Chapter 8
Mathematics Teaching Assistants
Learning to Teach: Recasting Early Teaching Experiences as Rich Learning Opportunities

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This article focuses on the need to provide professional development activities for mathematics teaching assistants (MTAs) that integrate content-specific approaches to how students understand, think about, and learn mathematical ideas. Research findings at the K-12 level suggest that such an approach will lead to increased undergraduate achievement, however more empirical research is needed to establish specific procedures and guidelines.

Graduate students typically have significant training in their field of research but little preparation to teach. Although many departments and universities provide instruction in the mechanics of teaching, the influences on teachers’ practices are varied and complex. Even though basic pedagogical skills and content knowledge play roles in teachers’ effectiveness, other aspects of knowledge have important influences on teachers’ practices and their students’ achievement. In the area of mathematics education, for example, there is growing evidence that content-specific knowledge of the ways students understand, think about, and learn particular mathematical ideas has an especially powerful impact on what teachers do to create learning opportunities for their students. Most of the research into this area of knowledge (referred to as pedagogical content knowledge (Grossman, Wilson, & Shulman, 1989; Shulman, 1986)), has occurred at K-12 levels. Given that such knowledge is apt to shape college teachers’ practices, we assert that the design of professional development for graduate students in mathematics (and other disciplines) can be enhanced through the application of lessons derived from an analysis of research on pedagogical content knowledge. Taking this approach to graduate student professional development generates several questions: What knowledge of student thinking would help teaching assistants (TAs) provide their students with rich opportunities to learn? How might TAs use that knowledge? How might TAs gain that knowledge? In this paper we take a theoretical look at these ques-
tions, drawing on findings from research on K-12 mathematics teacher practices and professional development to inform a conceptualization of how the preparation we provide could support the growth of TAs' knowledge of student thinking in their discipline.

In the first section of this paper, we examine the work of mathematics teaching assistants (MTAs), identifying aspects that are shaped by their knowledge of student thinking and what the research literature says about such knowledge. Next, we look at how MTAs’ knowledge could be developed and enhanced in the context of their teaching responsibilities. We end by examining how such learning could be supported in professional development programs for MTAs. This analysis provides a “reverse-engineering” perspective on the professional development of MTAs, starting with the knowledge needed in their teaching, moving to the ways in which they might acquire that knowledge, and ending with suggestions about the kinds of professional development that might support them to develop such knowledge. Thus we are recasting MTA professional development from the perspective of instruction in teaching to that of providing the tools to learn from teaching experiences. Our goal is to provide those who plan and carry out MTA professional development activities a new lens through which to view the development of their TAs.

A Scene From The Life of a Calculus Teaching Assistant

We begin with a short, fictional vignette as context for our examination of how MTAs use and acquire knowledge of student thinking. We invite the reader to consider what Beth, an experienced MTA, knows about student thinking, how she draws on that knowledge, and how she might have developed that knowledge.

Beth sits in her office a few moments before she has to go teach calculus. The class is in the midst of the section on sequences. She remembers that at this point last year, her students somehow got stuck in their heads that only monotone sequences (ones that are either increasing or decreasing) converge. This year she wants to help them avoid this difficulty so she carefully chooses her examples to include a non-monotone, convergent sequence. Beth is confident that this example will make the relevant characteristics of convergence clear to her students.

In class, after handling a variety of simple examples and talking through both informal and formal definitions of convergence, she puts her new example on the board:

\[ 1, \quad 0, \quad \frac{1}{2}, \quad 0, \quad \frac{1}{3}, \quad 0, \quad \frac{1}{4}, \quad 0, \ldots \]

Beth says, “OK, now I want you to take a minute to discuss whether this sequence converges or diverges.”

Beth then wanders by a pair of students who are deep in conversation. George thinks it diverges: “The terms aren’t going to zero.” Maria thinks it converges: “No, you see they both go to zero.”

On the spot, Beth is faced with a host of questions. George is exhibiting the ideas she saw last year, but Maria, who has the correct answer, appears not to be reasoning correctly either. What is Maria thinking?

What questions could Beth ask that would prompt the two students to come to a better understanding of convergence? Should she have one or both of them explain their reasoning more fully—and if she asks both of them to do so, who should she ask first? What sequence should she give them to clarify their thinking? How can she use their knowledge of limits (in the context of derivatives and integrals) to elucidate the ideas when it comes to sequences?

