Collegiate mathematics teaching: An unexamined practice

Natasha M. Speer a,*, John P. Smith III b, Aladar Horvath c

a Mathematics and Statistics, 234 Neville Hall, University of Maine, Orono, ME 04469, United States
b Department of Counseling, Educational Psychology and Special Education, College of Education, Michigan State University, United States
c Division of Science and Mathematics Education, Michigan State University, United States

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ABSTRACT

Though written accounts of collegiate mathematics teaching exist (e.g., mathematicians’ reflections and analyses of learning and teaching in innovative courses), research on collegiate teachers’ actual classroom teaching practice is virtually non-existent. We advance this claim based on a thorough review of peer-reviewed journals where scholarship on collegiate mathematics teaching is published. To frame this review, we distinguish between instructional activities and teaching practice and present six categories of published scholarship that consider collegiate teaching but are not descriptive empirical research on teaching practice. Empirical studies can reveal important differences among teachers’ thinking and actions, promote discussions of practice, and support learning about teaching. To support such research, we developed a preliminary framework of cognitively oriented dimensions of teaching practice based on our review of empirical research on pre-college and college teaching.

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Scholars in mathematics education have generated a substantial body of literature that provides insight into how K-12 teachers use particular instructional activities in their classrooms. This research has contributed to the education community's understanding of teachers' enactment of, for example, small group problem solving and whole-class discussion. Using these and other instructional activities involves the use of various teaching practices, including, for example, selecting specific content to be discussed, choosing particular examples or tasks, and deciding if and how to modify a lesson in the middle of class. Researchers have examined teachers' beliefs, content and pedagogical knowledge, and other factors that shape how these and other teaching practices are used to carry out various instructional activities. In addition to contributing to our understanding of teacher thinking and practice, this research has informed the design of teacher preparation and professional development programs for K-12 teachers.

At the collegiate level, however, very little empirical research has yet described and analyzed the practices of teachers of mathematics. Because the term "teaching practice" has not been widely used in collegiate mathematics education scholarship, this claim may initially strike readers as simply ill-informed. Certainly there is published scholarship on collegiate teaching. But while some mathematicians have written about their teaching, others have analyzed aspects of their teaching and their students’ learning in innovative collegiate courses, and a diverse body of other scholarship mentions collegiate mathematics teaching, very little research has focused directly on teaching practice—what teachers do and think daily, in class and out, as they perform their teaching work.

The absence of such research has restricted our understanding of collegiate mathematics teaching and of the resources that collegiate teachers, especially beginners, might access to learn about the work of teaching. Much of what exists are materials...
and programs designed by experienced instructors and/or based on other forms of scholarship. These may be valuable resources for novice instructors of collegiate mathematics; however, as has been the case for K-12 teacher professional development, additional insights into teachers' thinking and teaching practices can improve the design of such materials and programs. Major professional societies have called for increased attention on teaching and the creation of professional development resources for teachers of college mathematics (Ewing, 1999; Fulton, 2003), however, the research base on college teachers and their practices that could inform efforts in these areas is small and limited in scope (Speer, Guttmann, & Murphy, 2009). As a result, the design of most existing programs and resources has not benefited from analyses of the practices of college mathematics teachers or examinations of the influences on and development of those practices. In short, the community's efforts to support instructors as they learn to teach college mathematics is often not informed by data and research on what is involved in teaching college mathematics. Research of this sort could be a valuable resource to people who design professional development opportunities for novice college teachers.

For example, research on K-12 mathematics teaching has shown that many factors in addition to knowledge of mathematics (as traditionally understood) and general pedagogical skills shape teachers' practice (Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Ma, 1999). These factors (e.g., pedagogical content knowledge, specialized content knowledge) contribute to the variation among teachers' practices and their students' learning opportunities (Fennema et al., 1996; Hill, Rowan, & Ball, 2005). We contend that this is also likely to be true at the collegiate level and that the practices of collegiate teachers are worthy and fruitful targets of research.

Our central claim—that collegiate teaching practice remains a largely unexamined topic in mathematics education—depends on a particular, yet broad, conception of practice that includes teachers' planning, minute-to-minute classroom decisions, construction and scoring of assessments, and evaluation of student's spoken and written work. We distinguish the work that teachers do within their practice from the instructional activities they use, whether lecture, group problem solving, test review, or computer lab work. Teachers' instructional activities frame and shape, but do not determine, their teaching practice. Our argument is silent on the question of whether any particular instructional activity is the best way to organize mathematics learning in collegiate classrooms.

In the opening sections of this article, we describe our search for descriptive empirical research on collegiate teaching practice. We summarize the few studies we found and, perhaps more importantly, describe categories of closely related work—studies of collegiate mathematics teaching that were either not empirical or not focused on the description of teaching practices. These categories are: reflections on teaching mathematics, that either (1) take the form of memoirs or (2) are more analytic in nature, (3) analyses of the impact of particular instructional activities on student learning, (4) research on the impact of reform calculus programs, (5) analyses of the impact of reshaping classroom norms in innovative courses, and (6) studies of student learning that include prescriptions for teaching. In each case, we highlight how research in the category concerns teaching but does not address issues of teaching practice that we have targeted.

In the second part of the article, we propose seven dimensions of teaching practice that we consider central at the collegiate level and that could shape the nature and design of descriptive empirical studies. Those aspects of practice could be examined using research methods commonly used in research on K-12 teaching, such as classroom observation and interviews with teachers. Four kinds of sources informed the development of this framework: (1) studies of collegiate mathematics teaching (both empirical and not), (2) studies of K-12 teaching practice, (3) standards for K-12 teaching (National Council of Teachers of Mathematics, 1991, 2007), and (4) the first and third authors' experiences in teaching collegiate mathematics.

1. Examples from research on K-12 teaching practice

If collegiate mathematics teaching practice has largely been unexamined, what makes it a worthwhile focus for empirical research, and what has been lost in its absence? We appeal to the much more extensive corpus of research on classroom teaching practice at the pre-college level. Studies that have targeted the classroom practices of K-12 teachers have been productive in understanding the choices and acts of teaching, the factors that shape them, and the design and practice of teacher education. We expect that similar research at the collegiate level holds equal promise for understanding teachers' choices (and their rationales for them) and for aiding beginners by informing the design of professional development. To cite some examples, studies of pre-college teachers' classroom practice have shown that (1) teachers' presentation of problems can substantially raise or lower the level of mathematical thinking required of students (Henning, 1997; Rittenhouse, 1998; Tjebbe, 1997; Stigler & Hiebert, 1999); (2) the timing and nature of teachers' questions affect students' engagement and contributions to class discussions (Fravillig, Murphy, & Fuson, 1999; Lobato, Clarke, & Ellis, 2005); (3) skilled teachers balance students' exploration of new content with their own contributions to move learning forward (Ball, 1993b; Fravillig et al., 1999; Leinhardt & Steele, 2005); and (4) teachers' use of representations of mathematical ideas have been crucial for supporting or limiting their students' learning (Ball, 1993a; Borko et al., 1992). While the analytic focus and results of these studies have varied, they collectively show that there is more to teaching than knowing and organizing content. In particular, teaching involves more than identifying the target content and using certain instructional activities (such as explorations of realistic problem situations, small group problem solving, or whole group discussion) to convey that content. Teaching also concerns how mathematical work with students is planned and carried out within those activity structures.

