s x and y, and several of you have talked about, you know, you plug in some values, and then, there’s some talk about the rate of change. Can we, can we get a little bit more clarity on what these things are, on what they are not? How to think about them? Or how you think about them? Charles?

Despite Rossa’s initial request for reactions to the student who first responded, none of the student follow-ups addressed the first student or each other. Rossa’s revoicings of Ryan’s comments were entirely free of evaluation and made without explicit connection to any other comments — he was not eliciting or drawing on students’ contributions to the discussion in ways that guided the discussion in a mathematically productive direction. While Rossa possessed the general pedagogical knowledge and skills to orchestrate a discussion (by asking for comments and reflecting back comments to the class), he appeared to lack knowledge of how students might be thinking about the ideas and/or which ideas he might profitably introduce into the discussion. In this matter, more comments and ideas were placed before the class, without connection or cross-critique. This led to Rossa’s perception that these kinds of conversations were “out of control,” as he described in the second part of his story:
By asking others what they thought about these (“bad”) ideas I hoped to get some other student to re-direct the “poor” ideas into a more profitable direction, or to get some student to say that (. . . and possibly why) the idea in question was misguided, instead of me saying it. But this approach to try to redirect using other students’ opinions took very long and, more often than not, backfired by accelerating movement away from any profitable direction. I felt very much at their mercy, out of control.

Since Rossa knew well the mathematics with which students were working, he knew where he wanted the classroom discussions to end up. Coordinating a classroom discussion along a profitable trajectory, however, requires not just plans for an end, but also plans for the means to reach that end. Rossa’s lack of experience with discussions of this sort left him without a clear sense of what such trajectories might look like and, perhaps more importantly, he had little knowledge about what sorts of ideas students would offer to the discussions. In particular, Rossa did not know beforehand what kind of “bad” ideas might arise in a discussion, so he was not prepared to respond to them except to follow the rule of “not telling” and asking (and hoping) that other students in the class would respond to them. His lack of foreknowledge of which ideas might arise, however, left him with only the most general forms of teacher moves to use in his attempts to redirect the discussion. Wagner observed, and the classroom dialogue offered above exemplifies, that most of Rossa’s efforts to keep the discussion moving were initiated by repeatedly asking students, “What do you think?”

Planning for classroom discussions requires teachers to consider beforehand the ideas they believe need to come from students to reach the instructional goal, as well as to consider how to elicit such ideas if they are not readily contributed by the students. Instructor notes provided with the IO-DE materials frequently name the kinds of contributions that experience has shown students are likely to make, but Rasmussen (personal communication) has observed that first-time instructors often do not know what to do with such information. This was the case for Rossa whose lack of knowledge of likely student ideas or how to make productive use of them even when highlighted by the instructor notes substantially hindered his ability to plan for such discussions. In other words, Rossa’s prior, traditional classroom experience did not support the development of sufficient pedagogical content knowledge that would enable him to construct a reliable, detailed “lesson image” (Morine-Dershimer, 1978/1979) of an IO-DE lesson. Such knowledge would enable him to imagine what mathematical ideas were necessary to lead to the desired end, what moves he could make if those ideas did not readily arise, and how to reply to the “bad” ideas that were inevitable along the way.

Rossa was keenly aware that the class discussions were failing to help him create the kind of learning environment that he imagined was expected when using the IO-DE materials. He also recognized that what was missing was structure for the discussions and that it was his responsibility to provide it:

This led to one of my very significant moments in the course! It took a few meetings to pinpoint this conflict/difficulty of mine, and it took someone on the outside (Joe Wagner) to point out to me that I was wrong! They needed my guidance! It wouldn't bother them if I guided them with some judgment, but that they were actually looking to me to get that guidance! So, by doing what I did, I was not doing my job!

The three parts of Rossa’s story illustrate his need for knowledge other than what he had readily at his disposal. Believing that he was to lead without “telling” left Rossa perplexed about what his role in the classroom should be. He started the course with the rather extreme but not uncommon belief that almost any efforts on his part to direct class discussions in a particular way would interfere with the students’ “discovery” processes. Through conversations with Wagner he came to realize that guidance from his part played a crucial role in directing class discussion and he began to take on that role more assertively. Taking on such a role, however, required not merely this insight, but further knowledge of how to walk a careful line of “guiding” without “telling” — an instructional goal not typically a part of an instructor’s repertoire when teaching primarily through lecture. Consequently, as Wagner’s observations and Rossa’s further stories (below) reveal, Rossa’s insight about his classroom role did not immediately resolve his problems in orchestrating class discussions.