Although teaching assignments for MTAs in the U.S. vary greatly by institution and course (see Belnap & Allred, 2009, this volume), the preceding scenario is illustrative of some typical experiences for MTAs and we will analyze aspects of the scenario in the balance of this paper. Let us begin by looking back to the vignette and considering the knowledge of student thinking Beth used. What knowledge did she use to come up with her non-monotone, convergent example? What did Beth draw on to pose the questions she is considering when the vignette ends? What knowledge would she be drawing upon to answer those questions? Let us take another step back. This situation could be a “learnable moment” for Beth. What knowledge might she gain through these interactions, about mathematical content and about student thinking? How should she approach the situation if one of her goals is to acquire additional knowledge of student thinking to inform her decisions when she teaches this topic next semester? Finally, let us take one last step back. How might Beth have developed the knowledge she already has about student thinking about sequences? What professional development might Beth have already participated in to help her make the teaching moves that she did? What professional development could have been provided beforehand to help Beth use this situation as an opportunity to develop her knowledge of student thinking? An overview of this structure is provided in Figure 1.

This Framework provides the structure for this paper. In the first section, we justify our focus on teachers’ knowledge of student thinking by reviewing the research literature on the roles of K-12 teachers’ knowledge (particularly, knowledge of student thinking), relationships between teachers’ knowledge and their students’ achievement, and how professional development can play a role.
in helping teachers develop knowledge of student thinking. The subsequent section takes us on a brief tour through the "Using Knowledge" column of the Framework, looking at how MTAs might use their knowledge of student thinking while they teach. We then move to the "Gaining Knowledge" column, theorizing about how the activities of TAs provide them with opportunities to gain knowledge. Finally, we turn to the last column, making recommendations about how professional development might be structured to help TAs learn from their experiences.

**Research on Mathematics Teachers' Knowledge**

It is often presumed that what is needed to teach mathematics is knowledge of mathematical content plus presentation skills. Research over the past several decades has demonstrated that this view is incomplete and that there are many mathematics-specific aspects of knowledge essential to the effective teaching of mathematics (Hill, Ball, & Schilling, 2008; Ball & Bass, 2000; Carpenter, Fennema, Peterson, & Carey, 1988; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996; Grossman et al., 1989; Hill, Rowan, & Ball, 2005; Shulman, 1986). One aspect of this knowledge is pedagogical content knowledge (PCK). PCK is a hybrid of knowledge of content and knowledge of teaching and learning. It includes, for example, the concepts that are especially difficult for students, the problems that are especially illuminating, misconceptions students are likely to have, and in which order certain topics are typically presented. At the early elementary level, several programs of research have successfully tied increases in one aspect of teacher’s PCK—knowledge of student thinking—to improvements in student performance (Fennema et al., 1996). We hypothesize that similar relationships hold at the college level. In other words, improving college mathematics instructors’ knowledge of student thinking might lead them to provide better opportunities for students to learn which would, in turn, lead to improved student achievement.

In this section we take a tour through the areas of research most relevant to TAs teaching at the college level. We start by looking at the types of knowledge that might matter for MTAs. This leads us into a discussion of teachers’ knowledge of student thinking and how such knowledge translates into improved student performance. We then give a brief overview of programs designed to improve teachers’ knowledge of student thinking and their findings, extrapolating from them the knowledge teachers need to improve student understanding at the college level. Finally, we briefly cover some of the literature on learning on-the-job, which must play a crucial role in TA professional development given the financial realities of higher education. Taken together, these areas of research inform our focus on student thinking and our discussion of the professional development needed to help TAs build the knowledge described in subsequent sections of this paper.

Throughout this literature review, we limit our analysis to work done within a specific theoretical tradition, centered on the role of just one component of teachers’ thinking. In particular, we focus on research conducted from a cognitive perspective, where the emphasis is on how individual teachers think and make instructional decisions, and the influence that PCK has on those processes. Many factors shape teachers’ use of their knowledge (e.g., their beliefs, the constraints and affordances of the context in which they teach, etc.) and other researchers have approached questions of teacher practice and professional development from different perspectives (see, for example, Hativa, Barak, & Simhi (2001) for efforts focused on general teaching skills or McGivney-Burelle, DeFranco, Vinsonhaler & Santucci (2001) for an examination of the roles of beliefs). However, given the success of the cognitively based mathematics education research in fueling improved student learning at the elementary level, we have chosen to focus specifically on research and programs directed at building an individual teacher’s knowledge of student thinking.

**Identifying Knowledge that Matters in Teaching**

Recent years have seen an explosion in research on the knowledge teachers need in order to teach effectively. Early work in this area had proven challenging (Ball, Lubienski, & Mewborn, 2001). For example, studies in the 1970s showed the counter-intuitive finding of no correlation between the number of mathematics courses teachers had taken and their students’ performance (Begle, 1979). Since then, educational researchers have searched for and found other aspects of knowledge that teachers draw on that do correlate with their students’ achievement.