Though we cite the productiveness of research on pre-college teaching in arguing for research on collegiate teachers' practice, we also acknowledge that there are important differences between college and pre-college teachers and teaching. Collegiate teachers, for example, are less likely to face limits in their content knowledge. On the other hand, they also have less
time with students, making experimenting with new content and activities potentially harder. Even with these differences, collegiate teaching is far from a completely constrained, mechanical process. In the space that is not constrained, collegiate teachers make judgments and decisions, before, during, and after teaching, based on their sense of the content, what their students do and do not understand, and what is possible in the time remaining in their courses. This is the space of teaching practice that we consider worthy of examination and analysis.

2. Teaching practice and instructional activities

“Teaching” is a broad term that can refer to everything teachers do with their students and curriculum materials. In order to clarify what we mean by teaching practice and to focus attention on aspects of it, we first distinguish it from instructional activities. We use these terms in ways similar to others in mathematics education (e.g., Lampert & Graziani, 2009; Roskam et al., 2009) who distinguish between the activity structures that teachers use to organize student learning (e.g., small group problem solving) and the specific work that teachers do (e.g., choosing tasks, asking questions of the students) when they use those structures. In collegiate mathematics education research, “teaching” has conflated these two constructs. Our reading of the collegiate mathematics education literature suggests that this distinction between instructional activities and teaching practice has not been applied and its absence may have contributed to minimal attention to teaching practice and empirical studies of practice. In particular, the effects of instructional activities on learning have been examined, where the actions of teachers using those activities have not.

Instructional activities are the organized and regularly practiced routines for bringing together students and instructional materials (textbooks, whiteboards, overhead projectors, computer-generated graphic displays, etc.) to support students’ learning of mathematics. These activities include teacher-led discussion, lecture, small group problem solving, and individual student practice on exercises. Typically, class meetings are composed of a small number of such activities, chosen and sequenced by teachers. Pre-college lessons in U.S. classrooms typically include three instructional activities: checking homework, teacher presentation of new content, and in class practice (Stigler & Hiebert, 1999). The most common instructional activity in collegiate classrooms, in mathematics and more generally, is lecture (Lutzer, Rodi, Kirkman, & Maxwell, 2007; National Research Council, 1991), where teachers present the content to be learned, orally and written on some display device (overhead projector, blackboard, or whiteboard), and students listen and take notes. Other instructional activities used in collegiate mathematics classrooms may include, for example, homework review, small group problem solving, student presentation of problem solutions, and computer- or calculator-based lab activities.

In contrast, teaching practice concerns teachers’ thinking, judgments, and decision-making as they prepare for and teach their class sessions, each involving one or more instructional activities. It includes their planning work prior to classroom teaching, thinking and decision-making during lessons (e.g., adjustments to the lesson plan made “on the fly”), and their reflections on and evaluations of completed lessons. Teaching practice includes both what teachers do before they initiate instructional activities and what they do within them. At the heart of teaching practice are the anticipations of how students will react to the content presented (in any instructional activity), the adjustments that teachers often make when their anticipations are not fulfilled, and the thinking/decision-making that occurs at these times. When a class session includes only one instructional activity, teaching practice involves the choice of, preparation for, carrying out, and evaluation of that activity. When there are multiple instructional activities in a class session, it also involves teachers’ work across activities, e.g., to allot time between and sequence instructional activities.

The following example from the collegiate classroom illustrates this distinction more specifically. Suppose a calculus teacher has chosen to teach the chain rule via lecture presentation. After presenting the rule in general terms, that teacher might work through a sequence of examples to illustrate how the rule works with different classes of functions. The decision to present these examples as well as the thinking that went into selecting and sequencing them are important aspects of his/her practice. All are aspects of practice framed by the instructional activity of lecture. Perhaps the first example has been chosen to be accessible to all students and subsequent examples become progressively more complex for different types of functions. The teacher’s choice of these examples is based on his/her judgments of what students already know and find challenging; what increments in difficulty are appropriate; and what sorts of forthcoming work (homework and test problems) lie ahead for students. Of course, this work has not taken place in a vacuum. The teacher’s text provides candidate examples for each topic and more experienced colleagues may share them, along with an accompanying logic and argument for their effectiveness. But whatever their original source, our experience has indicated that (1) example sequences vary among teachers for the same topic, even among skilled and experienced teachers, and (2) teachers’ rationales for their sequences—though highly important—remain largely covert. It is less the specific examples in those sequences than the thinking that generated them that could make them useful contributions to the research literature and teacher professional development.

3. The search for empirical research on teaching practice

Our review of scholarship in collegiate mathematics education located only five research studies that have described and analyzed teachers’ classroom practice—as we have defined it. Before presenting those studies as examples of empirical research that describe collegiate teaching practice, we present our search methods, for two reasons. Methodologically, the nature and breadth of our search constitutes evidence for our central claim that collegiate mathematics teaching practice...
remains unexamined in empirical research. Substantively, our search did identify a variety of publications that concerned or discussed teaching at the collegiate level, but were not descriptive empirical studies of teaching practice. We review this literature below as research about collegiate mathematics teaching.

We searched for all discussions of mathematics teaching at the collegiate level that appeared in peer-reviewed publications. We carried out this search in three ways. First, we searched electronic databases using specific keywords (see below). Second, we “manually” searched journals where we expected that such research might appear. Third, we consulted colleagues who have pursued empirical research on collegiate mathematics education to identify articles that had targeted collegiate teachers’ classroom practice. More specifically, we sought publications that satisfied four additional criteria: (1) they reported research (i.e., disciplined inquiry framed by research questions), (2) the research was empirical (i.e., systematic data on teachers and teaching was collected and analyzed), (3) the research included descriptions of teachers’ classroom practice (i.e., teachers’ talk and actions was characterized in some detail), and (4) the research was carried out in mathematics courses taught to a wide range of students (including majors) rather than to specific populations, e.g., pre-service elementary teachers.

We searched three article databases (ERIC, JSTOR, RUME) and Google Scholar using the following search terms: “mathematics,” “college,” “undergraduate,” “university,” “teaching,” “teacher,” or “education.” For example, one specific triad was “mathematics, collegiate, teacher.” Next, we reviewed the following journals for relevant studies, from the present backward in time for a minimum of ten years: American Mathematical Monthly; College Math Journal; Educational Studies in Mathematics; For the Learning of Mathematics; International Journal of Mathematics Education in Science and Technology; Journal of Mathematical Behavior; Journal of Mathematics Teacher Education; Journal for Research in Mathematics Education; Mathematical Thinking and Learning; Notices of the American Mathematical Society; School Science and Mathematics; and Research in Collegiate Mathematics Education. The temporal range of our search of each journal is given in Appendix A.

