CURRENT AND FUTURE FACULTY MEMBERS’ MATHEMATICAL KNOWLEDGE FOR TEACHING CALCULUS

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Findings from research into “mathematical knowledge for teaching” have informed the design of preparation and professional development programs for K-12 teachers. At the college level there has been limited research into mathematical knowledge for teaching. We lack findings that demonstrate that expert teachers of college mathematics know and make use of knowledge beyond solely mathematical content. The goal of this study is to examine the knowledge of student thinking possessed by mathematicians who teach calculus. Data come from interviews on student thinking about core calculus concepts. Interviewees were research mathematicians who have been recognized for their teaching excellence and mathematics graduate students. Findings demonstrate that the mathematicians were more able to identify known student difficulties as well as to describe common strategies students use to successfully solve the problems. Implications for research and professional development for novice college mathematics instructors are discussed.

Key words: teacher knowledge, knowledge of student thinking, mathematical knowledge for teaching, graduate students

Problem Statement

Concerns about enrollment and retention rates as well as the depth and breadth of calculus students’ understanding sparked much activity over the past several decades (see, e.g., Bressoud, 2004, 2010; Lutzer, Rodi, Kirkman, & Maxwell, 2007; Lutzer & Maxwell, 2002). To meet this country’s needs for scientists and engineers, we must find ways to increase the quality of students’ understanding and the number of students who succeed in calculus.

There is now broader recognition at K-12 levels that in addition to knowledge of content, effective teaching relies on knowledge of (a) how students think and (b) mathematics that is “specialized” for the work of teaching (e.g., making sense of students’ written or spoken work). Researchers have demonstrated that teachers with stronger knowledge of these sorts help students learn more mathematics content. These findings have prompted the K-12 education community to include these kinds of knowledge in professional development for teachers.

There is some evidence that these elements of “mathematical knowledge for teaching” also play roles in the teaching practices of college mathematics instructors, especially those practices needed for inquiry-oriented approaches to instruction (Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). What the community lacks, however, is strong evidence that effective teachers of college mathematics possess this knowledge and use it in their instructional practices. Armed with information about the knowledge used by such instructors, professional development for novice college mathematics instructors (e.g., graduate students) could be designed to focus specifically on the development of such knowledge. This in turn could create better learning opportunities for students and lead to better achievement and retention.

The current study is focused on two research questions: What knowledge of student thinking and specialized content knowledge do experienced teachers of calculus possess? And, how does this compare to knowledge of novice teachers of calculus?
Research on Mathematical Knowledge for Teaching

Theoretical perspective
This project lies at the intersection of research on teachers’ knowledge and on teachers’ practices and was conducted using a cognitive approach. This approach has been used productively to examine teachers’ knowledge and its roles in teaching practices (Borko & Putnam, 1996; Calderhead, 1991, 1996; Escudero & Sanchez, 2007; Schoenfeld, 2000; Sherin, 2002). In such an approach, knowledge is seen as a key factor influencing teachers’ goals and the ways they work to accomplish those goals as they plan for, reflect on, and enact instruction.

Knowledge for teaching
No one questions the essential role that content knowledge plays in teachers’ practices. However, such knowledge in itself is not strongly linked to student achievement (Ball, Lubienski, & Mewborn, 2001; Wilson, Floden, & Ferrini-Mundy, 2002). Research findings suggest that other types of knowledge play substantial roles in teachers’ practices and learning opportunities they create for students.

Pedagogical content knowledge (PCK) refers to (among other content-specific things) knowledge of topics which typically cause students difficulty, the nature of those difficulties, and particularly useful examples for teaching (Shulman, 1986). Teachers’ knowledge of the different strategies their students would use to approach problems is positively correlated with students’ achievements (see, e.g., Fennema et al., 1996). For this project, analyses concentrate on knowledge of students’ ideas (KSI), a subset of PCK used in the research noted above.

Specialized content knowledge (SCK) is a form of knowledge, not necessarily developed in ordinary mathematics courses, that enables teachers to engage in teaching tasks (Ball & Bass, 2000; Hill et al., 2005, 2004). SCK is used to follow students’ thinking, evaluate validity of student-generated strategies, and make sense of student-generated solution paths (Hill, Ball, & Schilling, 2008). Teachers’ SCK has been shown to be positively related to student achievement gains in elementary mathematics (e.g., Hill et al., 2005).