The 1980’s saw the introduction of the term pedagogical content knowledge to describe knowledge needed for teaching a subject that was neither purely subject knowledge nor purely pedagogical knowledge (Grossman et al., 1989; Shulman, 1986). Two important components of PCK are knowledge of

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_Figure 1: Framework of Use, Acquisition, and Development of Knowledge of Student Thinking_
student thinking and knowledge of the curriculum. In the vignette above, Beth exhibits such knowledge when she chooses an example based on the misconceptions she knows her students are likely to have (knowledge of student thinking). She might also be drawing on knowledge of the curriculum; sequences with every other term equal to zero appear in the Taylor expansion of sine and cosine which her students will be studying shortly. Her decision-making process reflects interactions between her knowledge of mathematics and her knowledge of students' typical experiences while learning particular mathematical ideas. This portion of the vignette corresponds to the (planning, using knowledge) box in the framework in Figure 1. (We use the standard (row, column) notation when referring to specific boxes in Figure 1.)

The Role of Knowledge of Student Thinking

In the early 1980s, a group of researchers at the University of Wisconsin at Madison began to design and study professional development programs for in-service teachers structured around research on how early elementary students approach addition and subtraction problems. This work eventually evolved into the program known as Cognitively Guided Instruction (CGI). Improving teachers' knowledge of the relationships between problem types and students' solution strategies improved teachers' abilities to assess children's knowledge and adapt instruction based on that knowledge (Fennema, Franke, & Carpenter, 1993). These gains in knowledge and changes in instructional practices were linked to increases in student achievement (Fennema et al., 1996).

The CGI researchers developed an assessment tool to measure teachers' use of student thinking in their classrooms. Level 1 teachers characterize student thinking only in terms of procedures students had been taught. The remaining levels described teachers with increasing attentiveness to student thinking, culminating with Level IV teachers who let children's thinking drive instructional decisions and create opportunities to build greater knowledge of their particular students' thinking (Franke, Fennema, & Carpenter, 1997). These most advanced CGI teachers had created a feedback loop, using knowledge of student thinking to plan student activities and manage classroom discussion in ways which helped them learn more about student thinking—which further informed their instructional decisions.

This program of research examined all three columns in the framework from Figure 1 (using knowledge, gaining knowledge, as well as the professional development that can support teachers' learning) and focused in particular on how knowledge was manifested in teachers' in-class practices (as represented by the "instructing" row of the framework) and planning practices. Beth's teaching choices in the vignette could be evidence of Level IV teaching, as she used knowledge of student thinking to guide her construction of an example. The vignette closes before we can determine if she will use George and Maria's responses to build further knowledge of student thinking, but that potential exists. In the case of the CGI teachers, researchers have used their framework to connect teacher understanding of student thinking with student success:

This study provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. These findings, when viewed in conjunction with those of other studies, provide a convincing argument that one major way to improve mathematics instruction and learning is to help teachers understand the mathematical thought processes of their students. It also appears that this knowledge is not static and acquired outside of classrooms in workshops, but dynamic and ever growing, and can probably only be acquired in the context of teaching mathematics. (Fennema et al., 1996, p. 432)

Other researchers have come to similar conclusions: knowledge of student thinking can be a fruitful focus for professional development (see, e.g., The Purdue Problem-Centered Mathematics Project (Cobb, Wood, & Yackel, 1990) and Summer Math for Teachers (Schifter, 1993).

Although the above work focuses on the K-12 level, the guiding principle—that Beth and other college instructors will provide a more productive learning environment if they have greater understanding of and attend more to student thinking—would seem to apply at the college level as well. While studies have yet to verify this claim, some research suggests that many MTAs do not possess this rich knowledge of student thinking (Speer, Strickland, & Johnson, 2005; Speer, Strickland, Johnson, & Gucler, 2006). In that study, MTAs, including those with several years of teaching experience, were largely unable to identify solution strategies other than their own and were generally unaware of many of the conceptual difficulties detailed in the literature.

There is, however, hope. Research on TAs who have had significant experience facilitating group work in calculus courses suggests that TAs can gain rich knowledge of student thinking during their graduate school teaching experiences (Kung, in press). These TAs reported that observing students working on challenging problems afforded them the opportunity to develop knowledge of student thinking—which they used in teaching later courses.

The above research, when taken together, suggests two things. First, improving TAs' knowledge of student thinking is likely to improve their teaching and their students' learning. Second, this goal is necessary (novice MTAs largely do not possess this knowledge) and reachable (MTAs are able to gain such knowledge during graduate school).