As noted earlier, there exist many resources and programs designed to assist instructors as they learn to teach college mathematics. This set includes written guidebooks and instructional materials that can be used in professional development (e.g., Case, 1994; DeLong & Winter, 2002; Friedberg et al., 2001a, 2001b; Rishel, 2000), as well as programs offered through professional organizations (e.g., the Mathematical Association of America’s Professional Enhancement Program (PREP), workshops and mini-courses offered at national and regional mathematics conferences, etc.). There are also organizations that serve the needs of college instructors of mathematics (e.g., Project NExT (New Experiences in Teaching), Project Kaleidoscope). These represent just some of the valuable resources that exist for college teachers of mathematics. Their design has generally been based on the collective wisdom that members of the mathematics community have acquired from their teaching experiences. It may be that some of these resources (or aspects of them) are consistent with findings from empirical research on teachers’ thinking and their teaching practices. A serious examination of the extent to which current resources and programs reflect what is known from such research (at the college or K-12 level) is beyond the scope of this article, so we did not review artifacts from these resources in the analysis conducted for this study. However, even if there are extensive consistencies between the design of current resources and research on teaching practices, that does not diminish the need for a more substantial base of empirical research base. Expanding that research base would enable designers of professional development to deliberately set out to make decisions informed by findings and knowledge derived from empirical investigations.

4. Scholarship about collegiate mathematics teaching

In reaching our conclusion that few research studies have yet examined and analyzed teaching practice, we found a diverse body of scholarship that considered collegiate mathematics teaching. Because this literature has not been reviewed in detail elsewhere and because examining categories of related work can help to focus our notion of teaching practice and descriptive empirical research on practice, we present six categories of scholarship about collegiate mathematics teaching and characterize work in each. Broadly speaking, this scholarship fell into two categories: (1) reflections on past teaching and (2) studies of student learning. In the first, we found memoirs of distinguished mathematicians and analytic reflections on teaching in particular courses. In the second, we found studies of the impact of specific instructional activities, research on the impact of reform calculus programs, studies of the effects of reshaping the norms for work in collegiate classrooms, and prescriptive analyses of instruction. Where readers may question the relevance of studies of student learning in this context, we included them in our summary because studies of learning have frequently included some discussion or consideration of teaching. For reasons of space, we present illustrative rather than exhaustive results. In our review, we explain how the

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1 Collegiate mathematics education research is a rapidly developing field with many works-in-progress, including conferences papers and articles currently in review. We chose not to attempt a comprehensive review of this work, for two main reasons. First, it seems important to analyze the published scholarship that was available for readers to examine. Second, reviewing a constantly changing field of work-in-progress would have presented insurmountable challenges to carrying out a principled review.

2 In this example, the specific query using these terms would search for articles were “mathematics,” “collegiate,” and “teacher” appeared as keywords or in the article’s abstract.
studies we discuss fail to meet one or more of our criteria for descriptive empirical research that focused on collegiate teachers’ practice. See Table 1 for a summary of the categories and associated characteristics.

4.1. Category I: reflections on past teaching; memoirs of mathematicians

Many mathematicians have written about their experience of teaching collegiate mathematics, reflecting across years of classroom teaching. One example is Paul Halmos who has reflected (1975, 1985, 1994) on his conversion and commitment to teaching mathematics through problem solving, and specifically via the Moore method (Zitarelli, 2004). He argued that teaching mathematics via lecture was an insufficient means to understanding; students needed to do mathematics. Halmos believed that the “problem approach” was the right pedagogy for any content (not just mathematics). He practiced that approach in standard collegiate courses, and was not deterred by “coverage” concerns or students’ initial resistance (see Halmos, 1985).

Such memoirs refer to classroom events and contain judgments about teaching practices that worked (or not), but they are not empirical, because they are based on the authors’ subjective recollections of classroom events. These recollections do not constitute research data in the standard sense, as they are neither explicit (i.e., written down) nor sharable. Though engaging and useful, they often lack the descriptive detail necessary to support empirical research.

4.2. Category II: reflections on past teaching; analytic reflections on teaching particular courses

Collegiate teachers have also analyzed their own teaching experience, focusing on particular issues of teaching or learning in repeated cycles of teaching particular courses. For example, Harel (1998) drew on his experiences teaching linear algebra to assert the Necessity Principle: students are more likely to learn key mathematical concepts, e.g., linear independence, when the need for those concepts are clear and meaningful to them. He identified three types of needs—for the computation of exact values, for formalization, and for beauty or elegance. His Necessity Principle expresses a conjecture about students’ learning (i.e., that need or purpose is a precondition for learning) that has implications for teaching (i.e., the need for a concept must exist prior to its presentation).

Similarly, Epp (2003) described the evolution of her work teaching basic mathematical reasoning skills. In prior teaching of proof-writing classes, she had found her students did not understand quantification, logical equivalence, and negation, making the construction of mathematical sentences and chains of reasoning impossible. She reported improvement came with her willingness to work between mathematical and everyday language and situations. To interpret and write mathematical statements, her students first had to appreciate the difference in logical language spoken in these two worlds.

A third example is Schoenfeld’s numerous discussions (e.g., 1991, 1994; 1998) of teaching his problem solving course. These discussions have focused on three related themes in his teaching: (1) redirecting students’ sense of mathematical authority from him to the mathematics and their own understanding, (2) building a classroom mathematical community around individual, small group, and collective work to solve difficult problems, and (3) learning the power of Polya-like heuristics in problem solving. Some discussions (Schoenfeld, 1994, 1998) have focused on the nature of the course and his teaching in it, as Harel and Epp have done; others have described the teaching and learning in the course to exemplify some more general point about mathematics education, e.g., the importance of informal processes in doing mathematics (Schoenfeld, 1991).

As with memoirs, these more analytic reflections have been generated from authors’ recollections of their mathematics teaching. But unlike memoirs, analytic reflections have focused on challenges arising in specific courses and have been composed during, rather than at the end of the authors’ careers. Because, like memoirs, they are grounded in authors’ recollections rather than systematic data collection and analysis, they are not examples of empirical research (i.e., their recollections are not sharable data). Moreover, descriptions of teaching practice, when given, are selective (focusing on specific issues) rather than extensive and both memoirs and analytic reflections have been completed after rather than during the teaching of particular mathematics courses. In arguing that neither analytic reflections nor memoirs constitute descriptive empirical research on teaching practice, we do not question their importance as accounts of teaching. Such work is insightful, even as it does not focus directly on what teachers say and do in collegiate mathematics classrooms.
4.3. Category III: studies of student learning; research on the impact of instructional activities

Although lecture dominates collegiate mathematics teaching, other instructional activities have been examined for their effect on student learning and engagement, including computer-based lab work, small group cooperative learning or problem-solving, and the “workshop model.” Research has generally examined the effect on students’ engagement and achievement of adding such activities to lecture presentation, in contrast to lecture alone.