Rossa also experienced challenges stemming from his lack of another kind of knowledge: mathematical knowledge for teaching associated with these topics. In particular, labeling which ideas are “good” and which are “bad” may not be simple if done without understanding the students’ reasoning behind those ideas or their potential to support good reasoning. Knowing which solution path among many correct ones is best to pursue because of the potential for it to generalize later in the curriculum is precisely the kind of knowledge other researchers are designing instruments to assess (see, e.g., Ferrini-Mundy et al., 2003; Hill et al., 2004). Lacking this knowledge, Rossa was unable to anticipate...
or recognize fruitful mathematical trajectories and was unsure of which mathematical ideas were likely to lead students toward a particular mathematical result. This stands in contrast to Rossa’s experience in his traditional lecture courses in which “getting to” a particular mathematical result need only be done in one way — and that way is usually chosen by the instructor in order to serve the instructor’s purpose.

Rossa’s expression of his insecurity with class discussions was a theme in the post-class conversations he had with Wagner, especially early in the semester. In addition, Wagner identified Rossa’s struggles with his role within a student-centered classroom as one of the factors shaping the instructional practices Rossa was using during class. These issues exemplify the “Classroom Orchestration Goals” category in Wagner’s taxonomy, shown in Fig. 1.

The taxonomy of goals is intended to highlight significant questions that instructors such as Rossa may face in their preparations to teach in reform-oriented ways. Paralleling these questions alongside established goals in traditional classrooms highlights the ways in which the familiar patterns and norms of traditional classrooms may fail to carry over to inquiry-oriented instruction. We pose the inquiry-oriented goals in the form of questions because they represent the very sorts of questions that Rossa was unable to answer adequately, thus highlighting his need for new knowledge. The questions are not merely theoretical constructs; they represent the very real questions Rossa needed to learn to ask and to answer.

3.3. Story 2: What are they learning?

Rossa’s second story focused on the challenges he faced in identifying the knowledge students were acquiring through the IO-DE approach:

I saw and knew that there was learning going on, but I (as well as they) had a hard time identifying what it was, because it was different from other more traditional classes. A big consequence of that is: How do I evaluate their progress, both formally on exams and day by day in classroom discussions?

I could tell from the discussions that they were progressing, conceptually, somehow. But if you would have asked me, “What exactly did they learn?” I would not have been able to give a good answer. I realized that we were talking about important ideas in deeper ways than I had experienced in the course when I taught it before, using textbook and lecture: For example the idea that a DE can be represented as a flow field, and that solutions are paths through that flow field, and that a formula for such a path was an “exact solution” to the DE . . . . These ideas were becoming familiar to most of the students, and they were using them freely and intelligently in their discussions. But in terms of concrete ideas or techniques that I would know how to test for, it was not clear to me (and even less to them) what exactly they were now able to do that they couldn’t do before. In fact, several students stopped by and told me that they did not think they were learning anything.

It took the first test (5–6 weeks into the course) to make these items more visible. I was not able to produce a good set of test-questions myself, because I could not judge well what would be both reasonable to ask of them and what would at the same time check for “appropriate progress.”

I trusted Chris and Karen1 (who designed the materials), and I used their test questions. These questions and the students’ responses to these questions (the fact they were able to answer the questions) began to shed light

---

1 Karen Allen, then a graduate student working with Chris Rasmussen. Karen has extensive experience with the IO-DE materials and was an additional source of support to Rossa through several meetings during the semester.
for me on “what are they learning”... This felt very backwards to me! Usually I decide what they should learn, and then test if they did. Here, I asked questions the folks who had prepared the materials gave me, and realized more concretely from the students’ responses what they had learned.

Rossa’s second story raises important questions about assessment in this context, and those issues will be taken up more directly in the next story. Here we focus on Rossa’s attempts to determine the nature of the knowledge being learned by his students. Reform-oriented goals of “teaching for understanding” such as those associated with the IO-DE materials are challenging to enact in part because the notion of (conceptual) understanding is so complex. Rossa was accustomed to identifying “concrete ideas or techniques” to test for because those were the very things most often presented in his lectures. He realized, however, that students were learning something different in this new context, yet he had a difficult time identifying what it was.

He sensed that his students were learning (“they were progressing”), yet he was unable to identify the specifics — and this challenge was substantial enough to make it difficult for him to design the first exam for the course on his own. These challenges appear to be connected to both the context in which learning was happening and the actual content of that learning. We turn first to the challenges that stemmed from the learning context.