The question then becomes: How might TAs come to learn about student thinking? For possible answers, we turn to the literature on professional development in mathematics education, noting, however, that the research community has only focused specifically on teachers' knowledge of student thinking for the last twenty years, and so it is not surprising that we do not yet have an
extensive understanding of how that research can be used in professional development activities.

Programs to Develop Knowledge of Student Thinking

If knowledge of student thinking can be a powerful catalyst to improve student work, the natural question becomes, “How can a professional development program best help college mathematics instructors gain such knowledge?” In the tradition of Houle, we view such professional development as long-term, extending throughout one’s career (Houle, 1980); however, given the relatively short time that graduate students have access to professional development, we focus here on more limited programs that concentrate on building knowledge of student thinking.

One successful effort to support teachers’ acquisition of knowledge of student thinking is the CGI program mentioned earlier. The core of the program is to provide in-service teachers with a set of frameworks that categorize key problems in the area of addition and subtraction problems, along with the strategies children use to solve them (Carpenter et al., 1988; Carpenter et al., 1989). Program activities typically include discussions of the research basis for the frameworks, structured interviews with children to assess how their strategies fit within the framework, discussions of those interviews, and practice writing word problems that fit various parts of the framework. Follow up activities are also typically provided and usually include meetings among a team of CGI teachers all at the same school.

The results of the CGI professional development efforts are impressive. “Teachers realized that they needed to listen to their students’ mathematical explanations, create strategies and questions to elicit those explanations, and understand enough about children’s thinking and the content to know what to do with what they heard” (Franke & Kazemi, 2001). Furthermore, as is the hope for Beth above, being attentive to students’ thinking created opportunities for the teachers to continue to develop their knowledge while teaching. In other words, their teaching practices became generative of new knowledge relevant to teaching. “It transformed teachers into learners. They learned in the context of their practice about the teaching and learning of mathematics…” (p. 104). The effects of the program were strong enough that in a follow-up study, four years later, some teachers were still engaged in generative growth of the sort targeted in their original professional development activities.

Learning from Teaching Experiences

Just as doing mathematics creates opportunities to learn mathematics, “doing teaching” creates opportunities to learn to teach. Understanding how people learn from their experiences and how such learning can be supported through instruction is the focus of work in the area of experiential learning (Dewey, 1938; Kolb, 1984). Two lines of inquiry shed light on the mechanisms by which mathematics teachers learn from their experiences of “doing teaching” and the features of professional development that support such learning. Sherin (2002) examined the implementation of high school algebra curriculum materials and found that the teachers used three different types of content knowledge: subject matter knowledge, knowledge of the new curriculum materials, and knowledge of student learning. These types of knowledge were neither completely distinct nor static for these teachers, and they drew upon different types of knowledge at different times. Analysis of the data indicates that observing students using novel approaches can catalyze changes in teachers’ thinking. She concludes that, as was the case for Beth, “novel student ideas have emerged as a key trigger for teacher learning during instruction” (p. 145). This program of research illustrates how professional development can help teachers gain knowledge in the context of instructing and reflecting on that instruction (captured in the Figure 1 framework by the intersection of the “instructing” and “reflecting” rows with the “gaining knowledge” and “professional development support” columns.)

Finally, Little and Horn (2007) have examined discourse among teachers as a way of investigating the types of interactions that prove powerful enough to generate new understanding. This is yet another example of how teachers can learn through reflection on their instructional practices. While many conversations involved teachers “normalizing” colleagues’ experiences (i.e., reassuring them that their difficulties were not uncommon), only some of these conversations went further to open up “opportunities for learning in, from, and for practice” (p. 4). Productive groups followed the normalizing comments by explicitly eliciting greater detail and initiating an analysis of the situation. In these “transformative” instances, these questions served as a crucial transition between non-productive and productive interactions. This work suggests that there are types of conversations Beth might have with other TAs to help them all interpret and learn from the interactions they have with students. PD could be structured to help facilitate such conversations.

Taken together, these studies provide a basis from which to extrapolate to the context of novice TAs teaching in college classrooms. Although differences certainly exist between the K-12 setting of the studies above and the postsecondary context of TAs, the commonalities in mathematics and teaching provide a fertile ground for examining the work of MTAs and the types of professional development that might be provided to help them learn more from their own teaching. As such, the literature sheds light on the opening vignette and demonstrates how such activities might help Beth pursue her own teaching questions in ways that are generative of knowledge that can improve her students’ opportunities to learn.
How TAs Use Knowledge of Student Thinking

In this section, we take a closer look at the “using knowledge” column of the framework and examine the activities of TAs with a focus on the many ways in which their teaching could be informed by knowledge of student thinking.