One well-known model of small group learning in mathematics is the “workshop model” developed by Triesman (1985, 1992) at the University of California, Berkeley. A key goal has been to develop minority students’ sense of community and belonging in the college setting—factors that deeply influenced their success in calculus (Triesman, 1985, 1992). The model involves small group problem solving as a major component of work in discussion/recitation sections or as an addition to regular lecture and recitation instruction. In the workshops, students work to solve challenging calculus problems in cooperative groups and participation in the workshop improves student achievement and retention in calculus (Fullilove & Treisman, 1990). Although certain teaching practices are associated with the workshop model, those have not been the focus of published research. What has been examined, however, is the impact of the set of instructional activities that make up the workshop model (i.e., groupwork, extended time on challenging problems, etc.) on student achievement, retention, and attitudes (Asera, 2001; Hsu, Murphy, & Treisman, 2008).

Ahmadi’s (2002) study of group problem solving in collegiate courses (business calculus and finite mathematics) is another example of research on the effects of incorporating other instructional activities into lecture-based courses. The group problem solving students who solved homework problems independently, discussed and resolved their solutions, and turned in a single write-up had more positive effects in course completion and achievement, conceptions of mathematics, and attitudes towards the subject. However, the positive effects of including this instructional activity in lecture-based courses has not been universal. Herzig and Kung (2003) found no significant effects on student achievement and attitude toward mathematics in calculus when traditional instruction in discussion sections (teaching assistants working problems at the board) was replaced with group problem solving. They reported that achievement in sections led by more experienced teaching assistants was generally higher than for those led by those with less experience, independent of instructional approach. As found elsewhere (see below), studies of the impact of instructional activities often suggest that particular acts of teaching within those activity structures are equally, if not more important than the activities themselves in shaping students’ learning opportunities.

In contrast to memoirs and analytic reflections (Categories I and II above), research in this category is empirical and descriptive in nature, but it has not focused on teachers’ practice, either within “traditional” or “alternative” instructional activities. Instead, the instructional activity itself has been seen as the key independent variable influencing the dependent variables of interest—most often, student achievement. Despite the evidence that teaching practice may influence the results (e.g., Herzig & Kung, 2003), variations in how teachers work within instructional activities and the impact of those variations on student learning have not been examined. In short, research of this sort has been empirical and sometimes descriptive of student learning, but not of teaching.

4.4. Category IV: studies of student learning; research on the impact of reform calculus

Despite the prevalence of pre-calculus courses in U.S. colleges and universities, calculus remains the paradigmatic collegiate mathematics course, and for many college students, their only mathematics course. Significant work was completed in the 1980s and 1990s to reform, reshape, and revitalize calculus curriculum and pedagogy (see e.g., Douglas, 1986; Robert & Speer, 2001; Tucker, 1990 for reviews). Extensive attention was then given to comparing the achievement and attitudes toward mathematics of students working with reform curricula to those using traditional curricula (see Ganter, 2001; Smith & Star, 2007 for reviews). These comparisons have generally, but not exclusively favored reform approaches. For example, Bookman and Friedman (1994) reported students in Project CALC (that focused on realistic problems, small group interactions, clear expression of reasoning, and technology to support exploration) outperformed their peers in traditional sections on assessment problems that were stated primarily in words (so they required formulation in symbolic terms) and whose solution required written explanation as well as symbolic reasoning. Those authors also reported that Project CALC students’ attitudes toward mathematics became more positive than their traditional peers, after an initial period of questioning and objecting to the demands of the new program (Bookman & Friedman, 1994, 1998).

Where research in Category III focused on the impact of new instructional activities on student learning, research on the effects of reform calculus has primarily targeted the impact of written curricula on learning. In neither case, however, has much attention been given to teaching, despite some reports (e.g., Brown & Borko, 1996) that the character of teaching may have a greater impact on student learning than curriculum type (reform vs. traditional). Research on the impact of reform calculus has been empirical and descriptive, but has focused on student achievement and attitude outcomes, much less on students’ learning processes (Smith & Star, 2007), and not at all on the effects of teaching. Descriptions of teachers’ practice in reform calculus have been limited to listing the instructional features emphasized by the program, e.g., pose questions that involve multiple representations, require students to explain their answers orally, have students provide detailed written explanations. But examinations of what instructors actually do with the written curricula—has been noticeably absent.
4.5. Category V: studies of student learning; effects of reshaping classroom norms

Recently, some researchers have examined the use of curricula in collegiate courses where a major goal has been to change the nature of students and teachers’ discussions to support deeper and richer learning and reasoning capacity. For example, Rasmussen and his colleagues have analyzed how aspects of teaching in differential equations classes have oriented student learning (Rasmussen, 2001). Using video and audio records of class discussions and students’ written work, they have focused on how the teacher’s efforts to establish and maintain particular norms for classroom work have created different (and more productive) opportunities for student learning (Rasmussen, Yackel, & King, 2003; Yackel, Rasmussen, & King, 2000). When the teacher expected students to propose solutions to problems and explain their reasoning, the nature of classroom discussion and learning changed dramatically (Stephan & Rasmussen, 2002). Though these studies have asserted and illustrated the teacher’s essential role in setting classroom norms and practices (e.g., by considering the influence of particular kinds of teacher questions and prompts), they have not examined teachers’ practice in detail. The focus instead has been on the character of students’ reasoning and participation in the discussions. This research has been classroom-based and empirical, but without a focus on teaching practice.

4.6. Category VI: research on student learning: prescriptive analyses of instruction

The final category of research about teaching has focused on the cyclical design of instructional materials for use in collegiate courses, where materials are developed, used in classrooms, and revised based on evidence of their effectiveness. One example of this research has been grounded in a particular view of how students learn mathematical concepts—the APOS (Action, Process, Object, Schema) framework (Dubinsky & McDonald, 2001). Dubinsky and colleagues have argued that APOS is an invariant sequence of stages that describes the development of students’ understanding of many different mathematical concepts (Asiala et al., 1996; Dubinsky, 1991; Weller et al., 2003). In the Action stage, students manipulate symbols or transform some representation by following specific rules or procedures. Repeating these actions in different contexts supports the development of a more abstract understanding of the general Process. When students come to see the Process as an entity that they can act on, they have reached the Object stage for that concept. In the final Schema stage, students not only have Action, Process, and Object understandings for a concept but can also see how these are related to the corresponding elements in other concepts and can determine which share a particular schema and which do not.

Research based on the APOS framework has been carried out for a variety of concepts in collegiate mathematics, including function (Breidenbach, Dubinsky, Hwaks, & Nichols, 1992; Dubinsky & Harel, 1992), limit (Dubinsky, Weller, McDonald, & Brown, 2005a; Dubinsky, Weller, McDonald, & Brown, 2005b), and infinity (Cottrill et al., 1996). Results have shown that students can progress through these stages of learning and have identified some specific cognitive challenges that they face as they do so. But this research has not focused on the practices of teachers who guide students in their use of the APOS instructional materials. Instead, the analytic focus has been on the instructional materials themselves and the evidence of their efficacy in promoting students’ learning of the target concepts.