The question of “what students are learning” in traditional lecture classrooms is most directly answered with “the material presented in the lecture or textbook.” In the absence of such lectures or a textbook, however, what Rossa’s students learned was a function of the activities with which they engaged. When learning takes place in such a context, appropriate selection of learning (and assessment) activities requires careful attention to the kinds of ideas that those activities are able to elicit. Rossa, however, was not accustomed to attending to the relationship between classroom activities and the subsequent ideas and ways of thinking that emerged from students. Hence, designing assessments based on what students have done and had opportunities to learn while working on the activities was quite foreign to Rossa. He said that things “felt backwards” when the students’ work on the test was the source of insight to him into what they could be expected to know. This experience highlights Rossa’s unfamiliarity with attending both to the kinds of ideas and knowledge that classroom activities and discussions can support, as well as the real-time outcomes of those activities as students revealed their thinking in the classroom.

The content that students were learning was also different from Rossa’s previous experiences with this course. He said he “realized they were talking about important ideas in deeper ways...” but he was not able to specify what was being learned. The intended purpose of the “inquiry” or “discovery” environment went beyond the goal of facilitating the emergence of the final idea that was discovered. The IO-DE curriculum also intended for students to have opportunities to learn (for example) ways of thinking, ways of arguing, and ways of justifying that are typical of the mathematics community and that lead to the ideas that are discovered. These things were not part of the explicit content of other curricula for this course which Rossa had experienced, and this created challenges for him in terms of identifying and describing this content in a way that could be assessed on an exam.

While most traditional curricula for a course in differential equations focus on defining and categorizing types of differential equations and acquiring and practicing specific techniques for solving them, the IO-DE curriculum places more expansive expectations on students. Students continually examine differential equations from graphical, numerical, and analytical perspectives, often rooted in real-world problem situations. Activities and assessments require students to identify, describe, and explain relationships among all these perspectives, offering mathematically meaningful justifications for their conclusions. Students are, indeed, required to develop and demonstrate their ability to carry out a variety of solution methods, but such activities are almost always placed within a larger context of deeper analyses of problem situations; the relationship between the situations, the differential equations and their solutions; and the mathematical and practical significance of solutions.

The challenges Rossa faced with regard to curricular goals suggest, among other things, his need for particular forms of pedagogical content knowledge and mathematical knowledge for teaching. In the traditionally structured course, Rossa began with a list of ideas he wished students to learn, and then he organized class lectures and assignments to provide students with opportunities to learn those ideas. In other words, he knew in advance where he wanted students to end up. Moreover, since he held primary responsibility for determining how the content was made available to students (via his lectures), he could structure class to ensure that students had had opportunities to learn that content by the time of the exam. Having a similar sense of where students are going in an inquiry-oriented course requires an ability (a) to anticipate what students are likely do with particular activities, (b) to anticipate which ideas are likely to emerge when students engage with them, and (c) to be able to recognize what students have opportunities to learn when
discussions do not play out as originally envisioned. The first two of these teacher-tasks rely heavily on pedagogical content knowledge, especially knowledge of student thinking (typical strategies, typical difficulties, etc.) while the third one relies also on mathematical knowledge for teaching that enables one to analyze and recognize the mathematical ideas as they emerge in class discussions. Such forms of knowledge for teaching have been identified by Hill et al. (2005, 2004) as essential to the “work of teaching” and associated with positive student achievement.

In his efforts to figure out what students were learning, Rossa also appeared to sense his own inability to discern the mathematical content that emerged. Mathematicians regularly engage in forms of argumentation and justification typical of the mathematics community, yet whose characteristics are rarely given explicit attention. Learning to attend to these processes — which serve as frameworks for connecting mathematical ideas and promoting genuine sense-making — relies on mathematical knowledge for teaching. Such knowledge is surely used by mathematicians, but it is not often attended to directly in a way required of teachers such as Rossa as they learn to identify more fully the knowledge their students are constructing.

This story offers a direct acknowledgment from Rossa that he was lacking essential forms of knowledge for effective teaching. Certainly teachers want to have clear expectations beforehand of the kinds of knowledge students can construct when engaging in classroom activities, yet Rossa’s unfamiliarity with the curriculum and student-centered teaching practices left him unable to form such expectations — even as he had a clear sense that students were learning the mathematics in a way he found desirable. In other words, they were acquiring mathematical knowledge that was different from Rossa’s prior expectations of student learning, and it was mathematical knowledge that Rossa himself could not readily identify. Because of this, Rossa was limited in his ability to select clear goals for his teaching and for his students. Rossa’s concerns in his second story are highlighted in Wagner’s “Cognitive/Learning Goals” category, depicted in Fig. 2.

