College Students’ Understanding of Geometric Dilations
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Introduction
According to the NCTM Standards (2000) students in grades 6-12 should be able to understand and represent transformations which include translations, reflections, rotations, and dilations of objects in the plane by using sketches and coordinate systems. Findings indicate that pre-school through high school students have great difficulty describing and constructing transformations. However, they are important in higher mathematics. Little is known about college students’ understanding of geometric transformations. In addition, much of the focus has been on pre-school students’ high school students’ understanding of Euclidean Transformations, which include translations, reflections, rotations, and glide reflections (Hollebrands, 2004; Kider, 1976; Moyer, 1976). Little is known about student understanding of geometric dilations otherwise known as size transformations. These findings here are part of a larger study on college students’ understanding of Euclidean Transformations.

What is a dilation?
Under a dilation, an object changes its size centered about a stationary point, other wise known as a focal point. A dilation is a size transformation from the plane to the plane with center O and scale factor r. This transformation assigns each point P in the plane the point A, other wise known as the image of A, such that O, A, and A’ are collinear and OA’ = r × OA, where O is not between A and A’ (Billstein, 2010). If the scale factor r > 1 then the imaged object is greater than the pre-image. If 0 < r < 1 then the imaged object is smaller than the pre-image. If r = 1 then the imaged object remains unchanged. Participants were asked to perform a dilation using a scale factor of r = ½.

Research Questions:
• How do college students construct dilations?
• How successful are college students at constructing dilations?
• What do students understand about the construction of dilations?
• Does directing students using informal mathematical language change students’ construction of dilations?

Methods
Participants in this study:
• Included 11 students from a geometry course for pre-service elementary school teachers, 2 Calculus I students and 7 Calculus III students.
• Complet ed a written test during a clinical interview (Hunting, 1997), which was audio and video recorded. Interviews ranged in time from 20 to 60 minutes.
• Were given protractors and rulers to utilize.
• Were first asked to dilate a triangle with directions given in a formal mathematical language similar to that used in the textbook. Participants were then asked to dilate the same triangle, using the same focal point with directions given in informal mathematical language.

Formal Mathematical Language: “Dilate triangle ABC with center O and scale factor ½.” These directions were given to participants when they were first asked to complete the task. Such directions mirror those in geometry textbooks at the high school and college level.

Informal Mathematical Language: “Dilate triangle ABC with the center of projection at O and scale factor ½.” If a participant performed the dilation incorrectly, they were re instructed using the prompt above. This was done to evaluate how a participant understood the question without using formal mathematical language that may influence participant misunderstanding of what the question was asking.

Data Analysis: Categories and Codes

<table>
<thead>
<tr>
<th>Incorrect</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Center</td>
<td>Participant dilated the triangle using an incorrect r and placed the imaged triangle over the center of dilation because the directions said center O.</td>
</tr>
<tr>
<td>Center</td>
<td>Participant drew the corresponding legs of the dilated triangle scaled with a factor of ½. The dilated triangle was placed on top of the center of dilation such that A’ ≠ O because “it is our starting point.”</td>
</tr>
<tr>
<td>Vertices</td>
<td>Participant drew the corresponding legs of the dilated triangle scaled with a factor of ½. The dilated triangle was placed on top of the center of dilation such that A’ ≠ O because “it is our starting point.”</td>
</tr>
<tr>
<td>Other</td>
<td>Participant drew the corresponding legs of the dilated triangle scaled by ½. The dilated triangle was “upside down from 0 if an angle” however did not involve the original triangle.</td>
</tr>
</tbody>
</table>

Results
Below is a graph depicting participants performance while constructing a dilation of scale factor ½ with directions given in formal mathematical language.

Conclusions
• As with students in the primary school grade levels, very few college students can construct a dilation using pencil, paper, rulers and protractors.
• Results indicate directions given in a formal mathematical language, such as those seen in a geometry textbook referencing dilations, can hinder the understanding and ultimately performance of college students.
• Findings show that students incorrectly utilize the center of dilation as part of the dilated triangle instead of using the center of dilation as a stationary reference point which projects the dilated triangle.
• Informal mathematical directions asking students to construct a dilation changes student performance.
• It is important to note that both Calculus I and Calculus III participants incorrectly perform dilations using the same incorrect methods as Pre Service Teachers (data not shown here).

Plans for Further Research
• Analyze college students performance on other transformations, which include translations, reflections and rotations to see if there is a common trend among college level students’ performance.
• Investigate possible correlations within individual participant performance between different transformations.
• Analyzing why students perform dilations differently when directed with an informal mathematical language.
• Further analysis will be conducted in order to verify no other categories and codes exist.
• Interview more college Calculus students to verify claims.
• Comparing and contrasting Pre-Service Teacher performance, Calculus I student performance and Calculus III student performance.

Literature cited

Dilate triangle picture from http://libguides.mpsaz.net/