5. Research on collegiate teaching practice

As our review of scholarship about teaching collegiate mathematics has shown, there is a diverse and growing literature where researchers have considered aspects of collegiate mathematics teaching. Yet the focus has not been direct: researchers’ questions, methods, and analyses have not generally targeted what teachers say, do, and think about in collegiate classrooms in an extensive or detailed way. However, some examples of research on collegiate teaching practice do exist. Here we summarize five studies that fit our definition—empirical analyses that describe teaching at a sufficiently fine level of detail that teachers and other researchers can inspect and learn from the instructional choices and reasoning of others. In these summaries, we consider the researchers’ questions and data collection methods because these features help to distinguish these studies from the work summarized above. Because they are so few in number, we do not consider that these studies undermine our main claim that collegiate teaching practice is largely an unstudied topic.

5.1. Example 1: an analysis of teaching of problem solving

In addition to Schoenfeld’s discussions of his teaching and students’ work in his problem solving class (1991, 1994, 1998) described above, one study targeted his teaching practice directly. Arcavi, Meira, Kessel, and Smith (1998) observed and documented one complete semester of class meetings and analyzed his teaching practice in the first two weeks of the course. In addition to the empirical data they collected, Arcavi and colleagues’ goal was to understand how Schoenfeld’s goals and instructional decisions came together to create a classroom culture where students engaged in genuine mathematical inquiry and how such a culture was achieved in a short period of time. Admittedly, this course was atypical of most at the collegiate level because there was no prescribed body of mathematics to cover. Instead, the content was a set of problem solving heuristics applied to problems from many different mathematical areas. The results showed that detailed observation and analysis could reveal important aspects of teaching that escape the attention of attentive and reflective teachers (in this case, Schoenfeld himself).
The analysis supported Schoenfeld’s prior claims that students who are skilled only in applying known techniques to solve routine problems can relatively quickly engage in mathematics more deeply, solve more challenging problems effectively, look for and solve other related problems. The analysis identified six different instructional activities used in the course—lecture, reflective teacher presentations, student presentations, small group work, whole-class discussions, and individual student work. These activities structured Schoenfeld’s teaching practice, but did not determine it. For example, the instructional activity of whole-class discussion was used to create several kinds of learning opportunities for students. At times, it was used to reach closure on a problem on which students had already made substantial progress. At other times, whole-class discussion was used as a forum for discussing students’ efforts on a problem with which they were struggling. Analysis of one such episode provided insights into Schoenfeld’s decisions including where he placed students’ suggestions on the blackboard and the order in which he pursued their suggestions. These decisions were all key components of how he structured the discussion to create opportunities for students to learn about a particular problem solving heuristic that would enable them to make progress on the problem.

Similarly, Schoenfeld used the instructional activity of lecture to accomplish a variety of goals. Near the start of the course he caricatured a mathematics lecture as a way of letting students know how their experiences in the problem solving course would be different from experiences that they may have had in past courses. At other times, he used lecture to provide students with information they needed to solve particular problems or to fill gaps in their mathematical backgrounds.

This analysis demonstrated that important elements of practice are situated within instructional activities and must be interpreted and analyzed with that nesting in mind. Knowing that Schoenfeld used whole-class discussions and lecture in the situations described above does not provide the same type of insights as does that description coupled with information about his decisions about what to do (and why) while using those instructional practices. In addition, the analysis revealed that particular acts of teaching, especially early in a course, can shape how students respond to, engage with, and work on mathematics problems.

5.2. Examples 2 and 3: analyses of teaching familiar content using a different approach

The second and third examples come from analyses of teaching practice in differential equations, a course typically required for mathematics majors. Wagner, Speer, and Rossa (2007) analyzed the practice of an experienced professor using a new curriculum to teach familiar content. The materials and classroom practices recommended by the curriculum designers differed substantially from those he had used in his prior teaching of differential equations. The materials included collections of problems and activities designed to guide students, working in small groups, through the generation of the core concepts of a dynamical systems approach to differential equations. The problems, activities, and expected student constructions were informed by research about student learning (Rasmussen, 2001; Rasmussen & King, 2000). The professor could easily recognize and locate the standard topics in differential equations in the course materials, but they were designed to introduce some ideas in a different order than found in traditional textbooks. As a result, he was not familiar with the conceptual path along which students were expected to work and progress.

The professor worked collaboratively with a colleague (the study’s first author) to address and resolve his teaching challenges. The analysis examined how these challenges were related to the new knowledge he needed to teach with this curriculum—knowledge that had not been necessary (or seen as necessary) in more traditional curricula. The focus was on the particular kinds of knowledge for teaching mathematics (e.g., knowledge of typical student difficulties, particular pedagogically useful examples and hints, and student solution strategies) and how its absence made some kinds of instructional decisions extremely difficult. The authors’ data included video records of class sessions, audio records of post-class interviews with the professor, and the professor’s written reflections on his teaching challenges. These provided information about his decisions, the resources (mathematical as well as pedagogical) he drew on to make those decisions, and the aspects of teaching he found most difficult.

In a related study, Speer and Wagner (2009) examined how a professor’s use of whole-class discussions sometimes failed to support students’ learning. The focus of the analysis was on the pedagogical content knowledge and other kinds of knowledge for teaching that were needed to recognize student contributions as potentially productive for advancing the mathematical discussion. Without such knowledge, the professor sometimes missed opportunities to move the discussion towards the mathematical goals for the lesson or inadvertently pushed the discussion away from the core goals he had in mind for the class. Data for the analysis were video recordings of class discussions and audio recordings of post-class interviews that included explorations of the professor’s thinking and decision-making during the whole-class discussions.

5.3. Example 4: an analysis of lecture presentation in real analysis

The fourth example, also a case study of one professor’s practice in a particular course, illustrates the potential utility of research on practice where the dominant instructional activity is lecture presentation. Weber (2004) analyzed an experienced professor’s use of “definition-theorem-proof” (DTP) lecture in introductory real analysis, drawing on classroom observations and records of weekly meetings where the professor described, “what he hoped to accomplish in the upcoming weeks and he explained why he taught the previous week’s lectures in the way that he did” (p. 117). The research goal was quite consistent with ours: to examine the actual, day-to-day, week-to-week character of teaching. The results indicated that the professor deployed three different “teaching styles” within the DTP framework. Those styles were matched to the kinds of knowledge...
that students must develop to write effective proofs, and the order in which he used the styles reflected his views of how students learned the content. One style was the logico-structural and it was focused on the syntactic skills needed to do things that are foundational to proof writing such as unpacking definitions. The procedural lecture style’s purpose was to help students develop the “strategic knowledge” that would enable them to use their skills to construct proofs. When using the semantic style, the professor’s goal was to help students develop the type of rich conceptual understanding of ideas that they would need when a procedural approach was inadequate.

Each of the three styles represented a different approach to presentation of the content, all within the instructional activity of lecture. The analysis included the professor’s descriptions of the goals for these different styles, his decision-making about when to use which type, and his thoughts about how each style created different kinds of learning opportunities for the students. Weber derived a list of (some of) the professor’s beliefs that shaped decisions he made about the need for different styles of presentation as well as the particulars of how and when to employ them. The analysis of what he did in the classroom combined with why he did such things provided insights into how he envisioned the students’ development during the course and how different aspects of learning to prove were chosen as developmentally appropriate at different points in time.