What is learned in traditional classrooms is generally thought to be the “knowledge” that is presented to students, and learning is presumably enhanced by clear, well-organized lectures. In reform-oriented classrooms, the teacher’s notes no longer serve as a template for identifying the knowledge that students construct. The boundaries of “what is being learned” are understood to be much broader and require different and deeper forms of attention to the activities with which students engage and the form and content of students’ contributions to those activities. Learning to attend to and identify within those contributions the forms of knowledge that students rarely have opportunity to express in traditional classrooms requires experience that only direct engagement with students’ ideas can provide. Such experience, of course, is not usually available to teachers using more traditional instructional methods.

3.4. Story 3: How much are they learning?

Rossa’s exposure to a much wider variety of student knowledge than he was accustomed to seeing in his classroom created novel challenges for him when it came to assessment. In the previous story, he mentioned the difficulties he faced in his first attempts to construct an exam for the IO-DE class. In the first part of this next story, he notes the additional challenges he faced when assessment naturally occurred in real time during class discussions and activities:

How much should they learn? How much insight is enough? As discussions continued even on topics we had already discussed, new issues arose every time, even though I had considered the issues resolved. In this way I had many surprises about what they didn’t understand the last time a topic came up – maybe I had changed my expectations for what I considered to be good explanations.
The challenges Rossa articulated in this story stem from both the timing of the assessment of student learning and the kind of access he had to the student learning he was assessing. We turn first to issues related to the timing of assessment.

In previous incarnations of the course, assessment occurred primarily through homework and exams. Both of these forms of assessment happen after some amount of student learning is presumed to occur. The IO-DE setting created opportunities very early in students’ learning for Rossa to see and hear how students were thinking about the ideas. These formative assessment opportunities included time while students worked on problems in groups as well as the class discussions that occurred about the problems and ideas. This new window into students’ thinking — both before and after more “standard” assessments such as homework and exams — made assessment more complex for Rossa. For example, he found that as particular mathematical ideas reappeared throughout the course, “new issues” arose that made it possible for him to see that students’ understanding was not as solid as he had believed it to be. When student understanding is measured primarily through traditional homework or exam items, successful performance on such assessments is often taken as sufficient to consider issues of understanding to be “resolved.” In student-centered classrooms in which students are primarily responsible for carrying on discussion, argumentation, and justification, Rossa needed to take on a new responsibility for attending to and assessing his students’ understanding during class, often revealing that their ideas were not nearly as “resolved” as he had anticipated.

The new classroom structures (e.g., group work, discussions) also altered the nature of the student thinking to which Rossa had access. In his traditional lecture environment, the forms of assessment most likely to take place during class time were questions he asked of the class or questions asked of him by students. In that setting, Rossa asked questions to assess how well students were following the lecture. In this way he may have gained further insight about students’ understanding by virtue of the number and nature of the questions students asked. The questions teachers ask, however, constrain the types of student knowledge they can reveal, yet teachers’ own expectations of students’ knowledge further constrain the questions they ask. While teacher-initiated and teacher-directed questioning also takes place in reform-oriented classrooms, the social and collaborative nature of the discussions provides a wealth of additional information about student knowledge and understanding as it reveals itself apart from the constraints of focused questioning. Students can and do, for example, reveal ways of thinking about things that Rossa (in his traditional classroom environment) might never learn about because such understandings lie outside of teachers’ expectations. Only in the freer-flowing environment of extensive classroom discussions are such constraints diminished and students are freer to express their ideas. This situation gave Rossa opportunities to see his students’ thinking and reasoning processes as never before. Rossa’s insights into those processes left him questioning his own assessment skills: he wondered whether students really did not understand, or if he was actually changing his expectations as the semester progressed.

The problem of assessing student progress in real-time posed very concrete dilemmas for Rossa as he strove to keep the course moving forward:

I often was reluctant to move on, because I had the feeling that “they didn’t get what I wanted,” but then, in discussion with Joe (after classes) it turned out that I didn’t really know what else I wanted! I somehow wanted them to make more connections, or be able to produce more careful arguments than before (changing expectations), but I had no clear idea of what, concretely, I was looking for to allow me to feel that we are ready to move on. As a consequence, things dragged on a lot.