Graduate students who have instructional responsibility plan for that instruction, enact that instruction in classrooms with students, and plan for their next class by taking into consideration what happened during class that day. As we saw in the case of Beth, teachers engage in a teaching cycle, planning, instructing, reflecting, and planning again. Knowledge of student thinking is potentially useful in all three phases of this process. Here we detail how TAs might make use of such knowledge. Throughout, we will refer to Beth’s situation in the vignette, without restricting ourselves to that particular situation.

Reflecting. In the vignette we stepped into the teaching cycle at the reflecting stage. Beth, while reflecting on her previous teaching experiences, used her general knowledge of student thinking to interpret students’ actions as a misunderstanding of the definition of convergence of a sequence. Beth’s knowledge of student thinking provided a lens that enabled her reflection on past classes to shed light on how students approach sequences. Beth might return to the act of reflection at the end of the vignette, processing how her class transpired in real time. She might consider the students’ reactions and her teaching decisions, using knowledge of how students think about the ideas in question as an interpretive lens.

Planning. Now we move to the top of the “using knowledge” column and look at Beth’s use of knowledge as she planned for class. In the vignette, Beth considered experiences from her previous time teaching sequences as she planned the examples to use with her class. Recalling the specific difficulties that students had with situations involving particular kinds of sequences, she decided to construct an example to use in class. The example that Beth created was intended to help students make the relevant distinctions and strengthen their understanding of convergence. When she made the decision to include such an example, Beth considered a variety of factors, including where students were in the process of learning the idea, which kinds of sequences they had already worked with, and what ideas the created example was likely to trigger.

Instructing. Once in the classroom, plans may play out just as the teacher had envisioned or modifications may occur in the midst of class. As each part of the plan is implemented, the outcome is compared with the goals for the class and then the decision is made about whether or not to alter the plan and how. For example, in the middle of class students may ask questions that indicate they have misunderstood an idea from a previous class (or course). In such situations, teachers must decide whether to change their lesson plan to address the students’ difficulties. Such decisions can be shaped in significant ways by what a teacher knows about how typical students think about the ideas, how the ideas relate to students’ prior knowledge, and how the current ideas will be built upon in future lessons. Teachers who possess richer and more detailed knowledge of student thinking are positioned to make decisions that are better informed and more closely tied to how students are likely to be experiencing the lesson.

In the vignette, the initial part of class seemed to go as Beth had envisioned. As students discussed her special example, however, she faced some unanticipated decisions. When the discussion between the two students generated opposite answers, Beth needed to decide, on the fly, how to respond in ways that would work toward her goal of resolving the confusion that she believed (from her knowledge of student thinking) students were having about the nature of monotonic sequences and convergence. Deciding what to do next could be guided by interpreting the students’ comments correctly, using knowledge of typical student misconceptions. For example, when Maria says, “They both go to zero,” she is probably interpreting the sequence in question as a combination of its two disjoint subsequences, a misconception detailed in the literature on student thinking (Tall & Vinner, 1981). Knowing this, Beth’s response might be, “Interesting, is this one sequence or two?” then she could leave the students to investigate the source of their conflict. In doing so, she would be using her knowledge of student thinking to guide her instructional choices.

TA Activities to Generate Knowledge of Student Thinking

As illustrated above, a TA can have ample opportunities to use knowledge of student thinking in the course of going through the teaching cycle. Where might a TA gain that knowledge? Given the realities of graduate school, the most realistic answer is “on-the-job.” This section is a tour of the “Gaining Knowledge” column of the framework, providing an examination of how the activities TAs engage in might provide opportunities for them to learn.

Here, we analyze the teaching activities of college mathematics teachers, looking for experiences that could be generative of knowledge of student thinking. Beth’s situation illustrates some of these: reflecting on past experiences and listening to students discuss a problem during instruction. Here we explore some of the many other opportunities in typical MTAs’ activities. Although we are guided in our analysis by the literature discussed above, this section is largely a theoretical exercise as we unpack the activities of TAs from our reverse-engineering perspective.
Writing and Selecting Problems

As MTAs look at potential problems or contemplate how to write a problem to address a particular topic, they might consider how the problem is situated in the overall curriculum and how aspects of the curriculum fit together. This kind of work would be included in the Planning, Gaining box of the Framework from Figure 1. For example, when choosing a problem for a calculus quiz, an MTA might come across one that initially seems appropriate. The problem, however, relies on both the product rule and the chain rule. It occurs to the MTA that students have not yet reached the chain rule section of the course, and thus the MTA determines that the problem is not appropriate for this quiz.