This study provides support for the main conjecture that underlies our call for empirical studies of collegiate teaching practice: that collegiate teachers work in different ways in their classrooms, even when their teaching might be described as “lecturing,” and that their practice and reasoning is worthy of study because it can help others (teachers and researchers) understand how and why teaching happens in certain ways. As with the two previous examples, this study also shows how productive data from classroom observations and discussions with teachers can be in producing useful descriptions of and insights into practice.

5.4. Example 5: an analysis of a novice calculus instructor’s beliefs and practices

Unlike the research on experienced instructors summarized above, our final example is a study of a novice teacher of college mathematics. Speer (2008) investigated the influences of a mathematics graduate student’s beliefs on the instructional practices he used while orchestrating small group problem solving in a calculus course. Having students work collaboratively on challenging problems in groups was a new instructional activity for the instructor. Using video recordings of classes and interviews with the instructor about specific interactions during those classes, the analysis revealed several collections of his beliefs (about teaching, learning, students, and mathematics) that were especially influential on decisions he made as he spoke with and guided the students’ efforts to solve problems. For example, he had particular beliefs about what counted as evidence that students’ understood the mathematical ideas they were working with. These beliefs shaped how he responded to students’ answers and helped explain why he reacted to both correct and incorrect answers with a series of follow-up questions. The fine-grained analyses of video of the teacher’s practices and corresponding interviews illuminated connections between specific beliefs and particular instructional practices that might not have been as evident from observations alone. These findings, illustrating how something as fundamental as one’s beliefs about learning can shape moment-to-moment instructional practices, can help both instructors and researchers understand more about teachers’ thinking and practices.

6. A preliminary framework for analyzing teaching practice

Though our purpose thus far has been to support our claim that research in mathematics education has largely ignored the practice of teaching collegiate mathematics, we do not want to point to a hole in the research literature without taking some constructive steps toward addressing the issue. Rather than simply calling for more careful empirical studies of collegiate teaching, we seek to support the development of such research by describing some aspects of practice that have been productive foci for research on K-12 teaching. We identify seven such dimensions: (1) Allocating time within lessons, (2) Selecting and sequencing content (e.g., examples) within lessons, (3) Motivating specific content, (4) Posing questions, using wait time, and reacting to student responses, (5) Representing mathematical concepts and relationships, (6) Evaluating and preparing for the next lesson, and (7) Designing assessment problems and evaluating student work.

We do not claim that these seven dimensions are sufficient to “cover” the complex work of collegiate teaching. Rather this preliminary framework is our effort to attend to some of the “core” of collegiate teaching—preparing for class, teaching class, evaluating that teaching before the next class, and assessing student learning. In addition, we have approached the development of the framework from a cognitive theoretical perspective. This is just one of the many theoretical orientations that could be used to examine research on teaching practice. We have, however, chosen to narrow our focus to connections between teachers’ thinking and their practices due to space considerations and the substantial body of research on K-12 teaching practices that has been conducted from this theoretical perspective.

In developing the framework, we tried to articulate dimensions that were relevant to the dominant instructional activity of lecture but were also general enough to be analytically relevant to a wide variety of instructional activities. With respect to assessment and evaluation, we have combined constructing problems with evaluating student work because we consider

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3 Interestingly, Weber also reported that the teacher was unaware of the research literature of students’ work to write proofs.
assessment a cyclic process of formulating expectations of what students have likely learned, constructing problems based on those expectations, and evaluating students’ work relative to them. That said, we also considered and set aside other dimensions, e.g., Characterizing the mathematical enterprise and Interacting with students during office hours, to focus on what we considered the core of collegiate teaching. Though both could be considered “motivating the subject,” we included Motivating specific content and excluded Characterizing the mathematical enterprise (except for a footnote) because we take the former to occur more frequently across courses and teachers.

6.1. Allocating time within lessons

While decisions about what content must be covered in mathematics courses are often made by others and/or shaped by course syllabi or pacing guides, collegiate teachers must still make their own decisions about how to allocate time among topics and within individual class periods. The tension between rich content and limited instructional time forces teachers to make hard choices about what to include and exclude (or put off until later) and how long to spend on particular topics and activities. Such choices are anticipatory; they are usually made before each class meeting and adjusted during class. Time allocation decisions are crucial for teachers who use multiple instructional activities because issues of sequence (e.g., which comes first?) and transition (how do I move between them?) must be addressed. But even where lecture is the sole instructional activity, teachers must decide how long to spend on each element of their presentation (e.g., on a definition, method, worked example, and/or theorem) in order to know what is feasible in one class session.

This dimension of practice is non-trivial because collegiate teachers, especially experienced ones, likely do not allocate time based solely on the content they must speak and write on the board. They likely also consider the difficulty of particular elements based on prior teaching and the likelihood and duration of students’ questions (see below). For example, expected student difficulty might influence the number of examples to be presented; motivate the identification of specific mistakes and misconceptions; and/or justify more detailed commentary on the topic. Compared to other dimensions of practice, teachers’ time allocation reasoning and decisions will generally be invisible to classroom observers. So examining this aspect of practice will require access to teachers’ thinking. Relevant data could come from teacher interviews and/or teacher logs where planning for particular classes is discussed or documented.

6.2. Selecting and sequencing content within lessons

Teachers also select the mathematical content they will present to students each class meeting. This aspect of practice is often partially “solved” by textbooks, course syllabi, and pacing guides, that can specify topics and lessons in some detail. But even when content choices are in part given, decisions remain for teachers in choosing and sequencing content. One category of choice relevant to lecture presentation is the choice of examples to illustrate focal concepts or methods. The universe of potential examples is very large, and it is plausible to think that teachers’ choices of examples have consequences for student learning, as Zodik and Zaslasky (2008) suggest from their examination of middle school teachers’ examples. Similarly, the choice of which elements of theory to present and at what level of detail is another broad category of content selection that arguably influences student learning.

Where textbooks provide sequences of examples for most concepts and methods, teachers may set aside some or all of those examples or use them in a different order. Such modifications are likely based on judgments from prior teaching experiences or attention to particular learning goals. Example choice and sequence calls on an important kind of knowledge and we suspect that experienced collegiate teachers (1) choose examples for specific reasons, e.g., to illustrate (or motivate) content and how that content “works” in specific situations (as Zodik & Zaslasky, 2008 report), (2) estimate the time required to present a set of examples and judge their relative difficulty for students, and (3) adjust their presentation when students indicate a quicker or slower grasp of the content.