At the same time I was aware that we needed to progress in the materials. I often simply made the decision to move on by looking at how much we still had to do, and how much time was left in the semester, rather than being happy with what I saw, feeling that we were ready to move on. The authors had provided some suggested timetable for each section, which told me clearly that the class was moving below the suggested pace.

In the previous section, we saw how Rossa struggled to analyze the class discussion to determine what content students were learning. Here, instead, we see the challenges he felt as he tried to determine at what point the content of the discussions was sufficient to meet the learning objectives of the course. This part of Rossa’s story suggests two particular aspects of the challenges he faced. The first aspect was the difficulties he encountered during class as he tried to figure out when learning was sufficient to move on. He sensed that he did not know what he “was looking for” to inform his decisions. Absent clear indicators or benchmarks, Rossa permitted discussions to continue in hopes that he would see or hear something that would make him feel secure in moving on.

These issues also fed into the challenges he faced between classes as he tried to monitor the pace of the overall course. When he felt the need to move on (for example, based on what was written in the pacing guide), he was unsure...
Fig. 3. Assessment goals.

<table>
<thead>
<tr>
<th>Assessment Goals</th>
<th>What is taken as evidence of learning? What determines pace?</th>
</tr>
</thead>
</table>
| Traditional Classroom | • Traditionally established: assessment occurs “after” learning through homework and exams  
  • Learning measured by success in solving problems and through exhibited knowledge of definitions, theorems, etc.  
  • Pace determined largely by \( \text{pace} = \frac{\text{remaining content}}{\text{remaining time}} \) |
| Inquiry-Oriented Classroom | • How/When does one assess in real time?  
  • What are signs that understanding is “sufficient” to move on? What questions, activities or problems can reveal such understanding?  
  • How rigorous should students’ explanations and justifications be? |

of how to apportion the remaining time and material. Rossa’s default solution to his dilemma was to determine the pace of the course as he had always done in the past: parcel out the material to be covered in the time that remained. In these new circumstances, however, Rossa now found such a solution unsatisfying.

This story illustrates again Rossa’s need for knowledge of students’ thinking. Here, however, the need is not just for knowledge of the understandings they have, but also knowledge of the kinds and quality of understanding that can and should be expected of students. Rossa opened this story with just that question: “How much should they learn? How much insight is enough?” On the one hand, Rossa was seeing a side of his students’ knowledge that he had not seen before, and much of that was surprising and impressive to him. On the other hand, he was also encountering the limitations of their understanding in ways he could not have observed through traditional assessment practices. He felt inclined to clarify and improve students’ understanding as much as possible, but the realities of a student-centered classroom revealed just how challenging such a goal is. New situations continually reveal new ways to use and understand existing ideas, so that no moment of “complete understanding” ever arrives. In his traditionally structured classrooms, Rossa did not face this dilemma because he had much more limited access to what students were thinking. Rossa’s own understanding of what understanding entails may have been at stake. His early expectations that issues of understanding could be considered “resolved” suggests an all-or-nothing view: either the student understands or the student does not. Revisiting issues again and again throughout the course — each time in different circumstances — reveals how it is possible that students can appear to “understand” in one situation while apparently failing to do so in another.

Wagner’s response to Rossa’s dilemma was to point Rossa toward the need for concrete answers to the questions he was posing. “How much should they learn?” and “How much insight is enough?” are very practical questions that demand practical answers in the form of teachers’ goals for instruction. As Rossa noted, however, he was not readily able to answer Wagner’s questions about what he “wanted” to hear from students that he would find acceptable to deem them ready to move on. His concerns about possibly changing his expectations are perhaps a natural response to listening and attending to what students are really saying. Almost every mathematical statement a student makes can open doors to the possibility of further understanding, yet following all such paths is simply not practical in any classroom. Deciding “when to move on” required Rossa to enter a classroom with clear objectives for what he wished to arise in classroom discussion, and what kinds of things he would take as evidence from his students that those objectives had been met. This necessarily entailed bypassing other issues that arose, and at times required him to overlook evidence of misunderstandings that may not have been practical to attend to at the time. These concerns are reflected in the “Assessment Goals” category of Wagner’s taxonomy (see Fig. 3).

This portion of the taxonomy highlights the need for reform-oriented teachers to know enough about how students think about the material being learned so that they might set realistic goals for student progress. This requires them to learn new ways of assessing students’ knowledge, both in real time and through constructed assessment instruments. Hill et al. (2004) identify the knowledge required to assess student understanding as a form of mathematical knowledge for teaching. Rossa’s case, however, highlights the further need for knowledge of not only how to assess, but also what to assess. Rossa struggled to decide which forms of emerging student knowledge were appropriate to pursue at any given time.