In these circumstances, MTAs also have opportunities to consider and learn about the relative importance of topics within the curriculum. When constructing her problem for class, for example, Beth had to decide that this particular aspect of understanding sequences was of sufficient importance to warrant time in class. In a similar fashion, when an instructor writes a quiz, decisions need to be made about which of the many aspects of the topic merit being assessed in the finite number of available questions. No assessment can include all aspects of a topic and no class can address all ideas related to a concept, so aspects must be prioritized. To make such decisions, MTAs need to decide how important the different aspects of the topic are in the field of mathematics in general as well as how necessary they are in equipping the students to be able to learn the topics that are still to come in the course. Making such decisions makes use of and builds knowledge of how the discipline and the particular course are structured and how the different concepts or topics connect to and support one another.

Observing Students Working on Problems

After planning for class, MTAs' opportunities to acquire knowledge continue during their in-class interactions with students, represented by the (Instructing, Gaining) cell in the Framework. As suggested in the vignette, when MTAs observe students who are working on problems, they have opportunities to access many kinds of knowledge of student thinking. Such access to how students think about content, in real time, creates rich opportunities for MTAs to build a mental catalog of ways that students think about and make sense of particular ideas. In the research and professional development programs discussed earlier (e.g., CGI), these are the very sorts of experiences that enabled teachers to acquire detailed knowledge of how children think about particular mathematical ideas. In Beth's case, she knew from a prior teaching experience that many students did not fully grasp the concept of convergence, but she did not have a clear idea of what specific thinking was behind students' incorrect reasoning. Observing George and Maria's argument could potentially shed light on that question, giving Beth the opportunity to gain knowledge about student thinking related to this particular content area; in the vignette, concepts of monotonicity, convergence, and sequences are all evoked.

In addition to learning how students think about particular content and the errors and strategies they use while working with that content, observing students as they work on problems creates opportunities to see how students think about prior content as they connect it with new ideas. For example, while watching students practice techniques for differentiation, MTAs may observe the varied ways in which students approach simplification of algebraic expressions or they may notice students' implicit use of the idea of limit. In these situations, MTAs have opportunities to learn how concepts from earlier in the course or prior courses interact with the learning of the content at hand. Armed with such knowledge, MTAs can anticipate difficulties students will encounter and design instruction that helps students overcome those lingering difficulties while also making progress on understanding new ideas.

Finally, observing students working on problems potentially allows TAs to build knowledge of the many coping strategies students use to "get through" problems, with or without understanding the underlying concepts. For instance, Beth might watch a student calculate the millionth term in the sequence and round to the nearest integer to find the limit of the sequence, a theoretically flawed procedure that would happen to work for the vast majority of sequences seen in a calculus course. In addition to gaining knowledge of students' coping strategies, MTAs who observe students working on problems have opportunities to acquire knowledge of the relative difficulty of different tasks.

Discussions with Students during Class

Discussing mathematics and mathematical problems with students during class also presents especially rich opportunities for MTAs to learn about student thinking. Asking open-ended questions, having students discuss a question in pairs and then share thoughts with the class, and other teaching techniques provide teachers with access to student thinking. Even while answering homework questions, TAs might elicit (and learn about) some student thinking with many of the same questions suggested by the CGI program.

Planning

As discussed above, the planning portion of the teaching cycle might provide opportunities for TAs to generate knowledge of student thinking. However, planning activities could be made more generative in several ways. Having groups of teachers do their planning together to learn from each others' insights into student thinking could provide an opportunity to build new knowledge of student thinking. A teacher's planning could also include reading research on student thinking and how it might be incorporated into their classroom. Finally, some Just In Time Teaching (JITT) techniques—which could be
Generative of new knowledge of student thinking—involves using on-line, pre-class student responses to plan instruction (Novak, Gavrin, & Wolfgang, 1999).

Grading Student Work

One of the major teaching activities TAs engage in is grading. Graduate students often have responsibility for grading homework, quizzes, and/or exams. While grading, MTAs assess the extent to which the student’s work is correct and/or represents an understanding of the concepts being tested in order to determine how many of the allotted points the answers merits. This kind of diagnostic work requires that MTAs try to imagine the student’s thought process that lead them to create the written work on the paper. Thus reflection on the grading process creates opportunities to build knowledge of both how students typically think about the ideas as well as the typical difficulties that students have while learning these topics. Although the details of students’ thinking may not be as clear as they can be during observations of students working on problems, grading can provide an opportunity to examine the results of their thinking.