Though we have not found empirical research that has specifically targeted the example choice dimension of practice, collegiate publications have supported our claim about its importance. Michener’s (1978) analysis of understanding in mathematics assigned an important role to examples, as motivation and illustration for key concepts and results. Selden and Selden (1998) have suggested that the distinction between examples that motivate concepts yet to be defined and examples that illustrate given definitions may strongly affect collegiate students’ understanding. Mason and Watson (2008) have argued that mathematicians have sets of examples that embody their understanding of concepts, but when presented in the classroom, those examples (like those in the text) often remain inaccessible to students. And at the pre-college level, Leinhardt and Steele (2005) have shown how a teacher’s sequence of example x-values influenced her elementary students’ learning about linear functions. A combination of observation and interview methods could reveal teachers’ reasoning about examples in ways that may contribute to our understanding of teaching and also be useful for beginning collegiate teachers.

6.3. Motivating specific content

Although course content is familiar and straightforward for collegiate teachers, that content (including its logical structure, internal connections, and historical development) is often entirely unfamiliar to students. A third dimension of practice therefore concerns how (and if) collegiate teachers introduce and motivate important chunks of content, e.g., parts of the course that extend over multiple class meetings, such as whole chapters or units. “Motivating the content” means providing
a rationale for a sequence of topics to increase students' engagement with that content. For example, a pre-calculus teacher could explain how the course is the study of different families of elementary functions structured from simple to complex and discuss some important dimensions of complexity. A calculus teacher might discuss how the difficulty of applying different techniques of integration varies across functions so that selecting a “good” technique always involves judgment. In both cases, motivation has a “meta” character: it portrays and unifies a body of content from some perspective, rather than presenting any new content.

Collegiate teachers' moves to motivate chunks of content may be substantially limited by the demands of content coverage. But some have developed a focus on motivation, apparently to positive effect. For example, Harel's (1998) teaching of linear algebra, as cited above, became more effective when he introduced new concepts according to the Necessity Principle. His approach takes the state of student's understanding seriously in introducing or relating content. Studying teachers' motivation of specific content could be undertaken with data from classroom observations, but to understand the rationale for teachers' specific motivational choices, data that capture teachers' reasoning, e.g., from interviews or other methods, will be needed.

6.4. Asking questions, using wait time, and reacting to student responses

Teachers of mathematics typically ask their students questions as an integral part of their teaching. We use the term “questions” to mean any teacher request for a response from students on an mathematical issue where the response is expected in a relatively short periods of time, typically only a few seconds. In posing mathematical questions, teachers must also make decisions about what to ask, how long to wait for student responses, and how to react to and evaluate those responses. The impact of teacher questions on student engagement and learning has been extensively analyzed, conceptually and empirically (e.g., Frelilg et al., 1999; Henningsen & Stein, 1997; Lobato et al., 2005; van Zee & Minstrell, 1997). We highlight four components of teacher questioning that have been shown to be important at the pre-collegiate level: (1) frequency, (2) character and intent, (3) wait time, and (4) reaction/evaluation.

Collegiate mathematics teachers may vary in how often they pose questions, from class to class and in comparison to pre-college teachers. Because questions are costly in terms of instructional time (time to pose, wait, listen, and finally respond), teachers may ask fewer questions than they would like. As with other teachers, concern for content coverage creates pressure for collegiate teachers to move along in their lessons. Though various factors may decrease the frequency of teacher questions, it remains the case that all teachers continuously choose whether or not to ask their students questions, minute to minute in every lesson.

When teachers do pose questions to their students, those questions can differ widely in their character and in teachers' goals or intentions. One issue concerns the degree to which questions seek information that the teacher does not know. At one end of this spectrum are “teacher questions,” which seek information that the teacher already knows (e.g., “so when I clear the parentheses, what do I get?”). Their function is to engage students in the teacher's presentation. At the other end are questions whose answers are unknown and when produced by students, are informative to the teacher. They are asked to reveal aspects of students' thinking and guide the teacher's near-term decisions, e.g., “should I move on because they understand or back up a bit?” In high school science, van Zee and Minstrell (1997) have analyzed one class of questions, reflective tosses, which ask students to clarify and enrich a just-offered response or restate what they have heard in a peer's response. Utilizing these types of questions was precisely one of the challenges experienced by the professor examined in Wagner et al.'s (2007) study of the use of a new differential equations curriculum. The professor's desire to rely heavily on student contributions to discussions meant he needed to use questions in different ways than he had in previous incarnations of the course. Intermediate between these extremes are questions for which teachers have their own “answers,” yet remain open to learning from students, e.g., “So if I am integrating this rational function, what's a reasonable technique to try?” With these questions, the teacher's goal is to gauge students' understanding to inform his/her next steps.

Third, teachers differ in how long they are willing to wait for student responses, before they decide to move on or answer the question themselves. In a series of studies, Rowe has shown that pre-college teachers typically wait only one second for a student response (wait time, Type 1) and only another second to react to the responses they have received (wait time, Type 2) (Rowe, 1986). However, when teachers are trained to use somewhat longer wait times, student become more engaged and make richer, more productive responses, as long as teachers do not offer indiscriminate praise (Rowe, 1974). Wait time influences students' sense of their teachers and his/her expectations of them. Very short Type 1 wait times may suggest that teachers are not seriously interested in their students' responses or thinking.

Fourth, teachers must weigh a range of considerations in reacting to students' responses. One aspect of teachers' reactions is their evaluative content—whether the response was taken as right or wrong, or valued or not. For each student contribution, teachers must listen and interpret, decide how to react, and then make their response. All components of teachers' reactions are oriented by the goals or intentions in posing the question in the first place (Schoenfeld, 1998). Three basic choices for teacher reaction are (within an admittedly very large space): (1) request further clarification (i.e., use a reflective toss), (2)

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4 Teachers may also engage in a related task of motivating the mathematical enterprise generally by offering their characterization of what “doing mathematics” involves. For reasons of space, we only mention, rather than develop this dimension of practice.

5 We distinguish these from both exercises and problems that engage students' thinking for longer periods of time (Schoenfeld, 1985).
seek a “second opinion,” or (3) evaluate the response, directly or indirectly. The content of students’ responses may orient teachers’ next moves, e.g., to proceed on to a particular new issue or back up and attempt repair of a problem evidenced in a flawed response. Though teachers’ intentions and goals in asking questions will require interviews to investigate, other aspects of teachers’ practices in posing questions, waiting, and reacting can be analyzed via detailed classroom observations. Understanding the logic behind their reactions will likely require both components: observation of the practice itself and interviews to uncover their judgments.

6.5. Representing mathematical concepts and relationships

Teachers must decide how to display mathematical ideas for students to explore, manipulate, and learn about. As a dimension of teaching practice, representing mathematics includes both what is displayed and how it is displayed. Though others have discussed the latter in collegiate settings (Mason, 2002), we focus on the former because it is so fundamental to teaching and learning. As many authors have noted, three representations have dominated the presentation of real-valued functions and their properties (the primary content of pre-calculus, calculus, and differential equations): tables of values, graphs, and algebraic expressions, especially equations (Harel & Dubinsky, 1992; Thompson, 1994).

Teachers’ choices and use of representations is influenced by their chosen (or assigned) texts but, as with examples, is not determined by them. They select representations likely with an eye to the mathematics itself (what does the graph make salient?) and/or how their students have worked with and understood these representations on similar topics. As the availability of advanced technology increases, teachers also make choices about whether they will continue to rely on static representations and/or explore more dynamic, computer-based representations. Teachers’ use of representations can be examined with data from classroom observations, but as with the choice and sequencing of content, the reasoning and judgment that underlies their practice will require interviews or other data on teachers’ thinking to reveal.