This issue was apparent in Rossa’s practice when, for example, he seemed to be searching or “fishing” for particular information during classroom discussions. Wagner commented to Rossa that he seemed to be going “fishing” when
leading classroom discussions, yet Rossa seemed uncertain of what he was fishing for — thereby highlighting Rossa’s need to set clearer objectives. Since understanding always lends itself to further growth, open-ended discussions and questioning often revealed room for such growth. In the absence of clear objectives concerning what sorts of student responses would serve as evidence of understanding sufficient for the course (or for the particular lesson or activity), Rossa continually looked (“fished”) for more. Because Rossa did not enter the classroom with clear objectives for what he wanted to emerge from class discussions, he found it difficult to decide in real time when a discussion had served its purpose and when it was time to move on.

3.5. Story 4: Mapping the course

Rossa’s concerns with timing raised in the previous story were further revealed in his efforts to adjust the sequence of the IO-DE materials or skip particular sections or activities in the interests of time. Decisions about skipping, reordering, or expanding on the materials require particular kinds of knowledge. Again we see how even with a thorough understanding of the mathematical content of a particular course, (pedagogical content) knowledge of how the subject matter fits together into a curriculum that is accessible and meaningful to students did not come automatically for Rossa. This is the theme of his next story, which begins here:

A fourth and last experience I was able to separate out of the overall experience was that I kept longing for some kind of overall “map of the course,” a “concept map” that outlined how all of the ideas addressed and dealt with in the activities fit together to form the course, to form the basic deep understanding of ODE’s [Ordinary Differential Equations] we were looking for. In a standard text book this might be a “table of contents” which you then cover from front to back and you assume that earlier ideas are assumed later on — or something like a diagram of topics, showing dependence between them and outlining a path through the material/course.

Example: I am looking at an activity and trying to figure out how it fits in with everything else we have done (and will do). Does the activity lead to a new idea/concept/technique? Every activity brought out some new idea! Are they all equally important? Important for what? Which one will come up for discussion again later? Will it be sufficient if students figure it out the next time around? What happens if I skip this activity? Can I treat it lightly now?

The IO-DE curriculum is highly structured. The activities and problems were carefully crafted and developed through several years of use to lead students along an expected pathway through major mathematical topics and ideas in ordinary differential equations. The activities and problems were divided into two major units (first-order equations and systems of equations), and further subdivided into sections involving particular “big ideas” of the curriculum. The order of activities was important, and many problems used earlier in the curriculum were explicitly referred to and tied to activities that appeared later. By examining the materials, an instructor could discern the general direction of the curriculum, but without actually working through all the problems and activities ahead of time, it might be difficult to discern the “hidden agenda” supported by those activities that revealed how they are connected across the semester. Still, the curriculum did lend itself to some flexibility, and it is quite possible to omit some activities without compromising the integrity of the course.

The practical needs of moving the course forward sometimes required Rossa to omit materials from the IO-DE curriculum or forgo discussion of some ideas. This was challenging for Rossa, in large part because he did not have extensive knowledge of how the problems and activities were related and fit together to form a coherent whole. Other researchers have described similar situations — situations where a teacher’s knowledge of the broader or deeper curricular or learning goals was insufficient to support instruction in ways consistent with the vision/goals of the designers (e.g., Cohen, 1990b; Cooney, 1985; Sherin, 1996). Rossa was not able to anticipate the kinds of ideas that students were apt to generate simply by examining the problems and activities of the curriculum. In addition, he lacked the pedagogical content knowledge about which of the students’ ideas were likely to be profitable if pursued, what direction they were likely to take, and what long term goals of the course they were most likely to support. As a result, he could not easily or comfortably make judgments about whether such ideas were worth pursuing as they arose, whether they should be postponed for a later time, or whether they should be bypassed entirely. This difficulty is similar to the one Rossa described in his first story — here at a different grain size. In that story, Rossa’s challenge was primarily concerned with orchestrating discussions and emerging ideas toward the objectives of the day’s lesson. In this case, Rossa showed similar concerns about how to orchestrate activities, ideas, and discussions toward the long-term goals for the semester.
Despite Rossa’s expert familiarity with the mathematical content and his equal familiarity with traditional curricular trajectories, such knowledge still left him lacking in an inquiry-oriented learning environment. This had both professional and more personal consequences for Rossa’s teaching experience:

Not being able to answer these kinds of questions kept me from making this course my own, from using my judgment to skip something, neglect something, etc., and it caused the overall feeling of not being fully in control, like “I sense that I always have to paddle hard, in the dark, careful not to make a lot of noise or jerky moves, hoping (indeed trusting) that the river would lead to where I was supposed to go (which in the end I think it did).