Research Design
Data come from task-based individual interviews with research mathematicians who have been recognized for their excellence in teaching (e.g., nominated for or won a teaching award) and graduate students in mathematics with less than two years of calculus teaching experience.

Tasks were taken from or modeled after tasks used in research on student thinking about limit, function (as it appears in calculus), derivative, and integral. Interviews consisted of three parts per task for each interviewee: Solve the task and describe the solution; Describe how students would solve the task, including difficulties/mistakes they might make and correct/incorrect ways of thinking they might display.

Data analysis was guided by research on student thinking but also relied on methods from Grounded Theory (Strauss & Corbin, 1990). Findings from research on student thinking were used to characterize the extent to which participants were knowledgeable of student thinking. Methods from Grounded Theory were then used to identify themes and to detect differences between findings from mathematicians and novice instructors.

Here we present findings from one task. Borrowed from Carlson (1998), this question taps into students’ abilities to interpret graphical information about two functions (see Figure 1).
The given graph represents speed vs. time for two cars. (Assume the cars start from the same position and are travelling in the same direction.)

Question: State the relationship between the position of car A and car B at t = 1 hour. Provide an explanation for your answer.

**Figure 1. Interview Task related to students’ difficulties with function graphs**

**Findings**

All participants generated correct solutions to the task. All participants were also able to describe at least some correct ways of thinking that students might use. For example, one participant said students might get the solution by thinking “speed is greater always … or if the velocity is greater always … the displacement at the end is going to be greater.” All participants were also able to describe some incorrect ways students might think about the task. These ways included ones documented in research on student thinking using this task. For example, participants noted that students might interpret the graph as if the dependent variable was distance instead of speed and conclude that the two cars have traveled the same distance at t = 1. In describing this kind of thinking, one participant stated, “A lot of students probably would say … the two graphs are intersecting at time equals to 1 so they are equal… and forget about what it is that is equal.”

There were differences in the extent to which the two populations (research mathematicians and graduate students) were able to generate possible student ways of thinking about the task. The mathematicians were generally able to describe more distinct ways than the graduate students. The mathematicians appeared to possess more knowledge of student thinking (PCK) from their experiences working with students. They also appeared more able to hypothesize other possible ways of thinking based on their knowledge of mathematics (e.g., using their SCK to create hypothetical ways one might approach the tasks even if they had not actually seen such an approach). Graduate students, in contrast, had a narrower set of ways of thinking from which to work and were less successful in generating a variety of potential approaches, sometimes focusing just on small variations to one approach they knew. For example, one graduate student said, “they might not, [they] can’t read … the graph they don’t know what does this graph represent [they] just don’t know how to read this graph I think…” He tried, with limited success, to construct possible students’ mistakes based on an inability to read the graph but was not able to describe other kinds of difficulties students might have.

**Conclusions**

The graduate students had some knowledge of students’ difficulties and students’ strategies to solve the above mentioned problem. However, when asked to construct possible students’
responses, graduate students had more difficulty than the mathematicians. All participants had the necessary subject matter knowledge to solve the task, however the mathematicians had a robust knowledge of students’ ideas, their difficulties with certain concepts, and were able to generate new ways students might think by using their content knowledge and specialized content knowledge.

Implications for Research and Practice
By comparing the knowledge of experienced college mathematics instructors to that of novices (and to what is known from research on student thinking for particular topics), we can identify areas where novice instructors might profit from professional development. The experienced instructors said they had learned about student thinking from their interactions with students and from examining students’ written work. Armed with findings about what novices do and do not know, professional development can be designed to help graduate students learn as much as possible from their interactions with students and student work so they can begin their faculty careers equipped with as much knowledge of student thinking as possible. This in turn may enable them to more quickly develop into accomplished teachers.

It also appears that the experienced instructors were well versed in the practices of anticipating or interpreting student ways of thinking by drawing on their knowledge of the mathematics content and their knowledge of student thinking. Further analysis may shed light on how experts do this particular kind of teaching-related work and may contribute to theories of how teacher generate new knowledge for teaching while engaged in the work of teaching.

Discussion Questions
1. In addition to examining samples of student work, are their other activities we could do in the interviews that would reasonably simulate the authentic work of teaching and generate data on mathematical knowledge for teaching?
2. What reasonably compact approaches might be best for presenting both the breadth and depth of an individual participant’s knowledge? What approaches might be best for describing these things for the two populations?

References