Office Hours and One-on-One Tutoring

Graduate students frequently interact with students outside the traditional classroom in office hours and in drop-in tutoring programs. This is another example from the (Instructing, Gaining Knowledge) box in the Framework in Figure 1. While the approaches taken during these encounters vary, the general goal is to figure out what students are struggling with and help in ways that are more tailored to their specific issues than is possible in a classroom setting. The kind of diagnostic work that occurs during a class or during grading could possibly occur in a more intense way in these one-on-one settings since graduate students could ask a series of questions of the student until the nature of the difficulty is uncovered. These interactions may make use of the graduate student’s knowledge of how students think about the topic while also helping to build on that graduate student’s knowledge of student thinking. Potentially, these situations are rich sources of such knowledge.

Implications for TA Preparation, Professional Development, and Graduate Programs

Teaching takes more than knowledge of mathematics and presentation skills, and it is impossible to completely prepare anyone for the enormously complex task of teaching. However, rather than just sending graduate students into the classroom with the expectation that they will eventually acquire the necessary knowledge, we can equip them with the skills and dispositions to seek out and acquire that knowledge in more efficient ways. Here we set out the principles suggested by the literature review and our analysis of TAs activities. Given the successes of K-12 professional development programs that focus on student thinking and the evidence that TAs do not begin their graduate careers with such knowledge, we suggest that professional development at the graduate student level needs to adopt a similar focus.

In this section we explore the last column of our Framework, that of “Professional Development to Support Learning.” We provide both some general thoughts as well as descriptions of several professional development activities that could be used with TAs. In addition, we consider how to make good use of graduate students’ non-classroom teaching-related assistantships as sites for learning about teaching. Throughout, implications will be stated for mathematics TAs, though many of the suggestions could easily be adapted for other subjects.

In an ideal world, interactive seminars taking several years would give novice MTAs ample opportunities to conduct numerous interviews of students, plan and carry out classroom activities with groups of students, and reflect on these activities in diverse groups including all levels of graduate students, professors, and mathematics education experts. The financial reality for most institutions is that graduate student TAs provide teaching labor and their services are needed in the classroom beginning in their first or second years. Thus, instead of providing a lengthy pre-service experience based on research on student thinking, circumstances dictate a more scaled-back, in-service approach focused on helping MTAs to learn while teaching.

Luckily, the research described above provides guidance as to how such activities might be structured. Much of what is done in pre-semester or ongoing professional development for TAs could be reshaped to include a focus on their acquisition of knowledge of student thinking. By giving TAs brief opportunities to engage with student thinking and providing targeted, informed support, TAs will be more likely to transform their early teaching experiences into learning experiences.

Predicting Student Thinking

One important function of knowledge of student thinking in mathematics is helping teachers anticipate the concepts that particular tasks will prompt students to think about. Being able to make accurate predictions about typical solution strategies and difficulties for a particular task enables teachers to tailor questions to tap into the particular concepts/skills they wish to assess and enables teachers to anticipate different ideas that students might need to understand in order to tackle the task successfully. These kinds of predictions come more easily for experienced teachers and this activity can help develop beginning teachers’ knowledge in this area and also sensitize them to the extent of such knowledge.
This activity encompasses all three rows (planning, instructing, reflecting) in the “professional development to support learning” column of the Framework from Figure 1. In this activity, MTAs would analyze a problem that they are going to use with their students—an aspect of planning. It could be a problem from a homework assignment, a quiz, or any other venue where the graduate students will have access to the students’ written solutions. It could be a problem given to the graduate students or one they have selected or written on their own. They examine the problem and write out predictions about the solution strategies that students will use—both correct and incorrect ones, the particular techniques/procedures that they will use (appropriately or inappropriately), and the specific difficulties or mistakes that will come up. Then the graduate students use the problem with their students, examine the students’ work, and compare/contrast what students actually did with what they predicted would occur, reflecting on the extent to which their predictions during the planning portion held up. Then graduate students could revise their initial list so it represents what the students actually did. This activity could also be done as a pre-semester activity using samples of existing student work. One possible variation on this activity would be to have graduate students make their predictions and talk with an experienced TA or professor to gather their predictions, then conduct the rest of the activity. This variation could help graduate students recognize that this type of knowledge is something acquired by instructors as they gain teaching experience. Graduate students could also share and discuss their lists so they can see which student strategies and difficulties are common across different topics.

Using Research Articles to Examine Students’ Thinking

While a central message of this article is that graduate students need to learn about student thinking from their teaching experiences, there is a substantial collection of research articles that represent what is known in the field of mathematics education about how students think about particular mathematical concepts. This activity provides graduate students with opportunities to become aware of this literature base and also to see how the findings of this kind of research may provide insight into how their students are learning/understanding particular mathematical ideas.