These reflections of Rossa’s, perhaps more vividly than all others, reveal Rossa’s own awareness that he lacked particular kinds of knowledge and expertise to teach in reform-oriented classrooms. He felt unable to trust his own judgment about important instructional decisions. Some of these dilemmas surface in traditional classroom experiences as well, but in those circumstances curricular trajectories and textbook designs provide guiding principles. Traditional curricular trajectories are typically more linear, with multiple content perspectives (e.g., analytical numerical, graphical) not as entwined as they are in the IO-DE curriculum, leaving it easier to see how later activities depend on earlier ones. Other instructional decisions were largely absent in teacher-centered classrooms. For example, decisions about which ideas proposed by students should be pursued did not arise as often because student ideas were not so often offered for classroom consideration.

The themes of Rossa’s story are best captured by the next category of Wagner’s taxonomy, “Content Goals,” found in Fig. 4.

This category of the taxonomy serves as a reminder of how much of the content of many mathematics courses has been determined by historical need, by tradition, and by longstanding assumptions about “what needs to be learned before what.” Teaching in reform-oriented classrooms required Rossa to make fresh considerations of what essential content looks like, not only because the nature of the content itself had changed, but because the manner and order of its emergence was much more dependent on student input. Important mathematical topics and ideas did not emerge in the form of completed theorems or precise definitions as they had in his traditional classroom, but came instead in the form of intuitions, insights, hypotheses and even “hunches” put forth by students that would only develop over time. Responding productively in these circumstances required quick judgments by Rossa about the potential of the students’ contributions—judgments that he felt unprepared to make.

4. Conclusions and implications

Rossa’s stories provide insight into the struggles a mathematician may face when trying to enact novel teaching practices. Analysis of the stories reveals the challenges associated with some key aspects of inquiry-oriented teaching and how those challenges can arise from the lack of particular types of knowledge useful for teaching.

In his first and fourth stories, Rossa described the struggles that emerged because he was unable to anticipate how students would respond to particular activities. In the first story, not knowing what ideas students would come up with or how to make use of the ideas that did arise made it difficult for Rossa to orchestrate student discussions in productive ways. He found it difficult to envision where the class discussion would or could go and how the discussions would come together to form a coherent whole that represented concepts students were to learn during that class. Similar issues surfaced in the fourth story as Rossa needed to make decisions about, and monitor the pace and scope of, the course as a whole. He could not always envision how the content or sequence of individual classes contributed to the
instructional goals of the entire course, and he expressed a lack of confidence in his own ability to create a path through the curriculum materials that would lead to the desired student learning.

The second and third stories illustrate other key aspects of teaching. Being able to reliably answer the questions, “What are they learning?” and “How much are they learning?” are central to every teacher’s practice. Rossa was skilled and experienced in answering these questions in the previous incarnations of this course, but the IO-DE curriculum made obtaining answers to these questions very challenging. The new ways in which time was spent in class (on group work or in class discussions, for example) made it difficult for Rossa to rely on his established strategies for monitoring what students were learning. The students’ activities also created learning opportunities that differed from those in Rossa’s previous renditions of the course. Both of these factors made it evident that Rossa’s existing pedagogical content knowledge and knowledge for teaching were not sufficient for him to determine what students were learning when what they were learning differed significantly from the content of a traditional “textbook and lecture” course.

The discussions and inquiry-based activities of the course also played a role in Rossa’s efforts to determine the extent of his students’ learning. Through classroom discussions, he had access to students’ thinking as their understanding was developing rather than merely through homework and exams administered after some amount of learning was assumed to be “complete.” Rossa struggled to make use of the new possibilities for assessment that the class structures provided, in large part because he did not know what “typical” or “productive” student thinking looked like in a not-fully developed form. This was compounded by the fact that Rossa also had access to students’ ways of arguing and ways of thinking that were previously unavailable to him.