Research on teachers’ deployment of mathematical representations at the pre-college level has been extensive. As elementary content moves beyond whole numbers and operations on them, work within and coordination between representations becomes essential for developing deeper understanding of that content (Ball, 1993a, 1993b; Post, Wachsmuth, Lesh, & Behr, 1985 [for fractions/rational numbers]; Leinhardt & Steele, 2005 [for linear functions]). At the collegiate level, Rasmussen and Marrongelle (2006) have described and analyzed two teachers’ use of graphical representations in differential equations courses via the term “pedagogical content tools.” Their observation-based analysis focused on the use of graphs for pedagogical purposes, not only to depict key mathematical ideas but to move the classroom discussion of the nature of those objects forward. Weber’s (2004) analysis, cited above, showed that an experienced real analysis professor used different representations in support of teaching of proof, depending on the nature of the statement to be proven. His “logical-structural” approach began with writing the definitions for hypothesis and conclusion before attempting to close the gap syntactically and his “semantic style” focused on graphical representations in the hopes that students would develop rich imagery to support their proof-writing efforts.

6.6. Evaluating completed teaching and preparing for the next lesson

Work within the first two dimensions, Allocating time within lessons and Selecting and sequencing content within lessons, generally takes place prior to teaching. But just as collegiate teachers design daily lessons before teaching them, they must also evaluate their plans, particular actions and choices, and their students’ contributions and questions before they teach their next lesson, even when course syllabi strongly influence its content. Depending on many factors, teachers’ evaluation of their just-completed lessons may or may not deeply influence what and how they teach their next one. They may, for example, plan “addenda” to or substantially reconfigure subsequent lessons to address perceived gaps in students’ understanding from a prior lesson or decide to skip or abbreviate subsequent content if students show more mastery than expected. As with the first two dimensions, some of teachers’ evaluation and preparation thinking may be revealed in classroom observations—should they choose to explain it to students. Lesson addenda, for example, may be easily recognizable (e.g., “before we move on, I want to go back and . . .”). But much of teachers’ evaluation of past teaching and preparation for next lessons will require interviews or other means of accessing teacher thinking.

6.7. Designing assessments and evaluating student work

Collegiate teachers are typically responsible for creating assessments to evaluate their students’ learning, though in some cases (e.g., large enrollment courses) tests and final examinations may be designed by others. Here we focus on work that teachers engage in when they create their own assessments, such as periodic quizzes. In an analogous way to selecting and sequencing content, teachers choose (or develop) and then sequence assessment problems for their match to particular elements of content, their estimated difficulty, and their collective coverage of the topics to be assessed. All four design components (match, difficulty, coverage, sequence) are likely influenced by teachers’ reasoning about a body of content.

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6 Mason’s discussion of methods of display focuses primarily on practical issues.
and their students’ apparent understanding of that content. For example, harder problems placed early in a quiz may well lower overall performance as students fail to manage their time well and/or become discouraged and lose confidence (see Schoenfeld, 1985, pp. 100–101). Teachers may also pursue specific goals beyond assessment of learning per se, e.g., to challenge a more successful class with more difficult problems or to build the confidence of a struggling class with easier ones.

We know very little about how collegiate teachers design assessments or evaluate students’ work on them. Moreover, we lack relevant lessons from studies of K–12 teachers’ practice, possibly because K–12 curricula include more pre-designed assessments. Assessment practice will not generally be accessible to classroom observation; interview techniques and/or observation of design sessions will be required. One possible approach would be to interview teachers before and after their students have worked on an assessment. That could reveal a great deal about the teachers’ design thinking, what they valued in students’ responses, how they tried to communicate that valuing, and what they learned from comparing expected and actual performance.

7. Discussion

Our goal has been to draw attention to the need for empirical research that examines and describes the work of teaching collegiate mathematics in detail. To this end, we have characterized prior research and scholarship about collegiate teaching, discussed five example studies that have examined teaching practice carefully and directly, and proposed some important dimensions of collegiate practice that researchers might target in future studies. Similar research in K–12 mathematics classrooms has proven productive for understanding the intellectual work of teaching and how teaching creates different contexts for students’ activity and learning. Though the importance of content coverage in collegiate courses and the frequent use of lecture presentation may constrain collegiate teaching in various ways, at least in terms of the diversity of instructional activities employed, studies of practice can still be productive in documenting teachers’ planning, moment-to-moment classroom decisions, assessment design, and post-teaching evaluations. Such research will require researchers to move into the classrooms and offices of collegiate teachers in order to collect data that can support analyses of practice.

All five examples summarized above have been case studies of individual teachers. While such work is valuable, future work that includes comparative analyses of different teachers teaching the same course and/or the same teachers teaching different courses could also make valuable contribute to the literature base. The results of such studies (both case studies and multi-teacher analyses) can support both deeper understandings of the tasks of teaching and the professional development of collegiate teachers, both novices and experienced teachers who wish to work on their practice. Such findings could help those charged with providing professional development select from among the existing resources the materials that target the specific aspects of teaching practices most appropriate for the teachers with whom they are working. Those who design materials and programs could develop resources to help teachers develop particular practices and/or existing materials could be enhanced as they are revised in light of findings from such studies.

Our framework is provisional by intention and necessity. We expect that additional studies of teaching practice will develop and refine issues within the dimensions we have proposed and possibly identify new dimensions as well. But because research on collegiate teaching practice is thus far in its infancy, we claim only to have identified components of practice that researchers could usefully focus on in many classrooms, including those where lecture is the dominant instructional activity. In focusing on teaching practice, we also acknowledge that other components interact with practice—namely, written curriculum, instructional activities, and assessment structures that are beyond the control of individual teachers—and that many consider these part and parcel of a broader view of teaching. Where those components certainly constrain and shape teachers’ practice and have important effects on students’ learning, focusing direct attention on practice—what teachers say and do before, during, and after teaching—is justified by its sheer absence. Focusing attention there need not and should not preclude attention to other related factors affecting teaching.

Before closing, it is sensible to ask why studies of collegiate teaching practice have been so scarce, despite the diversity of scholarship about teaching mathematics. Though we lack substantial evidence, our conversations with colleagues suggest that a cluster of factors may be responsible. First, lecture can be taken as a description of teaching practice, rather than a common instructional activity within which teaching takes place. Second, the professional culture of mathematics may obscure differences in teaching and forestall discussions of teaching within the set of shared norms. Strong content knowledge and the ability to structure it for students may be taken as sufficient for good teaching. Third, collegiate mathematics teachers have limited exposure to and knowledge of pre-college research, where aspects of practice have been productively analyzed. No matter what the reasons may be, we think it is clear that there is texture and diversity in collegiate mathematics teaching and believe that mathematics education, K–16, will be enriched if more researchers begin to examine and analyze it.

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Appendix A.

See Table A1.

References