There are articles that report on research about student thinking in many areas, including function, limit, and derivative (see, e.g., volumes of Research in Collegiate Mathematics for collections of articles and Making the Connection: Research and Practice in Undergraduate Mathematics Education (Carlson & Rasmussen, 2008) for relevant literature reviews). Such articles often present analyses that include categorizing student ideas or difficulties in various ways. After reading an article, graduate students could examine examples of students’ work on problems in the area of choice and reflect on the extent to which the categorization scheme used by the researchers fits the students’ work. This activity (representing the “reflecting” row in the Framework) could be done with student work from the graduate students’ classes or with existing or created work that represents students’ thinking about the ideas.

Interviewing Students About Their Thinking

Findings from the CGI-based research on teacher practice indicate that as teachers learn more about how students think about particular ideas, they are more inclined to both use that information in teaching decisions and to interact with students in ways that provide more access to that thinking. One of the major ways that teachers can both base their teaching on students’ thinking and learn more about that thinking is by asking many questions of their students. Having students explain their ideas, both correct and incorrect, provides teachers with insight into different ways of thinking. Incorporating questions of this sort into one’s practice is something that can be learned. This activity is designed to give graduate students opportunities to work on asking these kinds of questions, to reflect on what they learn about student thinking, and to develop an appreciation for the fact that there is much to know about how students think.

MTAs select or are given a problem from a topic that is coming up in their course. Students are recruited to work on the problem and to discuss their ideas with the MTA. During this time, the graduate student is NOT to coach students towards a particular solution, but is only to ask questions that prompt the students to explain what they were thinking and why. In educational research, this kind of interaction is called a clinical interview (Hunting, 1997). Graduate students could write up what happened during the interview, summarizing the students’ thinking. In a variation of this activity, all graduate students could use the same task in the interviews and then share the ways their students thought about the ideas, comparing and contrasting students’ ways of thinking. By concentrating on asking questions that elicit student thinking in an interview setting, graduate students may be more inclined and prepared to ask such questions in the context of their teaching, either for the particular topic from the interview or more generally. Graduate students who are not currently teaching can also participate in this activity—for them, it might be most useful to use a problem that connects to some major concepts in a course they are likely to teach in the future.

Assignment Decisions

Decisions about which courses are taught by novice versus experienced TAs are made in different ways across institutions. In some departments, TAs are assigned to teach the mathematically least advanced courses first and then move up through the curriculum over time. In other departments, the particular
challenges of teaching lower-level courses are seen as more appropriate for TAs with substantial teaching experience and thus beginning TAs are assigned to teach more mathematically advanced courses. In addition to (or perhaps in place of) these priorities, it could be constructive to consider which teaching assignments provide graduate students with the most useful access to student thinking and then assign novice TAs to those classes. Such teaching assignments might be those where TAs have increased on-one-time with students and have opportunities to reflect in groups on their interactions with other TAs and with facilitators who are trained to help move discussions from normalizing to transformative. Placing TAs into innovative classroom situations can be formative in preparing graduate students to be effective professors (Seymour, 2005).

As noted above in the discussion of possible professional development activities, graduate students with non-classroom assignments can learn from their experiences in a variety of ways. In fact, absent the time (and other) pressures of teaching a class, graduate students may be especially well-positioned to inquire into and learn about student thinking. As a result, there is much that can be done to structure learning opportunities for graduate students who are grading or tutoring. Graduate students doing this kind of work have a great deal of access to individual students’ thinking. Professional development programs should take advantage of such opportunities and help graduate students learn as much as possible while they are engaging with students in all arenas.

Conclusion

Being a professor entails many different experiences and responsibilities. Graduate school, no matter how intense, cannot possibly fully prepare future faculty for the varied demands of academic life. Given the multifaceted goals of graduate education and the finite time spent in graduate school, it would be constructive to focus attention on helping graduate students learn how to learn from their experiences. This approach to enhancing on-the-job learning has shown promise as a means for professional development and instructional improvement in K-12 settings. With attention to the particular features and constraints of the college setting, such approaches are apt to be effective in equipping graduate students with the tools to learn in the context of their experiences as they begin and continue their teaching careers.

Much remains to be examined in the area of pedagogical content knowledge. To inform the design of professional development activities as well as potential research programs, a richer understanding is needed of the developmental trajectory graduate students follow as they acquire knowledge of student thinking. For example, what do TAs enter graduate school knowing? How does a shift of focus from professional development for teaching to professional development for learning shape TAs’ practices? What do MTAs leave graduate school knowing and how might that be enhanced so that their students (now and in the future) can have the best possible learning opportunities? From findings at the K-12 level, there is reason to expect that this approach to graduate student professional development will lead to increased student achievement, but such an outcome needs to be established empirically with further research studies.

References


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