Rossa’s stories and our interpretation of those stories from a researcher perspective suggest that deep mathematical knowledge plus knowledge derived from teaching in a traditional manner does not necessarily equate to easy implementation of inquiry-oriented instructional practices. This finding adds credence to the claim of other researchers that there is knowledge particular to teaching that is distinct from knowledge of content and that is not easily constructed from knowledge of content. While many other researchers have established that knowledge (or lack thereof) can play a key role in teachers’ work, we hope that our findings help efforts to focus attention on understanding the diverse set of cognitive resources used in teaching and away from efforts at understanding teachers’ practices that privilege one type of knowledge over others.

We see implications of this work both for research in the area of teacher knowledge and for practice. From the analysis of Rossa’s experiences, it appears that there are some particular aspects of reform-oriented practice that are especially demanding of knowledge of sorts other than content. For example, the challenges he sensed in orchestrating discussions and pacing the course appeared to stem from his unfamiliarity with what students would do with particular tasks and what learning trajectories were plausible to expect from students’ work with the curriculum materials. The challenges he faced in determining what (and how much) students were learning also made evident areas of knowledge he needed. Further investigations that specifically focus on these aspects of pedagogical content knowledge and knowledge of mathematics for teaching may help us refine these knowledge constructs and may also help enrich our theories of how such knowledge shapes teachers’ practices.

The choice to study a teacher with very strong mathematical content knowledge created an unusual set of research circumstances. Since Rossa’s professional preparation was as a mathematician, the difficulties that surfaced could reasonably be attributed to issues with aspects of knowledge other than pure content — a useful feature of the research design for the purposes of data analysis. This suggests that one productive strategy for further examining/refining distinctions between content and non-content knowledge is to involve teachers who have a strong command of one kind of knowledge (either content or non-content) but minimal command over the other kind and study them as they negotiate a challenging teaching situation for the first time.

Close examination and identification of the kinds of knowledge teachers use while teaching in reform-oriented ways can directly inform educators, educational researchers, and curriculum designers who wish to develop support structures for mathematics teachers undergoing changes in their teaching practices. The taxonomy of goals was created in part to serve as a framework for interpreting the challenges that Rossa faced during his first semester teaching with the IO-DE materials. From the start, however, Wagner also intended for it to serve as a practical tool to name and highlight the very real questions to which teachers may need to attend in student-centered classrooms. For example, one cannot simply plan to “have a discussion” about some mathematical idea. Guiding a class discussion requires a teacher to have clear objectives for the outcomes of that discussion, some reasonable expectations for the kinds of ideas students are likely to raise and the directions they are likely to take, plans for how to steer those ideas in profitable directions, and plans for what to do if essential ideas do not arise from the students themselves. Only with such objectives in place is
an instructor prepared to determine during the real-time progress of the class when a particular discussion has fulfilled its purpose and it is “time to move on.” The taxonomy can be used to draw a teacher’s attention to questions such as these as he or she learns not only to teach but also to plan to teach in new ways.

Another potentially fruitful direction for research is to investigate the degree to which instructors can learn to make use of research-based knowledge of student thinking (following in the footsteps of CGI, for example). Wagner and co-workers are currently turning attention to developing better support materials for instructors who choose to use the IO-DE curriculum. While pedagogical content knowledge is often learned primarily through actual classroom experience, teachers who are new to the IO-DE curriculum or other innovative curricula can be given access to some such knowledge ahead of time. Implementation of the IO-DE curriculum has been studied for several years in a variety of classroom settings, so much has been learned about how students will respond to the activities it asks of them. Even though Rossa had access to some of this information, however, it was not clear to him how he could make use of it. Much remains to be learned about how to support instructors in benefiting from the availability of such knowledge.

This window into the relationship between aspects of a teacher’s knowledge and a teacher’s practice was only possible because Rossa was willing and able to identify and articulate his experiences during this process. Exposing not only one’s classroom practices, but also one’s thoughts, feelings, and struggles surrounding one’s teaching experiences is difficult for any teacher. Rossa’s willingness to work with Wagner demonstrates the value of collaborative efforts between mathematicians and mathematics education researchers — collaborations that remain all-too-rare in educational research.

Our primary goal, of course, is to provide learning opportunities for students in which they can develop rich knowledge of mathematical concepts, practices, and skills. Ultimately what is needed is effective support for teachers who desire to implement reform-minded practices in their classrooms. One productive path toward this goal is to link teacher knowledge more directly to teacher practice at all levels of schooling so as to identify the kinds of knowledge most needed for teaching. The study of mathematicians implementing change in their teaching practices offers insight into knowledge-practice connections specific to the college setting while also enriching the mathematics education community’s research base on knowledge needed for effective teaching at all levels of instruction.

References


